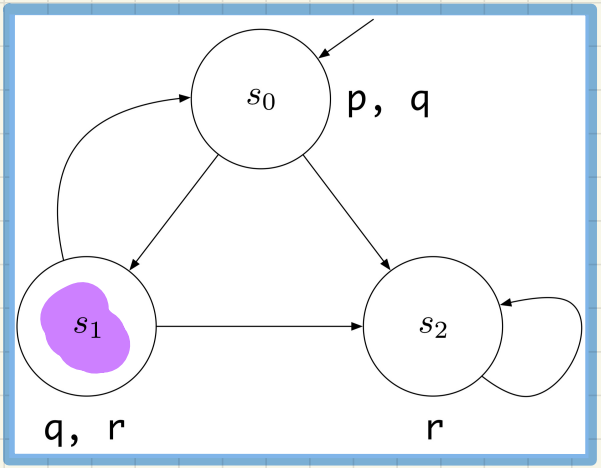
Model vs. Path Satisfaction: Exercises (1.1)



Recall: $\pi \models p \Leftrightarrow p \in L(s_1)$

Say: $\pi = s_0 \to s_1 \to s_2 \to s_2 \to \ldots$

$\pi^2 \pi \models \top$ True

$\pi^2 \pi \not\models \bot$ True

$\pi^2 \pi \models p \wedge q$ False

$\pi^2 \pi \models p \vee q$ True

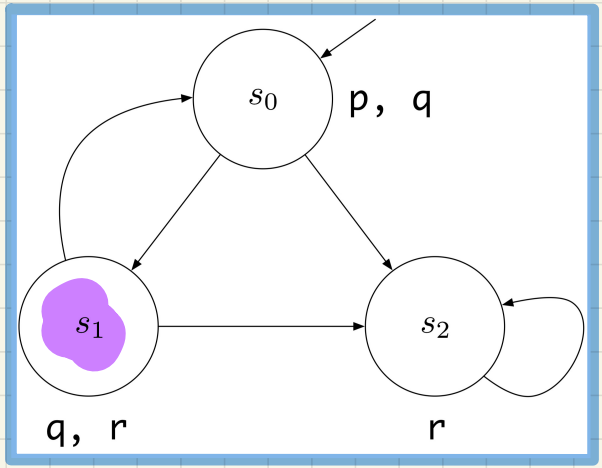
$\pi^2 \pi \models p \Rightarrow q$ True

$\pi^2 \pi \models r$ True

$\pi^2 \pi \models r \Rightarrow p \wedge q \wedge r$ False

Exercise: What if we change the LHS to π^2?

Model vs. Path Satisfaction: Exercises (1.2)



$s \models p \Leftrightarrow$ all π starting at s, $\pi \models p$

s_1 ~~s_0~~ $\models \top$ True

s_1 ~~s_0~~ $\not\models \bot$ True

s_1 ~~s_0~~ $\models p \wedge q$ False

s_1 ~~s_0~~ $\models p \vee q$ True

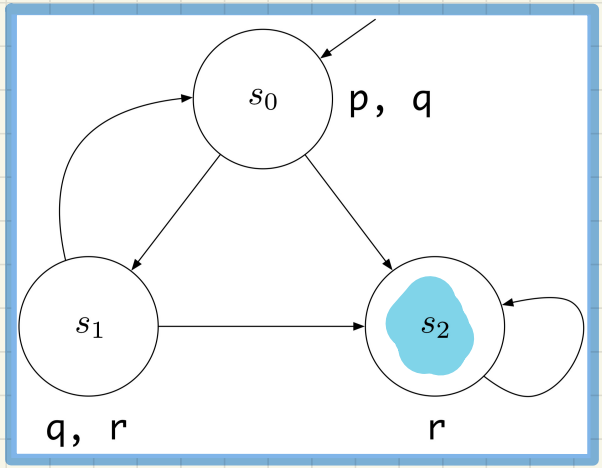
s_1 ~~s_0~~ $\models p \Rightarrow q$ True

s_1 ~~s_0~~ $\models r$ True

s_1 ~~s_0~~ $\models r \Rightarrow p \wedge q \wedge r$ False

Exercise: What if we change the LHS to s_1?

Model vs. Path Satisfaction: Exercises (2.1)



$(\pi^2)^2 = \pi^3$

Recall: $\pi \models \mathbf{X}\,\phi \Leftrightarrow \pi^2 \models \phi$

Say: $\pi = s_0 \to s_1 \to s_2 \to s_2 \to \ldots$

π^2 ; $(\pi^2)^2 = \pi^3$

$\pi^2\pi \models \mathbf{X}\,\top$ True

$\pi^2\pi \not\models \mathbf{X}\,\bot$ True

$\pi^2\pi \models \mathbf{X}\,(q \wedge r)$ False

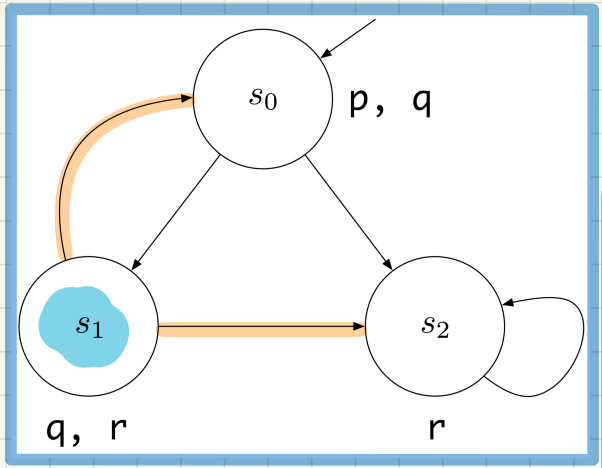
$\pi^2\pi \models \mathbf{X}\,q \wedge r$ — $\mathbf{X}\,q$: fake; r: fake on s_1; Fake

$\pi^2\pi \models \mathbf{X}\,(q \Rightarrow r)$ True

$\pi^2\pi \models \mathbf{X}\,q \Rightarrow r$ True — $\mathbf{X}\,q$: fake

Exercise: What if we change the LHS to π^2?

Model vs. Path Satisfaction: Exercises (2.2)



$s \models \phi \Leftrightarrow$ all π starting at s, $\pi \models \phi$

s_1 ~~s_0~~ $\models \mathbf{X}\ \top$ True

s_1 ~~s_0~~ $\not\models \mathbf{X}\ \bot$ True

s_1 ~~s_0~~ $\models \mathbf{X}\ (q \wedge r)$ False witness: $s_1 \to s_2 \to \cdots$ ($\neg q \wedge r$)

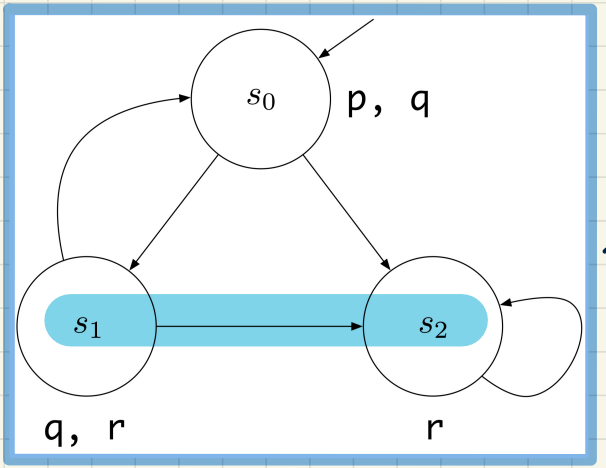
s_1 ~~s_0~~ $\models \mathbf{X}\ q \wedge r$ False witness: $s_1 \to s_2 \to \cdots$ (fake; r true on s_1)

s_1 ~~s_0~~ $\models \mathbf{X}\ (q \Rightarrow r)$ False witness: $s_1 \to s_0 \to \cdots$ ($\neg q$; $q \wedge \neg r$)

s_1 ~~s_0~~ $\models \mathbf{X}\ q \Rightarrow r$ True (fake)

Exercise: What if we change the LHS to s_1?

Model vs. Path Satisfaction: Exercises (3.1)



$\pi \models G\ \phi \Leftrightarrow \forall\ i \bullet i \geq 1 \Rightarrow \pi^i \models \phi$

Say: $\pi = s_0 \rightarrow s_1 \rightarrow s_2 \rightarrow s_2 \rightarrow \ldots$

$\pi^2 \pi \models G\ \top$ True

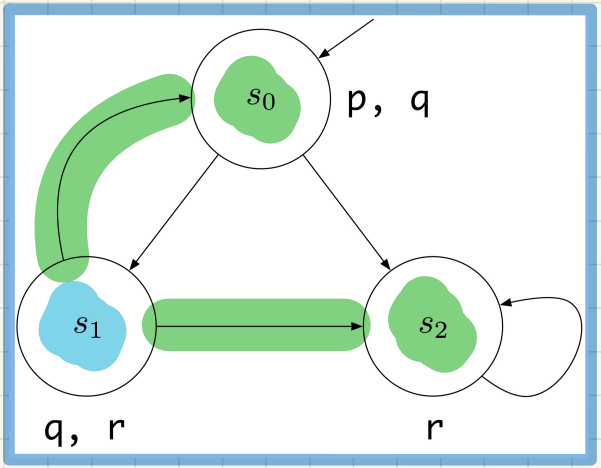
$\pi^2 \pi \not\models G\ \bot$ True

$\pi^2 \pi \models G\ \neg(p \wedge r)$ True ($\neg p$ sat. in s_1, s_2)

$\pi^2 \pi \models G\ r$ True

Exercise: What if we change the LHS to π^2?

Model vs. Path Satisfaction: Exercises (3.2)



$s \models \phi \Leftrightarrow$ all π starting at s, $\pi \models \phi$

s_1 ~~s_0~~ $\models \mathbf{G}\ \top$ True

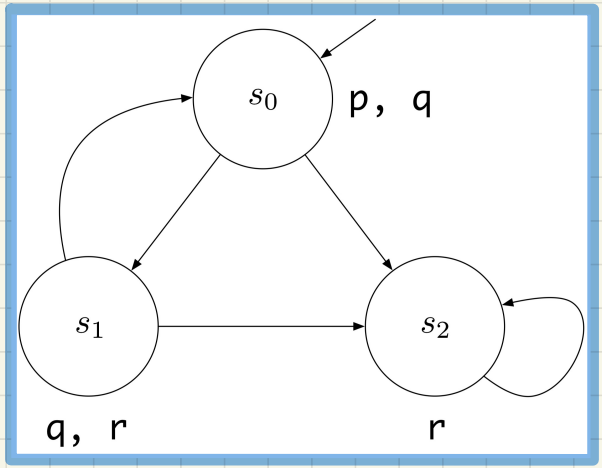
s_1 ~~s_0~~ $\not\models \mathbf{G}\ \bot$ True

s_1 ~~s_0~~ $\models \mathbf{G}\ \neg(p \wedge r)$ True (all states sat. $\neg p \vee \neg r$)

s_1 ~~s_0~~ $\models \mathbf{G}\ r$ False witness: $s_1 \to s_0 \to \cdots$ ($\neg r$)

Exercise: What if we change the LHS to s_1?

Model vs. Path Satisfaction: Exercises (4.1)



$$\pi \models \mathbf{F}\ \phi \Leftrightarrow \exists\, i \bullet i \geq 1 \wedge \pi^i \models \phi$$

Say: $\pi = s_0 \rightarrow s_1 \rightarrow s_2 \rightarrow s_2 \rightarrow \ldots$

π^2

$\pi^2\pi \models \mathbf{F}\ \top$ True

$\pi^2\pi \not\models \mathbf{F}\ \bot$ True

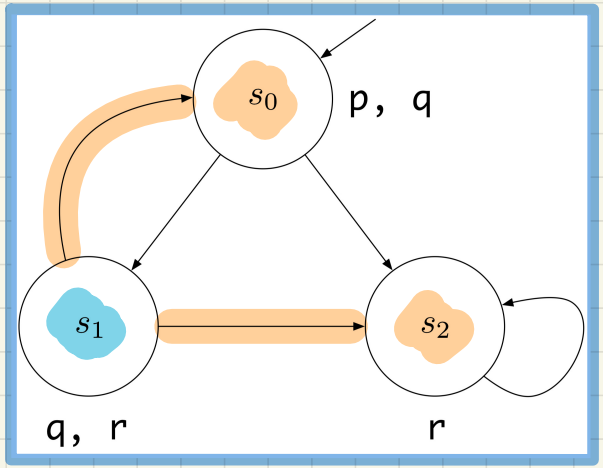
$\pi^2\pi \models \mathbf{F}\ \neg(p \wedge r)$ True witness: s_1

$\pi^2\pi \models \mathbf{F}\ r$ True witness: s_1

$\pi^2\pi \models \mathbf{F}\ (q \wedge r)$ True witness s_1

Exercise: What if we change the LHS to π^2?

Model vs. Path Satisfaction: Exercises (4.2)



$s \models \phi \Leftrightarrow$ all π starting at s, $\pi \models \phi$

s_1 ~~s_0~~ $\models$ F $\top$ True

s_1 ~~s_0~~ $\not\models$ F $\bot$ True

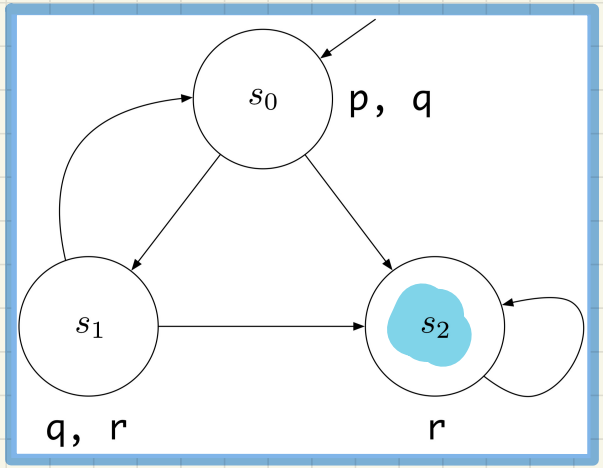
s_1 ~~s_0~~ $\models$ F $\neg(p \wedge r)$ True — sat. by s_0 and s_2

s_1 ~~s_0~~ $\models$ F r True — sat. by s_1

s_1 ~~s_0~~ $\models$ F $(q \wedge r)$ True — sat. by s_1

Exercise: What if we change the LHS to s_1?

Model Satisfaction: Exercises (5.1)



$s \models \phi \Leftrightarrow$ all π starting at s, $\pi \models \phi$

$s_2, s_0 \models \mathbf{FG}\ r$ True

$s_2 \to s_2 \to \cdots$ sat. $G\,r$

$s_2, s_0 \models \mathbf{FG}\ (p \vee q)$ False

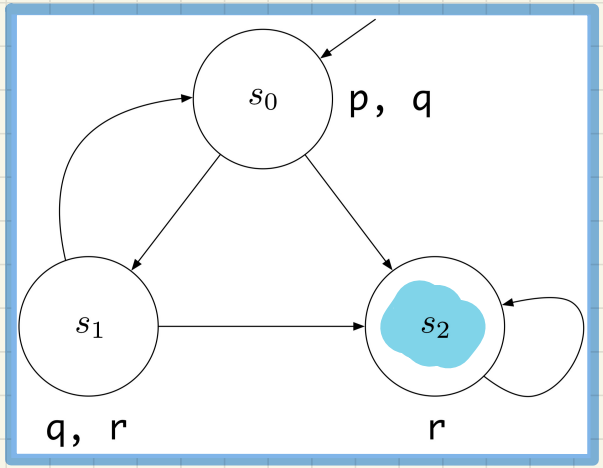
s_2 sat. $\neg p \wedge \neg q$

$s_2, s_0 \models \mathbf{FG}\ (p \vee r)$ True

s_2 sat. $G\,r$

Exercise: What if we change the LHS to s_2?

Model Satisfaction: Exercises (5.2)



$s \models \phi \Leftrightarrow$ all π starting at s, $\pi \models \phi$

s_2 $s_0 \models$ F $(\neg q \wedge r) \Rightarrow$ FG r

sat. by s_2 sat. by s_2

True

$T \Rightarrow T \equiv T$

Only one path to consider:

$s_2 \to s_2 \to \cdots$

s_2 $s_0 \models$ F $(\neg q \vee r) \Rightarrow$ FG r

sat. by s_2 sat. by s_2

True

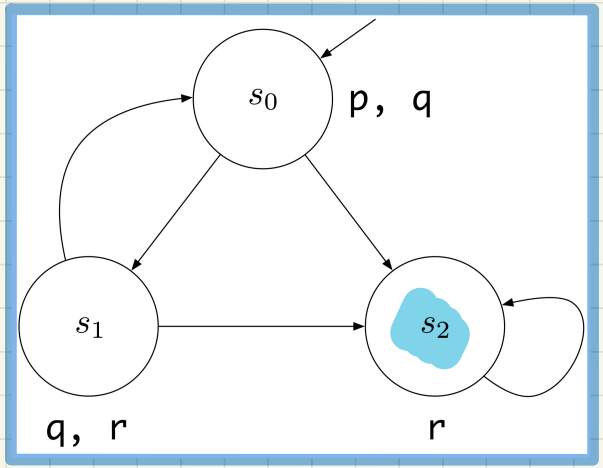
$T \Rightarrow T \equiv T$

Only one path to consider:

$s_2 \to s_2 \to \cdots$

Exercise: What if we change the LHS to s_2?

Model Satisfaction: Exercises (6.1)



$s \models \phi \Leftrightarrow$ all π starting at s, $\pi \models \phi$

s_2 ~~s_0~~ $\models$ GF p False

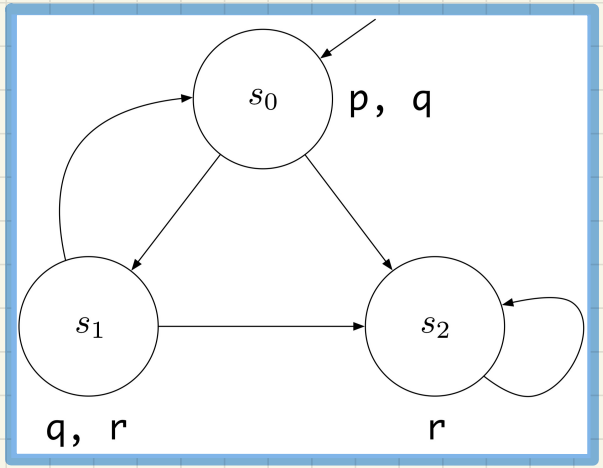
Witness: $s_0 \to s_0 \to \cdots$ (¬p ¬p)

s_2 ~~s_0~~ $\models$ GF (p $\vee$ q) False

Witness: $s_0 \to s_0 \to \cdots$ (¬p∧¬q ¬p∧¬q)

Exercise: What if we change the LHS to s_2?

Model Satisfaction: Exercises (6.2)



$s \models \phi \Leftrightarrow$ all π starting at s, $\pi \models \phi$

s_2 ~~s_0~~ $\models$ GF p $\Rightarrow$ GF r True

false false $\Rightarrow$ _ $\equiv$ True

Only one path to consider: $s_2 \to s_2 \to \cdots$

$\neg(GFp)$

s_2 ~~s_0~~ $\models$ GF r $\Rightarrow$ GF p False

True false True $\Rightarrow$ False $\equiv$ False

Witness: $s_2 \to s_2 \to s_2 \to \cdots$

Exercise: What if we change the LHS to s_2?