Parser: Syntactic Analysis

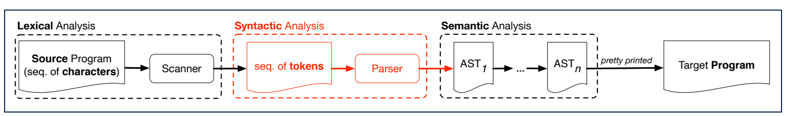
Readings: EAC2 Chapter 3

EECS4302 A:
Compilers and Interpreters
Summer 2025

CHEN-WEI WANG

Parser in Context

- Recall:



- Treats the input programas as a ***a sequence of classified tokens/words***
- Applies rules ***parsing*** token sequences as ***abstract syntax trees (ASTs)*** [***syntactic*** analysis]
- Upon termination:
  - Reports token sequences not derivable as ASTs
  - Produces an ***AST***
- No longer considers ***every character*** in input program.
- *Derivable* token sequences constitute a *context-free language (CFL)*.

Context-Free Languages: Introduction

- We have seen ***regular languages***:
  - Can be described using ***finite automata*** or ***regular expressions***.
  - Satisfy the ***pumping lemma***.
- Language with ***recursive*** structures are provably ***non-regular***. e.g., $\{0^n 1^n \mid n \geq 0\}$
- *Context-Free Grammars (CFG's)* are used to describe strings that can be generated in a ***recursive*** fashion.
- *Context-Free Languages (CFL's)* are:
  - Languages that can be described using CFG's.
  - A proper superset of the set of regular languages.

CFG: Example (1.1)

- The following language that is ***non-regular***

$$\{0^n \# 1^n \mid n \geq 0\}$$

can be described using a ***context-free grammar (CFG)***:

$$\begin{array}{lcl} A & \rightarrow & 0A1 \\ A & \rightarrow & B \\ B & \rightarrow & \# \end{array}$$

- A grammar contains a collection of ***substitution*** or ***production*** rules, where:
  - A **terminal** is a word $w \in \Sigma^*$ (e.g., 0, 1, *etc.*).
  - A ***variable*** or ***non-terminal*** is a word $w \notin \Sigma^*$ (e.g., A, B, *etc.*).
  - A ***start variable*** occurs on the LHS of the topmost rule (e.g., A).

CFG: Example (1.2)

- Given a grammar, generate a string by:
  1. Write down the ***start variable***.
  2. Choose a production rule where the ***start variable*** appears on the LHS of the arrow, and ***substitute*** it by the RHS.
  3. There are two cases of the re-written string:
     - **3.1** It contains **no** variables, then you are done.
     - **3.2** It contains **some** variables, then ***substitute*** each variable using the relevant ***production rules***.
  4. Repeat Step 3.
- e.g., We can generate an infinite number of strings from

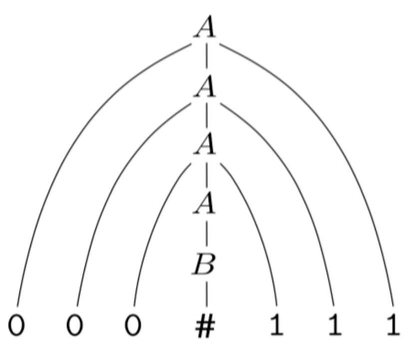
$$
\begin{array}{lcl}
A & \rightarrow & 0A1 \\
A & \rightarrow & B \\
B & \rightarrow & \#
\end{array}
$$

  - $A \Rightarrow B \Rightarrow \#$
  - $A \Rightarrow 0A1 \Rightarrow 0B1 \Rightarrow 0\#1$
  - $A \Rightarrow 0A1 \Rightarrow 00A11 \Rightarrow 00B11 \Rightarrow 00\#11$
  - ...

CFG: Example (1.2)

Given a CFG, a string's ***derivation*** can be shown as a ***parse tree***.

e.g., The derivation of $000\#111$ has the parse tree



CFG: Example (2)

Design a CFG for the following language:

$$\{w \mid w \in \{0,1\}^* \wedge w \texttt{ is a palidrome}\}$$

e.g., 00, 11, 0110, 1001, *etc.*

$$
\begin{array}{lcl}
P & \rightarrow & \epsilon \\
P & \rightarrow & 0 \\
P & \rightarrow & 1 \\
P & \rightarrow & 0P0 \\
P & \rightarrow & 1P1
\end{array}
$$

CFG: Example (3)

Design a CFG for the following language:

$$\{ww^R \mid w \in \{0,1\}^*\}$$

e.g., 00, 11, 0110, *etc.*

$$
\begin{array}{lcl}
P & \rightarrow & \epsilon \\
P & \rightarrow & 0P0 \\
P & \rightarrow & 1P1
\end{array}
$$

CFG: Example (4)

Design a CFG for the set of binary strings, where each block of 0's followed by at least as many 1's.

e.g., 000111, 0001111, *etc.*

- We use S to represent one such string, and A to represent each such block in S.

$$
\begin{array}{lll}
S & \rightarrow & \epsilon \quad \{\texttt{BC of } S\} \\
S & \rightarrow & AS \quad \{\texttt{RC of } S\} \\
A & \rightarrow & \epsilon \quad \{\texttt{BC of } A\} \\
A & \rightarrow & 01 \quad \{\texttt{BC of } A\} \\
A & \rightarrow & 0A1 \quad \{\texttt{RC of } A\texttt{: equal 0's and 1's}\} \\
A & \rightarrow & A1 \quad \{\texttt{RC of } A\texttt{: more 1's}\}
\end{array}
$$

CFG: Example (5.1) Version 1

Design the grammar for the following small expression language, which supports:

- Arithmetic operations: +, −, *, /
- Relational operations: >, <, >=, <=, ==, /=
- Logical operations: `true`, `false`, `!`, `&&`, `||`, `=>`

  Start with the variable ***Expression***.
- There are two possible versions:
  1. All operations are mixed together.
  2. Relevant operations are grouped together.

     Try both!

CFG: Example (5.2) Version 1

$$
\begin{array}{lll}
\textit{Expression} & \rightarrow & \textit{IntegerConstant} \\
 & | & -\textit{IntegerConstant} \\
 & | & \textit{BooleanConstant} \\
 & | & \textit{BinaryOp} \\
 & | & \textit{UnaryOp} \\
 & | & (\ \textit{Expression}\ ) \\
\textit{IntegerConstant} & \rightarrow & \textit{Digit} \\
 & | & \textit{Digit IntegerConstant} \\
\textit{Digit} & \rightarrow & 0\ |\ 1\ |\ 2\ |\ 3\ |\ 4\ |\ 5\ |\ 6\ |\ 7\ |\ 8\ |\ 9 \\
\textit{BooleanConstant} & \rightarrow & \texttt{TRUE} \\
 & | & \texttt{FALSE}
\end{array}
$$

CFG: Example (5.3) Version 1

$$
\begin{array}{lll}
\textit{BinaryOp} & \rightarrow & \textit{Expression} + \textit{Expression} \\
 & | & \textit{Expression} - \textit{Expression} \\
 & | & \textit{Expression} * \textit{Expression} \\
 & | & \textit{Expression} / \textit{Expression} \\
 & | & \textit{Expression}\ \&\&\ \textit{Expression} \\
 & | & \textit{Expression}\ ||\ \textit{Expression} \\
 & | & \textit{Expression} => \textit{Expression} \\
 & | & \textit{Expression} == \textit{Expression} \\
 & | & \textit{Expression}\ /=\ \textit{Expression} \\
 & | & \textit{Expression} > \textit{Expression} \\
 & | & \textit{Expression} < \textit{Expression} \\
\textit{UnaryOp} & \rightarrow & !\ \textit{Expression}
\end{array}
$$

CFG: Example (5.4) Version 1

However, Version 1 of CFG:

- ***Parses*** string that requires further ***semantic analysis*** (e.g., type checking):
  e.g., `3 => 6`
- Is ***ambiguous***, meaning?
  - Some string may have more than one ways to interpreting it.
  - An interpretation is either visualized as a ***parse tree***, or written as a sequence of ***derivations***.

  e.g., Draw the parse tree(s) for `3 * 5 + 4`

CFG: Example (5.5) Version 2

Expression	→	*ArithmeticOp*
	|	*RelationalOp*
	|	*LogicalOp*
	|	`(` *Expression* `)`
IntegerConstant	→	*Digit*
	|	*Digit IntegerConstant*
Digit	→	0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9
BooleanConstant	→	`TRUE`
	|	`FALSE`

CFG: Example (5.6) Version 2

ArithmeticOp	→	*ArithmeticOp* `+` *ArithmeticOp*
	|	*ArithmeticOp* `-` *ArithmeticOp*
	|	*ArithmeticOp* `*` *ArithmeticOp*
	|	*ArithmeticOp* `/` *ArithmeticOp*
	|	`(`*ArithmeticOp*`)`
	|	*IntegerConstant*
	|	`-`*IntegerConstant*
RelationalOp	→	*ArithmeticOp* `==` *ArithmeticOp*
	|	*ArithmeticOp* `/=` *ArithmeticOp*
	|	*ArithmeticOp* `>` *ArithmeticOp*
	|	*ArithmeticOp* `<` *ArithmeticOp*
LogicalOp	→	*LogicalOp* `&&` *LogicalOp*
	|	*LogicalOp* `||` *LogicalOp*
	|	*LogicalOp* `=>` *LogicalOp*
	|	`!` *LogicalOp*
	|	`(`*LogicalOp*`)`
	|	*RelationalOp*
	|	*BooleanConstant*

CFG: Example (5.7) Version 2

However, Version 2 of CFG:

- Eliminates some cases for further semantic analysis:
  e.g., `(1 + 2) => (5 / 4)` [no parse tree]
- Still ***parses*** strings that might require further ***semantic analysis***:
  e.g., `(1 + 2) / (5 - (2 + 3))`
- Still is ***ambiguous***.
  e.g., Draw the parse tree(s) for `3 * 5 + 4`

CFG: Formal Definition (1)

- A ***context-free grammar (CFG)*** is a 4-tuple (V, Σ, R, S):
  - V is a finite set of **variables**.
  - Σ is a finite set of ***terminals***. $[V \cap \Sigma = \varnothing]$
  - R is a finite set of ***rules*** s.t.

$$R \subseteq \{v \rightarrow s \mid v \in V \wedge s \in (V \cup \Sigma)^*\}$$

  - $S \in V$ is is the **start variable**.
- Given strings $u, v, w \in (V \cup \Sigma)^*$, variable $A \in V$, a rule $A \rightarrow w$:
  - $\boxed{uAv \Rightarrow uwv}$ menas that uAv ***yields*** uwv.
  - $\boxed{u \overset{*}{\Rightarrow} v}$ means that u ***derives*** v, if:
    - $u = v$; or
    - $u \Rightarrow u_1 \Rightarrow u_2 \Rightarrow \cdots \Rightarrow u_k \Rightarrow v$ [a ***yield sequence***]
- Given a CFG $G = (V, \Sigma, R, S)$, the language of G

$$L(G) = \{w \in \Sigma^* \mid S \overset{*}{\Rightarrow} w\}$$

CFG: Formal Definition (2): Example

- Design the ***CFG*** for strings of properly-nested parentheses.

  e.g., ( ), ( ) ( ), ((( ) ( ))) ( ), *etc.*

  Present your answer in a ***formal*** manner.
- $G = (\{S\}, \{(,)\}, R, S)$, where R is

$$S \rightarrow (\ S\ ) \mid SS \mid \epsilon$$

- Draw ***parse trees*** for the above three strings that G generates.

CFG: Formal Definition (3): Example

- Consider the grammar $G = (V, \Sigma, R, S)$:
  - R is

$$\begin{array}{lcl} Expr & \rightarrow & Expr + Term \\ & | & Term \\ Term & \rightarrow & Term * Factor \\ & | & Factor \\ Factor & \rightarrow & (Expr) \\ & | & \mathtt{a} \end{array}$$

  - $V = \{Expr, Term, Factor\}$
  - $\Sigma = \{\mathtt{a}, +, *, (,)\}$
  - $S = Expr$
- ***Precedence*** of operators $+$, $*$ is embedded in the grammar.
  - "Plus" is specified at a **higher** level (*Expr*) than is "times" (*Term*).
  - Both operands of a multiplication (*Factor*) may be **parenthesized**.

Regular Expressions to CFG's

- Recall the semantics of regular expressions (assuming that we do not consider $\varnothing$):

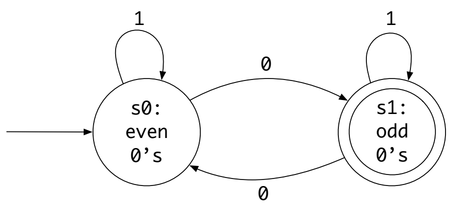
$$\begin{array}{lcl} L(\ \epsilon\ ) & = & \{\epsilon\} \\ L(\ a\ ) & = & \{a\} \\ L(\ E + F\ ) & = & L(E) \cup L(F) \\ L(\ EF\ ) & = & L(E)L(F) \\ L(\ E^*\ ) & = & (L(E))^* \\ L(\ (E)\ ) & = & L(E) \end{array}$$

- e.g., Grammar for $(00 + 1)^* + (11 + 0)^*$

$$\begin{array}{lcl} S & \rightarrow & A \mid B \\ A & \rightarrow & \epsilon \mid AC \\ C & \rightarrow & 00 \mid 1 \\ B & \rightarrow & \epsilon \mid BD \\ D & \rightarrow & 11 \mid 0 \end{array}$$

DFA to CFG's

- Given a DFA $M = (Q, \Sigma, \delta, q_0, F)$:
  - Make a ***variable*** R_i for each ***state*** $q_i \in Q$.
  - Make R_0 the ***start variable***, where q_0 is the ***start state*** of M.
  - Add a rule $R_i \rightarrow aR_j$ to the grammar if $\delta(q_i, a) = q_j$.
  - Add a rule $R_i \rightarrow \epsilon$ if $q_i \in F$.
- e.g., Grammar for



$$
\begin{aligned}
R_0 &\rightarrow 1R_0 \mid 0R_1 \\
R_1 &\rightarrow 0R_0 \mid 1R_1 \mid \epsilon
\end{aligned}
$$

CFG: Leftmost Derivations (1)

$$
\begin{aligned}
Expr &\rightarrow Expr + Term \mid Term \\
Term &\rightarrow Term * Factor \mid Factor \\
Factor &\rightarrow (Expr) \mid a
\end{aligned}
$$

- Given a string ($\in (V \cup \Sigma)^*$), a ***left-most derivation (LMD)*** keeps substituting the leftmost non-terminal ($\in V$).
- ***Unique LMD*** for the string `a + a * a`:

$$
\begin{aligned}
Expr &\Rightarrow Expr + Term \\
&\Rightarrow Term + Term \\
&\Rightarrow Factor + Term \\
&\Rightarrow a + Term \\
&\Rightarrow a + Term * Factor \\
&\Rightarrow a + Factor * Factor \\
&\Rightarrow a + a * Factor \\
&\Rightarrow a + a * a
\end{aligned}
$$

- This ***LMD*** suggests that `a * a` is the right operand of `+`.

CFG: Rightmost Derivations (1)

$$
\begin{aligned}
Expr &\rightarrow Expr + Term \mid Term \\
Term &\rightarrow Term * Factor \mid Factor \\
Factor &\rightarrow (Expr) \mid a
\end{aligned}
$$

- Given a string ($\in (V \cup \Sigma)^*$), a ***right-most derivation (RMD)*** keeps substituting the rightmost non-terminal ($\in V$).
- ***Unique RMD*** for the string `a + a * a`:

$$
\begin{aligned}
Expr &\Rightarrow Expr + Term \\
&\Rightarrow Expr + Term * Factor \\
&\Rightarrow Expr + Term * a \\
&\Rightarrow Expr + Factor * a \\
&\Rightarrow Expr + a * a \\
&\Rightarrow Term + a * a \\
&\Rightarrow Factor + a * a \\
&\Rightarrow a + a * a
\end{aligned}
$$

- This ***RMD*** suggests that `a * a` is the right operand of `+`.

CFG: Leftmost Derivations (2)

$$
\begin{aligned}
Expr &\rightarrow Expr + Term \mid Term \\
Term &\rightarrow Term * Factor \mid Factor \\
Factor &\rightarrow (Expr) \mid a
\end{aligned}
$$

- ***Unique LMD*** for the string `(a + a) * a`:

$$
\begin{aligned}
Expr &\Rightarrow Term \\
&\Rightarrow Term * Factor \\
&\Rightarrow Factor * Factor \\
&\Rightarrow (Expr) * Factor \\
&\Rightarrow (Expr + Term) * Factor \\
&\Rightarrow (Term + Term) * Factor \\
&\Rightarrow (Factor + Term) * Factor \\
&\Rightarrow (a + Term) * Factor \\
&\Rightarrow (a + Factor) * Factor \\
&\Rightarrow (a + a) * Factor \\
&\Rightarrow (a + a) * a
\end{aligned}
$$

- This ***LMD*** suggests that `(a + a)` is the left operand of `*`.

CFG: Rightmost Derivations (2)

$$\begin{array}{lcl} Expr & \rightarrow & Expr\ +\ Term \mid Term \\ Term & \rightarrow & Term\ *\ Factor \mid Factor \\ Factor & \rightarrow & (\ Expr\ ) \mid a \end{array}$$

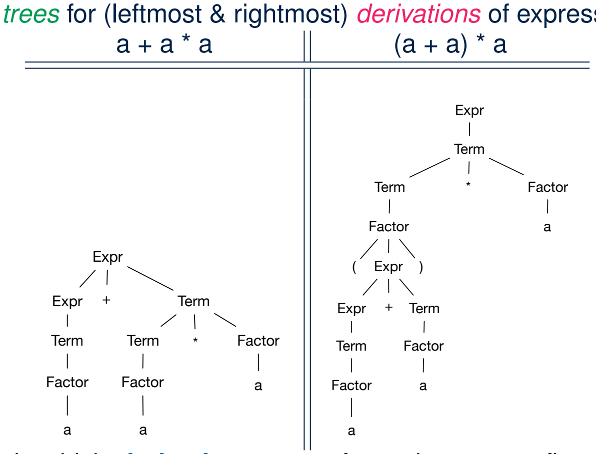
- ***Unique RMD*** for the string `(a + a) * a`:

$$\begin{array}{lcl} Expr & \Rightarrow & Term \\ & \Rightarrow & Term\ *\ Factor \\ & \Rightarrow & Term\ *\ a \\ & \Rightarrow & Factor\ *\ a \\ & \Rightarrow & (\ Expr\ )\ *\ a \\ & \Rightarrow & (\ Expr\ +\ Term\ )\ *\ a \\ & \Rightarrow & (\ Expr\ +\ Factor\ )\ *\ a \\ & \Rightarrow & (\ Expr\ +\ a\ )\ *\ a \\ & \Rightarrow & (\ Term\ +\ a\ )\ *\ a \\ & \Rightarrow & (\ Factor\ +\ a\ )\ *\ a \\ & \Rightarrow & (\ a\ +\ a\ )\ *\ a \end{array}$$

- This ***RMD*** suggests that `(a + a)` is the left operand of $*$.

CFG: Parse Trees vs. Derivations (1)

- *Parse trees* for (leftmost & rightmost) *derivations* of expressions:



- Orders in which ***derivations*** are performed are ***not*** reflected on parse trees.

CFG: Parse Trees vs. Derivations (2)

- A string $w \in \Sigma^*$ may have <u>more than one</u> ***derivations***.

  Q: distinct ***derivations*** for $w \in \Sigma^* \Rightarrow$ distinct ***parse trees*** for w?

  A: Not in general $\because$ Derivations with ***distinct orders*** of variable substitutions may still result in the ***same parse tree***.
- For example:

$$\begin{array}{lcl} Expr & \rightarrow & Expr\ +\ Term \mid Term \\ Term & \rightarrow & Term\ *\ Factor \mid Factor \\ Factor & \rightarrow & (\ Expr\ ) \mid a \end{array}$$

For string `a + a * a`, the ***LMD*** and ***RMD*** have ***distinct orders*** of variable substitutions, but their corresponding ***parse trees are the <u>same</u>***.

CFG: Ambiguity: Definition

Given a grammar $G = (V, \Sigma, R, S)$:

- A string $w \in \Sigma^*$ is derived *ambiguously* in G if there exist two or more ***distinct parse trees*** or, equally, two or more ***distinct LMDs*** or, equally, two or more ***distinct RMDs***.

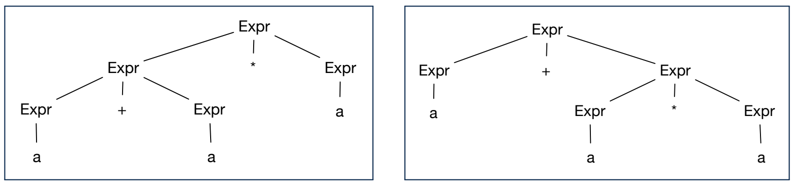
  We require that all such derivations are completed by following a <u>consisten</u> order (**leftmost** or **rightmost**) to avoid ***false positive***.
- G is *ambiguous* if it generates some string ambiguously.

CFG: Ambiguity: Exercise (1)

- Is the following grammar **ambiguous**?

$$Expr \rightarrow Expr + Expr \mid Expr * Expr \mid (Expr) \mid a$$

- Yes ∵ it generates the string `a + a * a` **ambiguously**:



- ***Distinct ASTs*** (for the ***same input***) imply ***distinct semantic interpretations***: e.g., a pre-order traversal for evaluation
- **Exercise**: Show ***LMDs*** for the two parse trees.

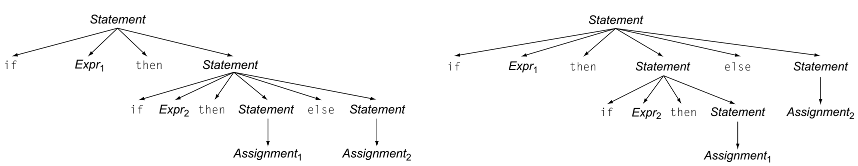
CFG: Ambiguity: Exercise (2.1)

- Is the following grammar **ambiguous**?

$$
\begin{array}{lcl}
Statement & \rightarrow & \texttt{if}\ Expr\ \texttt{then}\ Statement \\
& \mid & \texttt{if}\ Expr\ \texttt{then}\ Statement\ \texttt{else}\ Statement \\
& \mid & Assignment \\
& \ldots &
\end{array}
$$

- Yes ∵ it derives the following string **ambiguously**:

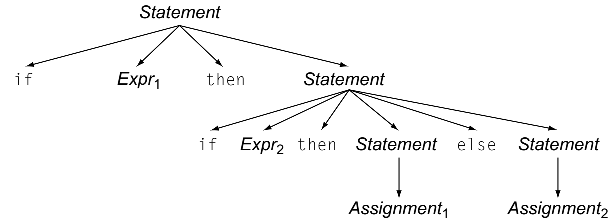
$\texttt{if}\ Expr_1\ \texttt{then}\ \texttt{if}\ Expr_2\ \texttt{then}\ Assignment_1\ \texttt{else}\ Assignment_2$



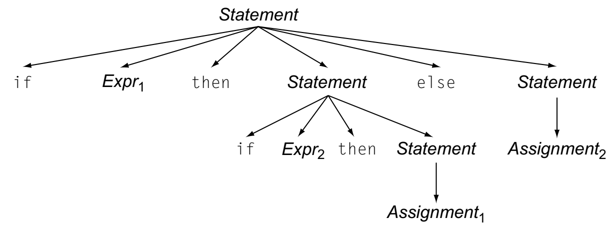
- This is called the ***dangling else*** problem.
- **Exercise:** Show ***LMDs*** for the two parse trees.

CFG: Ambiguity: Exercise (2.2)

(***Meaning 1***) $Assignment_2$ may be associated with the inner `if`:



(***Meaning 2***) $Assignment_2$ may be associated with the outer `if`:



CFG: Ambiguity: Exercise (2.3)

- We may remove the ***ambiguity*** by specifying that the *dangling* `else` is associated with the **nearest** `if`:

$$
\begin{array}{lcl}
Statement & \rightarrow & \texttt{if}\ Expr\ \texttt{then}\ Statement \\
& \mid & \texttt{if}\ Expr\ \texttt{then}\ WithElse\ \texttt{else}\ Statement \\
& \mid & Assignment \\
WithElse & \rightarrow & \texttt{if}\ Expr\ \texttt{then}\ WithElse\ \texttt{else}\ WithElse \\
& \mid & Assignment
\end{array}
$$

- When applying $\texttt{if}\ \ldots\ \texttt{then}\ WithElse\ \texttt{else}\ Statement$:
  - The ***true*** branch will be produced via *WithElse*.
  - The ***false*** branch will be produced via *Statement*.

  There is ***no circularity*** between the two non-terminals.

Discovering Derivations

- Given a CFG $G = (V, \Sigma, R, S)$ and an input program $p \in \Sigma^*$:
  - So far we **manually** come up a valid ***derivation*** s.t. $S \overset{*}{\Rightarrow} p$.
  - A *parser* is supposed to **automate** this ***derivation*** process.
    - Input: **A sequence of (t, c) pairs**, where each ***token*** t (e.g., `r241`) belongs to a ***syntactic category*** c (e.g., `register`); and a ***CFG*** G.
    - Output: A ***valid derivation*** (as an ***AST***); or A ***parse error***.
- In the process of constructing an ***AST*** for the input program:
  - ***Root*** of AST: The ***start symbol*** S of G
  - ***Internal nodes***: A ***subset of variables*** V of G
  - ***Leaves*** of AST: A ***token/terminal*** sequence

    ⇒ Discovering the *grammatical connections* (w.r.t. R of G) between the ***root***, ***internal nodes***, and ***leaves*** is the hard part!
- Approaches to Parsing: $[\, w \in (V \cup \Sigma)^*,\ A \in V,\ \boxed{A \to w} \in R \,]$
  - **Top-down** parsing

    For a node representing ***A***, extend it with a subtree representing ***w***.
  - **Bottom-up** parsing

    For a substring matching ***w***, build a node representing ***A*** accordingly.

TDP: Discovering Leftmost Derivation

```
ALGORITHM: TDParse
  INPUT: CFG G = (V, Σ, R, S)
  OUTPUT: Root of a Parse Tree or Syntax Error
PROCEDURE:
  root := a new node for the start symbol S
  focus := root
  initialize an empty stack trace
  trace.push(null)
  word := NextWord()
  while (true):
    if focus ∈ V then
      if ∃ unvisited rule focus → β1 β2 ... βn ∈ R then
        create β1, β2 ... βn as children of focus
        trace.push(βn βn−1 ... β2)
        focus := β1
      else
        if focus = S then report syntax error
        else backtrack
    elseif word matches focus then
      word := NextWord()
      focus := trace.pop()
    elseif word = EOF ∧ focus = null then return root
    else backtrack
```

backtrack ≜ pop *focus*.siblings; *focus* := *focus*.parent; *focus*.resetChildren

TDP: Exercise (1)

- Given the following CFG **G**:

$$
\begin{array}{lcl}
Expr & \to & Expr\ +\ Term \\
 & | & Term \\
Term & \to & Term\ *\ Factor \\
 & | & Factor \\
Factor & \to & (\ Expr\ ) \\
 & | & \mathtt{a}
\end{array}
$$

  Trace *TDParse* on how to build an AST for input `a + a * a`.
- Running *TDParse* with **G** results an ***infinite loop***!!!
  - *TDParse* focuses on the ***leftmost*** non-terminal.
  - The grammar **G** contains ***left-recursions***.
- We must first convert left-recursions in **G** to ***right-recursions***.

TDP: Exercise (2)

- Given the following CFG **G**:

$$
\begin{array}{lcl}
Expr & \to & Term\ \ Expr' \\
Expr' & \to & +\ Term\ \ Expr' \\
 & | & \epsilon \\
Term & \to & Factor\ \ Term' \\
Term' & \to & *\ Factor\ \ Term' \\
 & | & \epsilon \\
Factor & \to & (\ Expr\ ) \\
 & | & \mathtt{a}
\end{array}
$$

  Exercise. Trace *TDParse* on building AST for `a + a * a`.
  Exercise. Trace *TDParse* on building AST for `(a + a) * a`.
  Q: How to handle ϵ-productions (e.g., $Expr \to \epsilon$)?
  A: Execute *focus* `:=` *trace*`.pop()` to advance to next node.
- Running *TDParse* will ***terminate*** ∵ **G** is ***right-recursive***.
- We will learn about a systematic approach to converting left-recursions in a given grammar to ***right-recursions***.

Left-Recursions (LR): Direct vs. Indirect

Given CFG $G = (V, \Sigma, R, S)$, $\alpha, \beta, \gamma \in (V \cup \Sigma)^*$, G contains:

- A *cycle* if $\exists A \in V \bullet A \overset{*}{\Rightarrow} A$
- A ***direct*** LR if $A \rightarrow A\alpha \in R$ for non-terminal $A \in V$

e.g.,

$$\begin{array}{lll} Expr & \rightarrow & Expr + Term \\ & | & Term \\ Term & \rightarrow & Term * Factor \\ & | & Factor \\ Factor & \rightarrow & (Expr) \\ & | & \texttt{a} \end{array}$$

e.g.,

$$\begin{array}{lll} Expr & \rightarrow & Expr + Term \\ & | & Expr - Term \\ & | & Term \\ Term & \rightarrow & Term * Factor \\ & | & Term \ / \ Factor \\ & | & Factor \end{array}$$

- An ***indirect*** LR if $A \rightarrow B\beta \in R$ for non-terminals $A, B \in V$, $B \overset{*}{\Rightarrow} A\gamma$

$$\begin{array}{lll} A & \rightarrow & Br \\ B & \rightarrow & Cd \\ C & \rightarrow & At \end{array}$$

$A \rightarrow Br, B \overset{*}{\Rightarrow} Atd$

$$\begin{array}{lllll} A & \rightarrow & Ba & | & b \\ B & \rightarrow & Cd & | & e \\ C & \rightarrow & Df & | & g \\ D & \rightarrow & f & | & Aa \mid Cg \end{array}$$

$A \rightarrow Ba, B \overset{*}{\Rightarrow} Aafd$

TDP: (Preventively) Eliminating LRs

```
1  ALGORITHM: RemoveLR
2    INPUT: CFG G = (V, Σ, R, S)
3    ASSUME: G has no ε-productions
4    OUTPUT: G' s.t. G' ≡ G, G' has no
5            indirect & direct left-recursions
6  PROCEDURE:
7    impose an order on V: ⟨⟨A_1, A_2, ..., A_n⟩⟩
8    for i: 1 .. n:
9      for j: 1 .. i-1:
10       if ∃ A_i → A_j γ ∈ R ∧ A_j → δ_1 | δ_2 | ... | δ_m ∈ R then
11         replace A_i → A_j γ with A_i → δ_1 γ | δ_2 γ | ... | δ_m γ
12       end
13     for A_i → A_i α | β ∈ R:
14       replace it with: A_i → β A_i', A_i' → α A_i' | ε
```

- **L9** to **L12**: Remove ***indirect*** left-recursions from A_1 to A_{i-1}.
- **L13** to **L14**: Remove ***direct*** left-recursions from A_1 to A_{i-1}.
- ***Loop Invariant*** **(outer for-loop)**? At the start of i^{th} iteration:
  - No ***direct*** or ***indirect*** left-recursions for $A_1, A_2, \ldots, A_{i-1}$.
  - More precisely: $\forall j : j < i \bullet \neg(\exists k \bullet k \le j \land A_j \rightarrow A_k \cdots \in R)$

CFG: Eliminating ϵ-Productions (1)

- Motivations:
  - ***TDParse*** handles each ϵ-production as a special case.
  - ***RemoveLR*** produces CFG which may contain ϵ-productions.
- $\epsilon \notin L \Rightarrow \exists$ CFG $G = (V, \Sigma, R, S)$ s.t. G has no ϵ-productions.

  An *ϵ-production* has the form $A \rightarrow \epsilon$.
- A variable A is ***nullable*** if $A \overset{*}{\Rightarrow} \epsilon$.
  - Each terminal symbol is ***not nullable***.
  - Variable A is ***nullable*** if either:
    - $A \rightarrow \epsilon \in R$; or
    - $A \rightarrow B_1 B_2 \ldots B_k \in R$, where each variable B_i ($1 \le i \le k$) is a ***nullable***.
- Given a production $B \rightarrow CAD$, if only variable A is ***nullable***, then there are 2 versions of B: $B \rightarrow CAD \mid CD$
- In general, given a production $A \rightarrow X_1 X_2 \ldots X_k$ with k symbols, if m of the k symbols are ***nullable***:
  - $m < k$: There are 2^m versions of A.
  - $m = k$: There are $2^m - 1$ versions of A. [excluding $A \rightarrow \epsilon$]

CFG: Eliminating ϵ-Productions (2)

- Eliminate ϵ-productions from the following grammar:

$$\begin{array}{lll} S & \rightarrow & AB \\ A & \rightarrow & aAA \mid \epsilon \\ B & \rightarrow & bBB \mid \epsilon \end{array}$$

- Which are the ***nullable*** variables? [S, A, B]

$$\begin{array}{llll} S & \rightarrow & A \mid B \mid AB & \{S \rightarrow \epsilon \texttt{ not included}\} \\ A & \rightarrow & aAA \mid aA \mid a & \{A \rightarrow aA \texttt{ duplicated}\} \\ B & \rightarrow & bBB \mid bB \mid b & \{B \rightarrow bB \texttt{ duplicated}\} \end{array}$$

Backtrack-Free Parsing (1)

- `TDParse` automates the ***top-down***, ***leftmost*** derivation process by consistently choosing production rules (e.g., in order of their appearance in CFG).
  - This ***inflexibility*** may lead to ***inefficient*** runtime performance due to the need to *backtrack*.
  - e.g., It may take the ***construction of a giant subtree*** to find out a ***mismatch*** with the input tokens, which end up requiring it to *backtrack* all the way back to the ***root*** (start symbol).
- We may avoid backtracking with a modification to the parser:
  - When deciding which production rule to choose, consider:
    (1) the ***current*** input symbol
    (2) the consequential ***first*** symbol if a rule was applied for `focus`
    [*lookahead* symbol]
  - Using a ***one symbol lookahead***, w.r.t. a ***right-recursive*** CFG, each alternative for the ***leftmost nonterminal*** leads to a ***unique terminal***, allowing the parser to decide on a choice that prevents *backtracking*.
  - Such CFG is ***backtrack free*** with the ***lookahead*** of one symbol.
  - We also call such backtrack-free CFG a ***predictive grammar***.

The FIRST Set: Definition

- Say we write $T \subset \mathbb{P}(\Sigma^*)$ to denote the set of valid tokens recognizable by the scanner.
- **FIRST** $(\alpha) \triangleq$ set of symbols that can appear as the ***first word*** in some string derived from α.
- More precisely:

$$\text{FIRST}(\alpha) = \begin{cases} \{\alpha\} & \text{if } \alpha \in T \\ \{w \mid w \in \Sigma^* \land \alpha \overset{*}{\Rightarrow} w\beta \land \beta \in (V \cup \Sigma)^*\} & \text{if } \alpha \in V \end{cases}$$

The FIRST Set: Examples

- Consider this ***right***-recursive CFG:

0	*Goal*	→	*Expr*		6	*Term′*	→	× *Factor Term′*
1	*Expr*	→	*Term Expr′*		7		|	÷ *Factor Term′*
2	*Expr′*	→	+ *Term Expr′*		8		|	ϵ
3		|	- *Term Expr′*		9	*Factor*	→	(*Expr*)
4		|	ϵ		10		|	num
5	*Term*	→	*Factor Term′*		11		|	name

- Compute **FIRST** for each terminal (e.g., num, +, ():

	num	name	+	-	×	÷	(	)	eof	ϵ
FIRST	num	name	+	-	×	÷	(	)	eof	ϵ

- Compute **FIRST** for each non-terminal (e.g., *Expr*, *Term′*):

	Expr	*Expr′*	*Term*	*Term′*	*Factor*
FIRST	(, name, num	+, -, ϵ	(, name, num	×, ÷, ϵ	(, name, num

Computing the FIRST Set

$$\text{FIRST}(\alpha) = \begin{cases} \{\alpha\} & \text{if } \alpha \in T \\ \{w \mid w \in \Sigma^* \land \alpha \overset{*}{\Rightarrow} w\beta \land \beta \in (V \cup \Sigma)^*\} & \text{if } \alpha \in V \end{cases}$$

```
ALGORITHM: GetFirst
  INPUT: CFG G = (V, Σ, R, S)
  T ⊂ Σ* denotes valid terminals
  OUTPUT: FIRST : V ∪ T ∪ {ϵ, eof} ⟶ ℙ(T ∪ {ϵ, eof})
PROCEDURE:
  for α ∈ (T ∪ {eof, ϵ}): FIRST(α) := {α}
  for A ∈ V: FIRST(A) := ∅
  lastFirst := ∅
  while (lastFirst ≠ FIRST):
    lastFirst := FIRST
    for A → β1β2...βk ∈ R s.t. ∀βj : βj ∈ (T ∪ V):
      rhs := FIRST(β1) − {ϵ}
      for (i := 1; ϵ ∈ FIRST(βi) ∧ i < k; i++):
        rhs := rhs ∪ (FIRST(βi+1) − {ϵ})
      if i = k ∧ ϵ ∈ FIRST(βk) then
        rhs := rhs ∪ {ϵ}
      end
      FIRST(A) := FIRST(A) ∪ rhs
```

Computing the FIRST Set: Extension

- Recall: **FIRST** takes as input a token or a variable.

$$\textbf{FIRST} : V \cup T \cup \{\epsilon, eof\} \longrightarrow \mathbb{P}(T \cup \{\epsilon, eof\})$$

- The computation of variable ***rhs*** in algoritm `GetFirst` actually suggests an extended, overloaded version:

$$\textbf{FIRST} : (V \cup T \cup \{\epsilon, eof\})^* \longrightarrow \mathbb{P}(T \cup \{\epsilon, eof\})$$

FIRST may also take as input a string $\beta_1 \beta_2 \dots \beta_n$ (RHS of rules).

- More precisely:

$$\textbf{FIRST}(\beta_1 \beta_2 \dots \beta_n) = \left\{ \textbf{FIRST}(\beta_1) \cup \textbf{FIRST}(\beta_2) \cup \dots \cup \textbf{FIRST}(\beta_{k-1}) \cup \textbf{FIRST}(\beta_k) \;\middle|\; \begin{array}{l} \forall i : 1 \le i < k \bullet \epsilon \in \textbf{FIRST}(\beta_i) \\ \wedge \\ \epsilon \notin \textbf{FIRST}(\beta_k) \end{array} \right.$$

Note. β_k is the first symbol whose **FIRST** set does not contain ϵ.

Extended FIRST Set: Examples

Consider this ***right***-recursive CFG:

0	*Goal*	→	*Expr*	6	*Term′*	→	x *Factor Term′*
1	*Expr*	→	*Term Expr′*	7		|	÷ *Factor Term′*
2	*Expr′*	→	+ *Term Expr′*	8		|	ϵ
3		|	- *Term Expr′*	9	*Factor*	→	(*Expr*)
4		|	ϵ	10		|	num
5	*Term*	→	*Factor Term′*	11		|	name

e.g., **FIRST**(*Term Expr′*) = **FIRST**(*Term*) = {(, name, num}

e.g., **FIRST**(+ *Term Expr′*) = **FIRST**(+) = {+}

e.g., **FIRST**(- *Term Expr′*) = **FIRST**(-) = {-}

e.g., **FIRST**(ϵ) = {ϵ}

Is the FIRST Set Sufficient

- Consider the following three productions:

Expr′	→	+	*Term*	*Term′*	(1)
	|	-	*Term*	*Term′*	(2)
	|	ϵ			(3)

In TDP, when the parser attempts to expand an *Expr′* node, it ***looks ahead with one symbol*** to decide on the choice of rule: **FIRST**(+) = {+}, **FIRST**(−) = {−}, and **FIRST**(ϵ) = {ϵ}.

Q. When to choose rule (3) (causing ***focus := trace.pop()***)?

A?. Choose rule (3) when *focus* ≠ **FIRST**(+) ∧ *focus* ≠ **FIRST**(−)?

- ***Correct*** but ***inefficient*** in case of illegal input string: syntax error is only reported after possibly a long series of ***backtrack***.
- Useful if parser knows which words can appear, after an application of the ϵ-production (rule (3)), as leadling symbols.

- **FOLLOW** $(v : V) \triangleq$ set of symbols that can appear to the immediate right of a string derived from v.

$$\textbf{FOLLOW}(v) = \{ \boldsymbol{w} \mid w, x, y \in \Sigma^* \wedge v \overset{*}{\Rightarrow} \boldsymbol{x} \wedge S \overset{*}{\Rightarrow} \boldsymbol{xwy} \}$$

The FOLLOW Set: Examples

- Consider this ***right***-recursive CFG:

0	*Goal*	→	*Expr*	6	*Term′*	→	x *Factor Term′*
1	*Expr*	→	*Term Expr′*	7		|	÷ *Factor Term′*
2	*Expr′*	→	+ *Term Expr′*	8		|	ϵ
3		|	- *Term Expr′*	9	*Factor*	→	(*Expr*)
4		|	ϵ	10		|	num
5	*Term*	→	*Factor Term′*	11		|	name

- Compute **FOLLOW** for each non-terminal (e.g., *Expr*, *Term′*):

	Expr	*Expr′*	*Term*	*Term′*	*Factor*
FOLLOW	eof,)	eof,)	eof, +, -,)	eof, +, -,)	eof, +, -, x, ÷,)

Computing the FOLLOW Set

$\text{FOLLOW}(v) = \{ w \mid w, x, y \in \Sigma^* \land v \overset{*}{\Rightarrow} x \land S \overset{*}{\Rightarrow} xwy \}$

```
ALGORITHM: GetFollow
  INPUT: CFG G = (V, Σ, R, S)
  OUTPUT: FOLLOW : V ⟶ ℙ(T ∪ {eof})
PROCEDURE:
  for A ∈ V: FOLLOW(A) := ∅
  FOLLOW(S) := {eof}
  lastFollow := ∅
  while(lastFollow ≠ FOLLOW):
    lastFollow := FOLLOW
    for A → β1β2 ... βk ∈ R:
      trailer := FOLLOW(A)
      for i: k .. 1:
        if βi ∈ V then
          FOLLOW(βi) := FOLLOW(βi) ∪ trailer
          if ϵ ∈ FIRST(βi)
            then trailer := trailer ∪ (FIRST(βi) − ϵ)
            else trailer := FIRST(βi)
        else
          trailer := FIRST(βi)
```


Backtrack-Free Grammar

- A ***backtrack-free grammar*** (for a **top-down parser**), when expanding the *focus* ***internal node***, is always able to choose a unique rule with the ***one-symbol lookahead*** (or report a ***syntax error*** when no rule applies).
- To formulate this, we first define:

$$\text{START}(A \rightarrow \beta) = \begin{cases} \text{FIRST}(\beta) & \text{if } \epsilon \notin \text{FIRST}(\beta) \\ \text{FIRST}(\beta) \cup \text{FOLLOW}(A) & \text{otherwise} \end{cases}$$

  FIRST(β) is the extended version where β may be $\beta_1 \beta_2 \dots \beta_n$
- A ***backtrack-free grammar*** has each of its productions $A \rightarrow \gamma_1 \mid \gamma_2 \mid \dots \mid \gamma_n$ satisfying:

$$\forall i, j : 1 \leq i, j \leq n \land i \neq j \bullet \text{START}(\gamma_i) \cap \text{START}(\gamma_j) = \varnothing$$

TDP: Lookahead with One Symbol

```
ALGORITHM: TDParse
  INPUT: CFG G = (V, Σ, R, S)
  OUTPUT: Root of a Parse Tree or Syntax Error
PROCEDURE:
  root := a new node for the start symbol S
  focus := root
  initialize an empty stack trace
  trace.push(null)
  word := NextWord()
  while (true):
    if focus ∈ V then
      if ∃ unvisited rule focus → β1β2 ... βn ∈ R ∧ word ∈ START(β) then
        create β1, β2 ... βn as children of focus
        trace.push(βnβn−1 ... β2)
        focus := β1
      else
        if focus = S then report syntax error
        else backtrack
    elseif word matches focus then
      word := NextWord()
      focus := trace.pop()
    elseif word = EOF ∧ focus = null then return root
    else backtrack
```

backtrack ≜ pop *focus*.siblings; *focus* := *focus*.parent; *focus*.resetChildren

Backtrack-Free Grammar: Exercise

Is the following CFG ***backtrack free***?

11	*Factor*	→	name
12		|	name [*ArgList*]
13		|	name (*ArgList*)
15	*ArgList*	→	*Expr MoreArgs*
16	*MoreArgs*	→	, *Expr MoreArgs*
17		|	ϵ

- $\epsilon \notin$ FIRST(*Factor*) ⇒ START(*Factor*) = FIRST(*Factor*)
- FIRST(*Factor* → name) = {name}
- FIRST(*Factor* → name [*ArgList*]) = {name}
- FIRST(*Factor* → name (*ArgList*)) = {name}

∴ The above grammar is ***not*** backtrack free.
⇒ To expand an AST node of *Factor*, with a ***lookahead*** of name, the parser has no basis to choose among rules 11, 12, and 13.

Backtrack-Free Grammar: Left-Factoring

- A CFG is not backtrack free if there exists a ***common prefix*** (`name`) among the RHS of ***multiple*** production rules.
- To make such a CFG ***backtrack-free***, we may transform it using *left factoring*: a process of extracting and isolating ***common prefixes*** in a set of production rules.
  - Identify a common prefix α:

$$A \rightarrow \alpha\beta_1 \mid \alpha\beta_2 \mid \dots \mid \alpha\beta_n \mid \gamma_1 \mid \gamma_2 \mid \dots \mid \gamma_j$$

[each of $\gamma_1, \gamma_2, \dots, \gamma_j$ does not begin with α]

  - Rewrite that production rule as:

$$\begin{array}{lcl} A & \rightarrow & \alpha B \mid \gamma_1 \mid \gamma_2 \mid \dots \mid \gamma_j \\ B & \rightarrow & \beta_1 \mid \beta_2 \mid \dots \mid \beta_n \end{array}$$

  - New rule $B \rightarrow \beta_1 \mid \beta_2 \mid \dots \mid \beta_n$ may also contain ***common prefixes***.
  - Rewriting continues until no common prefixes are identified.

Left-Factoring: Exercise

- Use *left-factoring* to remove all ***common prefixes*** from the following grammar.

11	*Factor*	→	`name`
12		|	`name` [*ArgList*]
13		|	`name` (*ArgList*)
15	*ArgList*	→	*Expr MoreArgs*
16	*MoreArgs*	→	, *Expr MoreArgs*
17		|	ϵ

- Identify common prefix `name` and rewrite rules 11, 12, and 13:

$$\begin{array}{lcl} \textit{Factor} & \rightarrow & \texttt{name}\ \textit{Arguments} \\ \textit{Arguments} & \rightarrow & [\ \textit{ArgList}\ ] \\ & \mid & (\ \textit{ArgList}\ ) \\ & \mid & \epsilon \end{array}$$

Any more ***common prefixes***? [No]

TDP: Terminating and Backtrack-Free

- Given an arbitrary CFG as input to a ***top-down parser***:
  - **Q.** How do we avoid a ***non-terminating*** parsing process?
    A. Convert left-recursions to right-recursion.
  - **Q.** How do we minimize the need of ***backtracking***?
    A. left-factoring & one-symbol lookahead using **START**
- ***Not*** every context-free language has a corresponding ***backtrack-free*** context-free grammar.
  Given a CFL l, the following is ***undecidable***:

$$\exists cfg \mid L(cfg) = l \wedge isBacktrackFree(cfg)$$

- Given a CFG $g = (V, \Sigma, R, S)$, whether or not g is ***backtrack-free*** is ***decidable***:
  For each $A \rightarrow \gamma_1 \mid \gamma_2 \mid \dots \mid \gamma_n \in R$:

$$\forall i, j : 1 \leq i, j \leq n \wedge i \neq j \bullet \textsf{START}(\gamma_i) \cap \textsf{START}(\gamma_j) = \varnothing$$

Backtrack-Free Parsing (2.1)

- A *recursive-descent* parser is:
  - A top-down parser
  - Structured as a set of *mutually recursive* procedures
    Each procedure corresponds to a ***non-terminal*** in the grammar.
    See an example.
- Given a ***backtrack-free*** grammar, a tool (a.k.a. *parser generator*) can automatically generate:
  - **FIRST**, **FOLLOW**, and **START** sets
  - An efficient ***recursive-descent*** parser
    This generated parser is called an *LL(1) parser*, which:
    - Processes input from Left to right
    - Constructs a Leftmost derivation
    - Uses a lookahead of 1 symbol
- *LL(1) grammars* are those working in an *LL(1)* scheme.
  LL(1) grammars are ***backtrack-free*** by definition.

Backtrack-Free Parsing (2.2)

- Consider this CFG with **START** sets of the RHSs:

	Production	FIRST^{+}
2	$Expr' \rightarrow$ + $Term\ Expr'$	{ + }
3	$\mid$ - $Term\ Expr'$	{ - }
4	$\mid$ ϵ	{ ϵ, eof,) }

- The corresponding ***recursive-descent*** parser is structured as:

```
ExprPrim()
  if word = + ∨ word = - then /* Rules 2, 3 */
    word := NextWord()
    if(Term())
      then return ExprPrim()
      else return false
  elseif word = ) ∨ word = eof then /* Rule 4 */
    return true
  else
    report a syntax error
    return false
  end

Term()
  ...
```

See: parser generator

LL(1) Parser: Exercise

Consider the following grammar:

L	$\rightarrow$	R a	R	$\rightarrow$	aba	Q	$\rightarrow$	bbc
	$\mid$	Q ba		$\mid$	caba		$\mid$	bc
				$\mid$	R bc			

Q. Is it suitable for a ***top-down predictive*** parser?

- If so, show that it satisfies the *LL(1)* condition.
- If not, identify the problem(s) and correct it (them). Also show that the revised grammar satisfies the *LL(1)* condition.

BUP: Discovering Rightmost Derivation

- In TDP, we build the start variable as the ***root node***, and then work towards the ***leaves***. [**leftmost** derivation]
- In Bottom-Up Parsing (BUP):
  - Words (terminals) are still returned from **left** to **right** by the scanner.
  - As terminals, or a mix of terminals and variables, are identified as *reducible* to some variable A (i.e., matching the RHS of some production rule for A), then a layer is added.
  - Eventually:
    - ***accept***:
      The ***start variable*** is reduced and **all** words have been consumed.
    - ***reject***:
      The next word is not eof, but no further *reduction* can be identified.

Q. Why can BUP find the ***rightmost*** derivation (RMD), if any?

A. BUP discovers steps in a ***RMD*** in its *reverse* order.

BUP: Discovering Rightmost Derivation (1)

- ***table***-driven *LR(1)* parser: an implementation for BUP, which
  - Processes input from **L**eft to right
  - Constructs a **R**ightmost derivation
  - Uses a lookahead of **1** symbol
- A language has the *LR(1)* property if it:
  - Can be parsed in a single **L**eft to right scan,
  - To build a *reversed* **R**ightmost derivation,
  - Using a lookahead of **1** symbol to determine parsing actions.
- Critical step in a ***bottom-up parser*** is to find the ***next*** *handle*.

BUP: Discovering Rightmost Derivation (2)

```
ALGORITHM: BUParse
  INPUT: CFG G = (V, Σ, R, S), Action & Goto Tables
  OUTPUT: Report Parse Success or Syntax Error
PROCEDURE:
  initialize an empty stack trace
  trace.push(0) /* start state */
  word := NextWord()
  while(true)
    state := trace.top()
    act := Action[state, word]
    if act = ``accept'' then
      succeed()
    elseif act = ``reduce based on A → β'' then
      trace.pop() 2 × |β| times /* word + state */
      state := trace.top()
      trace.push(A)
      next := Goto[state, A]
      trace.push(next)
    elseif act = ``shift to s_i'' then
      trace.push(word)
      trace.push(i)
      word := NextWord()
    else
      fail()
```


BUP: Example Tracing (1)

- Consider the following grammar for parentheses:

1	*Goal*	→	*List*
2	*List*	→	*List Pair*
3		|	*Pair*
4	*Pair*	→	(*Pair*)
5		|	( )

- Assume: tables ***Action*** and ***Goto*** constructed accordingly:

State	Action Table: eof	Action Table: (	Action Table:)	Goto Table: *List*	Goto Table: *Pair*
0		s 3		1	2
1	acc	s 3			4
2	r 3	r 3			
3		s 6	s 7		5
4	r 2	r 2			
5			s 8		
6		s 6	s 10		9
7	r 5	r 5			
8	r 4	r 4			
9			s 11		
10			r 5		
11			r 4		

In ***Action*** table:

- s_i: shift to state i
- r_j: reduce to the LHS of production #j

BUP: Example Tracing (2.1)

Consider the steps of performing BUP on input ( ):

Iteration	State	word	Stack	Handle	Action
initial	—	(	$ 0	— *none* —	—
1	0	(	$ 0	— *none* —	*shift 3*
2	3	)	$ 0 (3	— *none* —	*shift 7*
3	7	eof	$ 0 (3) 7	( )	*reduce 5*
4	2	eof	$ 0 *Pair* 2	*Pair*	*reduce 3*
5	1	eof	$ 0 *List* 1	*List*	*accept*

BUP: Example Tracing (2.2)

Consider the steps of performing BUP on input (( )) ( ):

Iteration	State	word	Stack	Handle	Action
initial	—	(	$ 0	— *none* —	—
1	0	(	$ 0	— *none* —	*shift 3*
2	3	(	$ 0 (3	— *none* —	*shift 6*
3	6	)	$ 0 (3 (6	— *none* —	*shift 10*
4	10	)	$ 0 (3 (6) 10	( )	*reduce 5*
5	5	)	$ 0 (3 *Pair* 5	— *none* —	*shift 8*
6	8	(	$ 0 (3 *Pair* 5) 8	(*Pair*)	*reduce 4*
7	2	(	$ 0 *Pair* 2	*Pair*	*reduce 3*
8	1	(	$ 0 *List* 1	— *none* —	*shift 3*
9	3	)	$ 0 *List* 1 (3	— *none* —	*shift 7*
10	7	eof	$ 0 *List* 1 (3) 7	( )	*reduce 5*
11	4	eof	$ 0 *List* 1 *Pair* 4	*List Pair*	*reduce 2*
12	1	eof	$ 0 *List* 1	*List*	*accept*

BUP: Example Tracing (2.3)

Consider the steps of performing BUP on input `( ) )`:

Iteration	State	word	Stack	Handle	Action
initial	—	(	$ 0	— *none* —	—
1	0	(	$ 0	— *none* —	*shift 3*
2	3	)	$ 0 (3	— *none* —	*shift 7*
3	7	)	$ 0 (3) 7	— *none* —	*error*

LR(1) Items: Definition

- In **LR(1)** parsing, ***Action*** and ***Goto*** tabeles encode legitimate ways (w.r.t. a CFG) for finding ***handles*** (for ***reductions***).
- In a ***table***-driven **LR(1)** parser, the table-construction algorithm represents each potential ***handle*** (for a ***reduction***) with an **LR(1)** item e.g.,

$$[A \rightarrow \beta \bullet \gamma,\ \mathtt{a}]$$

where:

- A ***production rule*** $A \rightarrow \beta\gamma$ is currently being applied.
- A ***terminal symbol*** $\mathtt{a}$ servers as a ***lookahead symbol***.
- A ***placeholder*** $\bullet$ indicates the parser's ***stack top***.
  - ✓ The parser's ***stack*** contains β ("left context").
  - ✓ γ is yet to be matched.
  - Upon matching $\beta\gamma$, if $\mathtt{a}$ matches the current word, then we "replace" $\beta\gamma$ (and their associated states) with A (and its associated state).

LR(1) Items: Scenarios

An *LR(1) item* can denote:

1. **Possibility** $[A \rightarrow \bullet\beta\gamma,\ \mathtt{a}]$
   - In the current parsing context, an *A* would be valid.
   - $\bullet$ represents the position of the parser's ***stack top***
   - Recognizing a β next would be one step towards discovering an *A*.
2. **Partial Completion** $[A \rightarrow \beta \bullet \gamma,\ \mathtt{a}]$
   - The parser has progressed from $[A \rightarrow \bullet\beta\gamma,\ \mathtt{a}]$ by recognizing β.
   - Recognizing a γ next would be one step towards discovering an *A*.
3. **Completion** $[A \rightarrow \beta\gamma\bullet,\ \mathtt{a}]$
   - Parser has progressed from $[A \rightarrow \bullet\beta\gamma,\ \mathtt{a}]$ by recognizing $\beta\gamma$.
   - $\beta\gamma$ found in a context where an *A* followed by $\mathtt{a}$ would be valid.
   - If the current input word matches $\mathtt{a}$, then:
     - Current ***complet item*** is a *handle*.
     - Parser can ***reduce*** $\beta\gamma$ to *A*
     - Accordingly, in the ***stack***, $\beta\gamma$ (and their associated states) are replaced with *A* (and its associated state).

LR(1) Items: Example (1.1)

Consider the following grammar for parentheses:

1	*Goal*	→	*List*
2	*List*	→	*List Pair*
3		|	*Pair*
4	*Pair*	→	(*Pair*)
5		|	( )

Initial State: $[Goal \rightarrow \bullet List,\ \mathtt{eof}]$

Desired Final State: $[Goal \rightarrow List\bullet,\ \mathtt{eof}]$

Intermediate States: Subset Construction

Q. Derive all *LR(1) items* for the above grammar.

- $\text{FOLLOW}(List) = \{\mathtt{eof},\ \mathtt{(}\}$ $\quad \text{FOLLOW}(Pair) = \{\mathtt{eof},\ \mathtt{(},\ \mathtt{)}\}$
- For each production $A \rightarrow \beta$, given $\text{FOLLOW}(A)$, *LR(1) items* are:

$$\{\ [A \rightarrow \bullet\beta\gamma,\ \mathtt{a}] \mid a \in \text{FOLLOW}(A)\ \}$$
$$\cup$$
$$\{\ [A \rightarrow \beta \bullet \gamma,\ \mathtt{a}] \mid a \in \text{FOLLOW}(A)\ \}$$
$$\cup$$
$$\{\ [A \rightarrow \beta\gamma\bullet,\ \mathtt{a}] \mid a \in \text{FOLLOW}(A)\ \}$$

LR(1) Items: Example (1.2)

Q. Given production $A \to \beta$ (e.g., *Pair* → (*Pair*)), how many *LR(1) items* can be generated?

- The current parsing progress (on matching the RHS) can be:
  1. • (*Pair*)
  2. (•*Pair*)
  3. (*Pair*•)
  4. (*Pair*)•
- Lookahead symbol following *Pair*? **FOLLOW**(*Pair*) = {eof, (,)}
- All possible *LR(1) items* related to *Pair* → (*Pair*)?

✓	[•(*Pair*), eof]	[•(*Pair*), (]	[•(*Pair*),)]
✓	[(•*Pair*), eof]	[(•*Pair*), (]	[(•*Pair*),)]
✓	[(*Pair*•), eof]	[(*Pair*•), (]	[(*Pair*•),)]
✓	[(*Pair*)•, eof]	[(*Pair*)•, (]	[(*Pair*)•,)]

A. How many in general (in terms of A and β)?

$$\underbrace{|\beta| + 1}_{\text{possible positions of } \bullet} \times \underbrace{|\textsc{Follow}(A)|}_{\text{possible lookahead symbols}}$$

LR(1) Items: Example (1.3)

A. There are 33 *LR(1) items* in the parentheses grammar.

[*Goal* → • *List*,eof]
[*Goal* → *List* •,eof]

[*List* → • *List Pair*,eof]	[*List* → • *List Pair*,(]
[*List* → *List* • *Pair*,eof]	[*List* → *List* • *Pair*,(]
[*List* → *List Pair* •,eof]	[*List* → *List Pair* •,(]
[*List* → • *Pair*,eof]	[*List* → • *Pair*,(]
[*List* → *Pair* •,eof]	[*List* → *Pair* •,(]

[*Pair* → • (*Pair*),eof]	[*Pair* → • (*Pair*),)]	[*Pair* → • (*Pair*),(]
[*Pair* → (• *Pair*),eof]	[*Pair* → (• *Pair*),)]	[*Pair* → (• *Pair*),(]
[*Pair* → (*Pair* •),eof]	[*Pair* → (*Pair* •),)]	[*Pair* → (*Pair* •),(]
[*Pair* → (*Pair*) •,eof]	[*Pair* → (*Pair*) •,)]	[*Pair* → (*Pair*) •,(]
[*Pair* → • ( ),eof]	[*Pair* → • ( ),(]	[*Pair* → • ( ),)]
[*Pair* → (•),eof]	[*Pair* → (•),(]	[*Pair* → (•),)]
[*Pair* → ( ) •,eof]	[*Pair* → ( ) •,(]	[*Pair* → ( ) •,)]

LR(1) Items: Example (2)

Consider the following grammar for expressions:

0	*Goal* → *Expr*			6	*Term′*	→	× *Factor Term′*
1	*Expr* → *Term Expr′*			7		|	÷ *Factor Term′*
2	*Expr′* → + *Term Expr′*			8		|	ϵ
3	| - *Term Expr′*			9	*Factor*	→	(*Expr*)
4	| ϵ			10		|	num
5	*Term* → *Factor Term′*			11		|	name

Q. Derive all *LR(1) items* for the above grammar.
Hints. First compute **FOLLOW** for each non-terminal:

	Expr	*Expr′*	*Term*	*Term′*	*Factor*
FOLLOW	eof,)	eof,)	eof, +, -,)	eof, +, -,)	eof, +, -, ×, ÷,)

Tips. Ignore ϵ ***production*** such as *Expr′* → ϵ since the **FOLLOW** sets already take them into consideration.

Canonical Collection ($\mathcal{CC}$) vs. LR(1) items

1	*Goal* → *List*
2	*List* → *List Pair*
3	| *Pair*
4	*Pair* → (*Pair*)
5	| ( )

Recall:
LR(1) Items: 33 items
Initial State: [*Goal* → •*List*, eof]
Desired Final State: [*Goal* → *List*•, eof]

- The *canonical collection* [Example of $\mathcal{CC}$]
  $$\mathcal{CC} = \{cc_0, cc_1, cc_2, \ldots, cc_n\}$$
  denotes the set of **valid subset states** of a **LR(1) parser**.
  - Each $cc_i \in \mathcal{CC}$ $(0 \le i \le n)$ is a set of ***LR(1) items***.
  - $\mathcal{CC} \subseteq \mathbb{P}($***LR(1) items***$)$ $|\mathcal{CC}|$? $[\, |\mathcal{CC}| \le 2^{|LR(1)\ items|} \,]$
- To model a **LR(1) parser**, we use techniques analogous to how an ϵ-NFA is converted into a DFA (subset construction and ϵ-closure).
- **Analogies.**
  ✓ ***LR(1) items*** ≈ states of source *NFA*
  ✓ $\mathcal{CC}$ ≈ subset states of target *DFA*

Constructing $\mathcal{CC}$: The *closure* Procedure (1)

```
1   ALGORITHM: closure
2     INPUT: CFG G = (V, Σ, R, S), a set s of LR(1) items
3     OUTPUT: a set of LR(1) items
4   PROCEDURE:
5     lastS := ∅
6     while(lastS ≠ s):
7       lastS := s
8       for [A → ··· • C δ, a] ∈ s:
9         for C → γ ∈ R:
10          for b ∈ FIRST(δa):
11            s := s ∪ { [C → •γ, b] }
12    return s
```

- **Line 8**: $[A \rightarrow \cdots \bullet C\,\delta,\ a] \in s$ indicates that the parser's next task is to match $C\,\delta$ with a lookahead symbol a.
- **Line 9**: Given: matching γ can reduce to C
- **Line 10**: Given: $b \in \text{FIRST}(\delta a)$ is a valid lookahead symbol after reducing γ to C
- **Line 11**: Add a new item $[C \rightarrow \bullet\gamma,\ b]$ into s.
- **Line 6**: Termination is guaranteed.

  ∵ Each iteration adds ≥ 1 item to s (otherwise $lastS \neq s$ is *false*).

Constructing $\mathcal{CC}$: The *closure* Procedure (2.1)

1	$Goal \rightarrow List$
2	$List \rightarrow List\ Pair$
3	$\mid\ Pair$
4	$Pair \rightarrow \underline{(}\ Pair\ \underline{)}$
5	$\mid\ \underline{(}\ \underline{)}$

Initial State: $[Goal \rightarrow \bullet List,\ \texttt{eof}]$

Calculate cc_0 = ***closure***$(\{\ [Goal \rightarrow \bullet List,\ \texttt{eof}]\ \})$.

Constructing $\mathcal{CC}$: The *goto* Procedure (1)

```
1   ALGORITHM: goto
2     INPUT: a set s of LR(1) items, a symbol x
3     OUTPUT: a set of LR(1) items
4   PROCEDURE:
5     moved := ∅
6     for item ∈ s:
7       if item = [α → β • xδ, a] then
8         moved := moved ∪ { [α → βx • δ, a] }
9       end
10    return closure(moved)
```

Line 7: Given: item $[\alpha \rightarrow \beta \bullet x\delta,\ a]$ (where x is the next to match)
Line 8: Add $[\alpha \rightarrow \beta x \bullet \delta,\ a]$ (indicating x is matched) to *moved*
Line 10: Calculate and return ***closure***(*moved*) as the "***next subset state***" from s with a "transition" x.

Constructing $\mathcal{CC}$: The *goto* Procedure (2)

1	$Goal \rightarrow List$
2	$List \rightarrow List\ Pair$
3	$\mid\ Pair$
4	$Pair \rightarrow \underline{(}\ Pair\ \underline{)}$
5	$\mid\ \underline{(}\ \underline{)}$

$$\mathrm{cc}_0 = \left\{ \begin{array}{ccc} [Goal \rightarrow \bullet List,\ \texttt{eof}] & [List \rightarrow \bullet List\ Pair,\ \texttt{eof}] & [List \rightarrow \bullet List\ Pair,\ \underline{(}] \\ [List \rightarrow \bullet Pair,\ \texttt{eof}] & [List \rightarrow \bullet Pair,\ \underline{(}] & [Pair \rightarrow \bullet \underline{(}\ Pair\ \underline{)},\ \texttt{eof}] \\ [Pair \rightarrow \bullet \underline{(}\ Pair\ \underline{)}, \underline{(}] & [Pair \rightarrow \bullet \underline{(}\ \underline{)},\ \texttt{eof}] & [Pair \rightarrow \bullet \underline{(}\ \underline{)}, \underline{(}] \end{array} \right\}$$

Calculate $goto(cc_0,\ \underline{(})$. ["next state" from cc_0 taking $\underline{(}$]

Constructing $\mathcal{CC}$: The Algorithm (1)

```
ALGORITHM: BuildCC
  INPUT: a grammar G = (V, Σ, R, S), goal production S → S'
  OUTPUT:
    (1) a set CC = {cc0, cc1, ..., ccn} where cci ⊆ G's LR(1) items
    (2) a transition function
PROCEDURE:
  cc0 := closure({[S → •S', eof]})
  CC := {cc0}
  processed := {cc0}
  lastCC := ∅
  while(lastCC ≠ CC):
    lastCC := CC
    for cci s.t. cci ∈ CC ∧ cci ∉ processed:
      processed := processed ∪ {cci}
      for x s.t. [··· → ··· • x ...] ∈ cci
        temp := goto(cci, x)
        if temp ∉ CC then
          CC := CC ∪ {temp}
        end
        δ := δ ∪ (cci, x, temp)
```


Constructing $\mathcal{CC}$: The Algorithm (2.1)

1	*Goal*	→	*List*
2	*List*	→	*List Pair*
3		|	*Pair*
4	*Pair*	→	(*Pair*)
5		|	( )

- Calculate $\mathcal{CC} = \{cc_0, cc_1, \ldots, cc_{11}\}$
- Calculate the transition function $\delta : \mathcal{CC} \times (\Sigma \cup V) \to \mathcal{CC}$

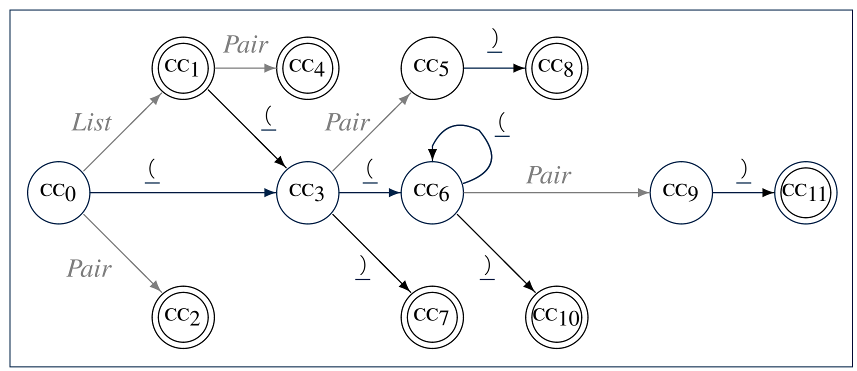
Constructing $\mathcal{CC}$: The Algorithm (2.2)

Resulting transition table:

Iteration	Item	*Goal*	*List*	*Pair*	(	)	eof
0	cc_0	∅	cc_1	cc_2	cc_3	∅	∅
1	cc_1	∅	∅	cc_4	cc_3	∅	∅
	cc_2	∅	∅	∅	∅	∅	∅
	cc_3	∅	∅	cc_5	cc_6	cc_7	∅
2	cc_4	∅	∅	∅	∅	∅	∅
	cc_5	∅	∅	∅	∅	cc_8	∅
	cc_6	∅	∅	cc_9	cc_6	cc_{10}	∅
	cc_7	∅	∅	∅	∅	∅	∅
3	cc_8	∅	∅	∅	∅	∅	∅
	cc_9	∅	∅	∅	∅	cc_{11}	∅
	cc_{10}	∅	∅	∅	∅	∅	∅
4	cc_{11}	∅	∅	∅	∅	∅	∅

Constructing $\mathcal{CC}$: The Algorithm (2.3)

Resulting DFA for the parser:



Constructing $\mathcal{CC}$: The Algorithm (2.4.1)

Resulting canonical collection $\mathcal{CC}$: [Def. of $\mathcal{CC}$]

$$cc_0 = \left\{ \begin{array}{ccc} [Goal \rightarrow \bullet List, \texttt{eof}] & [List \rightarrow \bullet List\ Pair, \texttt{eof}] & [List \rightarrow \bullet List\ Pair, \underline{(}] \\ [List \rightarrow \bullet Pair, \texttt{eof}] & [List \rightarrow \bullet Pair, \underline{(}] & [Pair \rightarrow \bullet \underline{(}\ Pair\ \underline{)}, \texttt{eof}] \\ [Pair \rightarrow \bullet \underline{(}\ Pair\ \underline{)}, \underline{(}] & [Pair \rightarrow \bullet \underline{(}\ \underline{)}, \texttt{eof}] & [Pair \rightarrow \bullet \underline{(}\ \underline{)}, \underline{(}] \end{array} \right\}$$

$$cc_1 = \left\{ \begin{array}{ccc} [Goal \rightarrow List \bullet, \texttt{eof}] & [List \rightarrow List \bullet Pair, \texttt{eof}] & [List \rightarrow List \bullet Pair, \underline{(}] \\ [Pair \rightarrow \bullet \underline{(}\ Pair\ \underline{)}, \texttt{eof}] & [Pair \rightarrow \bullet \underline{(}\ Pair\ \underline{)}, \underline{(}] & [Pair \rightarrow \bullet \underline{(}\ \underline{)}, \texttt{eof}] \\ & [Pair \rightarrow \bullet \underline{(}\ \underline{)}, \underline{(}] & \end{array} \right\}$$

$$cc_2 = \left\{ [List \rightarrow Pair \bullet, \texttt{eof}] \quad [List \rightarrow Pair \bullet, \underline{(}] \right\}$$

$$cc_3 = \left\{ \begin{array}{ccc} [Pair \rightarrow \bullet \underline{(}\ Pair\ \underline{)}, \underline{)}] & [Pair \rightarrow \underline{(} \bullet Pair\ \underline{)}, \texttt{eof}] & [Pair \rightarrow \underline{(} \bullet Pair\ \underline{)}, \underline{(}] \\ [Pair \rightarrow \bullet \underline{(}\ \underline{)}, \underline{)}] & [Pair \rightarrow \underline{(} \bullet \underline{)}, \texttt{eof}] & [Pair \rightarrow \underline{(} \bullet \underline{)}, \underline{(}] \end{array} \right\}$$

$$cc_4 = \left\{ [List \rightarrow List\ Pair \bullet, \texttt{eof}] \quad [List \rightarrow List\ Pair \bullet, \underline{(}] \right\}$$

$$cc_5 = \left\{ [Pair \rightarrow \underline{(}\ Pair \bullet \underline{)}, \texttt{eof}] \quad [Pair \rightarrow \underline{(}\ Pair \bullet \underline{)}, \underline{(}] \right\}$$

$$cc_6 = \left\{ \begin{array}{cc} [Pair \rightarrow \bullet \underline{(}\ Pair\ \underline{)}, \underline{)}] & [Pair \rightarrow \underline{(} \bullet Pair\ \underline{)}, \underline{)}] \\ [Pair \rightarrow \bullet \underline{(}\ \underline{)}, \underline{)}] & [Pair \rightarrow \underline{(} \bullet \underline{)}, \underline{)}] \end{array} \right\}$$

$$cc_7 = \left\{ [Pair \rightarrow \underline{(}\ \underline{)} \bullet, \texttt{eof}] \quad [Pair \rightarrow \underline{(}\ \underline{)} \bullet, \underline{(}] \right\}$$

$$cc_8 = \left\{ [Pair \rightarrow \underline{(}\ Pair\ \underline{)} \bullet, \texttt{eof}] \quad [Pair \rightarrow \underline{(}\ Pair\ \underline{)} \bullet, \underline{(}] \right\}$$

$$cc_9 = \left\{ [Pair \rightarrow \underline{(}\ Pair \bullet \underline{)}, \underline{)}] \right\}$$

$$cc_{10} = \left\{ [Pair \rightarrow \underline{(}\ \underline{)} \bullet, \underline{)}] \right\}$$

$$cc_{11} = \left\{ [Pair \rightarrow \underline{(}\ Pair\ \underline{)} \bullet, \underline{)}] \right\}$$

Constructing *Action* and *Goto* Tables (1)

```
1  ALGORITHM: BuildActionGotoTables
2    INPUT:
3      (1) a grammar G = (V, Σ, R, S)
4      (2) goal production S → S'
5      (3) a canonical collection CC = {cc0, cc1, ..., ccn}
6      (4) a transition function δ : CC × Σ → CC
7    OUTPUT: Action Table & Goto Table
8  PROCEDURE:
9    for cci ∈ CC:
10     for item ∈ cci:
11       if item = [A → β • xγ, a] ∧ δ(cci, x) = ccj then
12         Action[i, x] := shift j
13       elseif item = [A → β•, a] then
14         Action[i, a] := reduce A → β
15       elseif item = [S → S'•, eof] then
16         Action[i, eof] := accept
17       end
18     for v ∈ V:
19       if δ(cci, v) = ccj then
20         Goto[i, v] = j
21       end
```

- **L12, 13**: Next valid step in discovering A is to match terminal symbol $\texttt{x}$.
- **L14, 15**: Having recognized β, if current word matches lookahead $\texttt{a}$, reduce β to A.
- **L16, 17**: Accept if input exhausted and what's recognized reducible to start var. S.
- **L20, 21**: Record consequence of a reduction to non-terminal v from state i

Constructing *Action* and *Goto* Tables (2)

Resulting **Action** and **Goto** tables:

	Action Table			Goto Table	
State	eof	$\underline{(}$	$\underline{)}$	***List***	***Pair***
0		s 3		1	2
1	acc	s 3			4
2	r 3	r 3			
3		s 6	s 7		5
4	r 2	r 2			
5			s 8		
6		s 6	s 10		9
7	r 5	r 5			
8	r 4	r 4			
9			s 11		
10			r 5		
11			r 4		

BUP: Discovering Ambiguity (1)

1	*Goal*	$\rightarrow$	*Stmt*
2	*Stmt*	$\rightarrow$	`if expr then` *Stmt*
3		|	`if expr then` *Stmt* `else` *Stmt*
4		|	`assign`

- Calculate $\mathcal{CC} = \{cc_0, cc_1, \ldots, \}$
- Calculate the transition function $\delta : \mathcal{CC} \times \Sigma \rightarrow \mathcal{CC}$

BUP: Discovering Ambiguity (2.1)

Resulting transition table:

	Item	*Goal*	*Stmt*	if	expr	then	else	assign	eof
0	cc_0	∅	cc_1	cc_2	∅	∅	∅	cc_3	∅
1	cc_1	∅	∅	∅	∅	∅	∅	∅	∅
	cc_2	∅	∅	∅	cc_4	∅	∅	∅	∅
	cc_3	∅	∅	∅	∅	∅	∅	∅	∅
2	cc_4	∅	∅	∅	∅	cc_5	∅	∅	∅
3	cc_5	∅	cc_6	cc_7	∅	∅	∅	cc_8	∅
4	cc_6	∅	∅	∅	∅	∅	cc_9	∅	∅
	cc_7	∅	∅	∅	cc_{10}	∅	∅	∅	∅
	cc_8	∅	∅	∅	∅	∅	∅	∅	∅
5	cc_9	∅	cc_{11}	cc_2	∅	∅	∅	cc_3	∅
	cc_{10}	∅	∅	∅	∅	cc_{12}	∅	∅	∅
6	cc_{11}	∅	∅	∅	∅	∅	∅	∅	∅
	cc_{12}	∅	cc_{13}	cc_7	∅	∅	∅	cc_8	∅
7	cc_{13}	∅	∅	∅	∅	∅	cc_{14}	∅	∅
8	cc_{14}	∅	cc_{15}	cc_7	∅	∅	∅	cc_8	∅
9	cc_{15}	∅	∅	∅	∅	∅	∅	∅	∅

BUP: Discovering Ambiguity (2.2.1)

Resulting canonical collection $\mathcal{CC}$:

$$cc_0 = \left\{ \begin{array}{ll} [\textit{Goal} \rightarrow \bullet\, \textit{Stmt}, \texttt{eof}] & [\textit{Stmt} \rightarrow \bullet\, \texttt{if expr then } \textit{Stmt}, \texttt{eof}] \\ [\textit{Stmt} \rightarrow \bullet\, \texttt{assign}, \texttt{eof}] & [\textit{Stmt} \rightarrow \bullet\, \texttt{if expr then } \textit{Stmt}\ \texttt{else } \textit{Stmt}, \texttt{eof}] \end{array} \right\}$$

$$cc_1 = \left\{ [\textit{Goal} \rightarrow \textit{Stmt}\, \bullet, \texttt{eof}] \right\}$$

$$cc_2 = \left\{ \begin{array}{l} [\textit{Stmt} \rightarrow \texttt{if} \bullet \texttt{expr then } \textit{Stmt}, \texttt{eof}], \\ [\textit{Stmt} \rightarrow \texttt{if} \bullet \texttt{expr then } \textit{Stmt}\ \texttt{else } \textit{Stmt}, \texttt{eof}] \end{array} \right\}$$

$$cc_3 = \left\{ [\textit{Stmt} \rightarrow \texttt{assign}\, \bullet, \texttt{eof}] \right\}$$

$$cc_4 = \left\{ \begin{array}{l} [\textit{Stmt} \rightarrow \texttt{if expr} \bullet \texttt{then } \textit{Stmt}, \texttt{eof}], \\ [\textit{Stmt} \rightarrow \texttt{if expr} \bullet \texttt{then } \textit{Stmt}\ \texttt{else } \textit{Stmt}, \texttt{eof}] \end{array} \right\}$$

$$cc_5 = \left\{ \begin{array}{l} [\textit{Stmt} \rightarrow \texttt{if expr then} \bullet \textit{Stmt}, \texttt{eof}], \\ [\textit{Stmt} \rightarrow \texttt{if expr then} \bullet \textit{Stmt}\ \texttt{else } \textit{Stmt}, \texttt{eof}], \\ [\textit{Stmt} \rightarrow \bullet\, \texttt{if expr then } \textit{Stmt}, \{\texttt{eof}, \texttt{else}\}], \\ [\textit{Stmt} \rightarrow \bullet\, \texttt{assign}, \{\texttt{eof}, \texttt{else}\}], \\ [\textit{Stmt} \rightarrow \bullet\, \texttt{if expr then } \textit{Stmt}\ \texttt{else } \textit{Stmt}, \{\texttt{eof}, \texttt{else}\}] \end{array} \right\}$$

$$cc_6 = \left\{ \begin{array}{l} [\textit{Stmt} \rightarrow \texttt{if expr then } \textit{Stmt}\, \bullet, \texttt{eof}], \\ [\textit{Stmt} \rightarrow \texttt{if expr then } \textit{Stmt} \bullet \texttt{else } \textit{Stmt}, \texttt{eof}] \end{array} \right\}$$

$$cc_7 = \left\{ \begin{array}{l} [\textit{Stmt} \rightarrow \texttt{if} \bullet \texttt{expr then } \textit{Stmt}, \{\texttt{eof}, \texttt{else}\}], \\ [\textit{Stmt} \rightarrow \texttt{if} \bullet \texttt{expr then } \textit{Stmt}\ \texttt{else } \textit{Stmt}, \{\texttt{eof}, \texttt{else}\}] \end{array} \right\}$$

BUP: Discovering Ambiguity (2.2.2)

Resulting canonical collection $\mathcal{CC}$:

$$cc_8 = \{[\textit{Stmt} \rightarrow \texttt{assign}\, \bullet, \{\texttt{eof}, \texttt{else}\}]\}$$

$$cc_9 = \left\{ \begin{array}{l} [\textit{Stmt} \rightarrow \texttt{if expr then } \textit{Stmt}\ \texttt{else} \bullet \textit{Stmt}, \texttt{eof}], \\ [\textit{Stmt} \rightarrow \bullet\, \texttt{if expr then } \textit{Stmt}, \texttt{eof}], \\ [\textit{Stmt} \rightarrow \bullet\, \texttt{if expr then } \textit{Stmt}\ \texttt{else } \textit{Stmt}, \texttt{eof}], \\ [\textit{Stmt} \rightarrow \bullet\, \texttt{assign}, \texttt{eof}] \end{array} \right\}$$

$$cc_{10} = \left\{ \begin{array}{l} [\textit{Stmt} \rightarrow \texttt{if expr} \bullet \texttt{then } \textit{Stmt}, \{\texttt{eof}, \texttt{else}\}], \\ [\textit{Stmt} \rightarrow \texttt{if expr} \bullet \texttt{then } \textit{Stmt}\ \texttt{else } \textit{Stmt}, \{\texttt{eof}, \texttt{else}\}] \end{array} \right\}$$

$$cc_{11} = \{[\textit{Stmt} \rightarrow \texttt{if expr then } \textit{Stmt}\ \texttt{else } \textit{Stmt}\, \bullet, \texttt{eof}]\}$$

$$cc_{12} = \left\{ \begin{array}{l} [\textit{Stmt} \rightarrow \texttt{if expr then} \bullet \textit{Stmt}, \{\texttt{eof}, \texttt{else}\}], \\ [\textit{Stmt} \rightarrow \texttt{if expr then} \bullet \textit{Stmt}\ \texttt{else } \textit{Stmt}, \{\texttt{eof}, \texttt{else}\}], \\ [\textit{Stmt} \rightarrow \bullet\, \texttt{if expr then } \textit{Stmt}, \{\texttt{eof}, \texttt{else}\}], \\ [\textit{Stmt} \rightarrow \bullet\, \texttt{if expr then } \textit{Stmt}\ \texttt{else } \textit{Stmt}, \{\texttt{eof}, \texttt{else}\}], \\ [\textit{Stmt} \rightarrow \bullet\, \texttt{assign}, \{\texttt{eof}, \texttt{else}\}] \end{array} \right\}$$

$$cc_{13} = \left\{ \begin{array}{l} [\textit{Stmt} \rightarrow \texttt{if expr then } \textit{Stmt}\, \bullet, \{\texttt{eof}, \texttt{else}\}], \\ [\textit{Stmt} \rightarrow \texttt{if expr then } \textit{Stmt} \bullet \texttt{else } \textit{Stmt}, \{\texttt{eof}, \texttt{else}\}] \end{array} \right\}$$

$$cc_{14} = \left\{ \begin{array}{l} [\textit{Stmt} \rightarrow \texttt{if expr then } \textit{Stmt}\ \texttt{else} \bullet \textit{Stmt}, \{\texttt{eof}, \texttt{else}\}], \\ [\textit{Stmt} \rightarrow \bullet\, \texttt{if expr then } \textit{Stmt}, \{\texttt{eof}, \texttt{else}\}], \\ [\textit{Stmt} \rightarrow \bullet\, \texttt{if expr then } \textit{Stmt}\ \texttt{else } \textit{Stmt}, \{\texttt{eof}, \texttt{else}\}], \\ [\textit{Stmt} \rightarrow \bullet\, \texttt{assign}, \{\texttt{eof}, \texttt{else}\}] \end{array} \right\}$$

BUP: Discovering Ambiguity (3)

- Consider cc_{13}

$$cc_{13} = \left\{ \begin{array}{l} [\textit{Stmt} \rightarrow \texttt{if expr then } \textit{Stmt}\, \bullet, \{\texttt{eof}, \texttt{else}\}], \\ [\textit{Stmt} \rightarrow \texttt{if expr then } \textit{Stmt} \bullet \texttt{else } \textit{Stmt}, \{\texttt{eof}, \texttt{else}\}] \end{array} \right\}$$

  Q. What does it mean if the current word to consume is `else`?
  A. We can either ***shift*** (then expecting to match another *Stmt*) or ***reduce*** to a *Stmt*.
  Action[13, `else`] cannot hold ***shift*** and ***reduce*** simultaneously.
  ⇒ This is known as the *shift-reduce conflict*.

- Consider another scenario:

$$cc_i = \left\{ \begin{array}{l} [A \rightarrow \gamma\delta\bullet, \texttt{a}], \\ [B \rightarrow \gamma\delta\bullet, \texttt{a}] \end{array} \right\}$$

  Q. What does it mean if the current word to consume is `a`?
  A. We can either ***reduce*** to A or ***reduce*** to B.
  Action[i, `a`] cannot hold A and B simultaneously.
  ⇒ This is known as the *reduce-reduce conflict*.

Index (1)

Parser in Context

Context-Free Languages: Introduction

CFG: Example (1.1)

CFG: Example (1.2)

CFG: Example (1.2)

CFG: Example (2)

CFG: Example (3)

CFG: Example (4)

CFG: Example (5.1) Version 1

CFG: Example (5.2) Version 1

CFG: Example (5.3) Version 1

Index (2)

CFG: Example (5.4) Version 1

CFG: Example (5.5) Version 2

CFG: Example (5.6) Version 2

CFG: Example (5.7) Version 2

CFG: Formal Definition (1)

CFG: Formal Definition (2): Example

CFG: Formal Definition (3): Example

Regular Expressions to CFG's

DFA to CFG's

CFG: Leftmost Derivations (1)

CFG: Rightmost Derivations (1)

Index (3)

CFG: Leftmost Derivations (2)

CFG: Rightmost Derivations (2)

CFG: Parse Trees vs. Derivations (1)

CFG: Parse Trees vs. Derivations (2)

CFG: Ambiguity: Definition

CFG: Ambiguity: Exercise (1)

CFG: Ambiguity: Exercise (2.1)

CFG: Ambiguity: Exercise (2.2)

CFG: Ambiguity: Exercise (2.3)

Discovering Derivations

TDP: Discovering Leftmost Derivation

Index (4)

TDP: Exercise (1)

TDP: Exercise (2)

Left-Recursions (LF): Direct vs. Indirect

TDP: (Preventively) Eliminating LRs

CFG: Eliminating ϵ-Productions (1)

CFG: Eliminating ϵ-Productions (2)

Backtrack-Free Parsing (1)

The first Set: Definition

The first Set: Examples

Computing the first Set

Computing the first Set: Extension

Index (5)

Extended first Set: Examples
Is the first Set Sufficient?
The follow Set: Examples
Computing the follow Set
Backtrack-Free Grammar
TDP: Lookahead with One Symbol
Backtrack-Free Grammar: Exercise
Backtrack-Free Grammar: Left-Factoring
Left-Factoring: Exercise
TDP: Terminating and Backtrack-Free
Backtrack-Free Parsing (2.1)

Index (6)

Backtrack-Free Parsing (2.2)
LL(1) Parser: Exercise
BUP: Discovering Rightmost Derivation
BUP: Discovering Rightmost Derivation (1)
BUP: Discovering Rightmost Derivation (2)
BUP: Example Tracing (1)
BUP: Example Tracing (2.1)
BUP: Example Tracing (2.2)
BUP: Example Tracing (2.3)
LR(1) Items: Definition
LR(1) Items: Scenarios

Index (7)

LR(1) Items: Example (1.1)
LR(1) Items: Example (1.2)
LR(1) Items: Example (1.3)
LR(1) Items: Example (2)
Canonical Collection ($\mathcal{CC}$) vs. LR(1) items
Constructing $\mathcal{CC}$: The *closure* Procedure (1)
Constructing $\mathcal{CC}$: The *closure* Procedure (2.1)
Constructing $\mathcal{CC}$: The *goto* Procedure (1)
Constructing $\mathcal{CC}$: The *goto* Procedure (2)
Constructing $\mathcal{CC}$: The Algorithm (1)
Constructing $\mathcal{CC}$: The Algorithm (2.1)

Index (8)

Constructing $\mathcal{CC}$: The Algorithm (2.2)
Constructing $\mathcal{CC}$: The Algorithm (2.3)
Constructing $\mathcal{CC}$: The Algorithm (2.4)
Constructing *Action* and *Goto* Tables (1)
Constructing *Action* and *Goto* Tables (2)
BUP: Discovering Ambiguity (1)
BUP: Discovering Ambiguity (2.1)
BUP: Discovering Ambiguity (2.2.1)
BUP: Discovering Ambiguity (2.2.2)
BUP: Discovering Ambiguity (3)