

This exam contains 5 pages (including this cover page) and 5 problems.

**Check to see if any pages are missing.**

**Do not detach any question pages from the booklet.**

Enter **all** requested information on the top of this page before you start the exam, and put your **initials** on the top of every page, in case the pages become separated.

Attempt **all** questions. Answer each question in the boxed space provided.

The following rules apply:

- **NO QUESTIONS DURING THE EXAM.**
- **If a question is ambiguous or unclear, then please write your assumptions and proceed to answer the question.**
- Only writings within the designated answer boxes will be graded. Plan your answers on the sketch paper provided.
- **Write in valid Rodin ASCII syntax** wherever required.
- Where descriptive answers are requested, use complete sentences and paragraphs. Be precise and concise.
- In writing a sequent proof, only one inference rule can be applied at a time.
- Whenever the **ARI** inference rule is used, justify in writing its use.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive credit.** A correct answer, unsupported by calculations or explanation will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- All answers must appear in the boxed areas in this booklet.

Do not write in this table which contains your raw mark scores.

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	20	
Total:	60	

1. Given a model (with static and dynamic parts), what are the factors determining the number sequents generated for invariant preservation?

**Solution:**

- Number of (old and new) events
- Number of invariant conditions

[      of 10 marks]

2. Justify whether or not the following statement is true:

*A partial function is always a total function.*

**Solution:**

- The statement is false.
- A partial function  $f \in S \leftrightarrow T$  may have its domain  $\text{dom}(f) \subset S$ , which violates the requirement of a function being total (e.g.,  $\text{dom}(f) = S$ ).

[      of 10 marks]

3. Can the left sequent below be transformed to the two right sequents via **OR.L**?

$$\begin{array}{ccc}
 \boxed{a + 1 > 5 \vee a + 1 = 5} & ?? & \boxed{a > 0} \\
 \vdash & & \vdash \\
 \boxed{a > 0} & & \boxed{a + 1 > 5} \\
 & & \\
 & & \boxed{a > 0} \\
 & & \vdash \\
 & & \boxed{a + 1 = 5}
 \end{array}$$

**Solution:**

- No.
- By applying **OR.L**, the two disjuncts  $a + 1 > 5$  and  $a + 1 = 5$  should appear as separate antecedents, not separate goals. Also, the goal  $a > 0$  should not be transformed to a hypothesis.

[      of 10 marks]

4. Consider the following action which intends to update the balance function  $b \in \text{ACCOUNT} \rightarrow \mathbb{Z}$ :

$$b(a) := b(a) + v$$

In valid Rodin ASCII syntax, rewrite the right-hand side of “becomes” operator using set and/or relational operators.

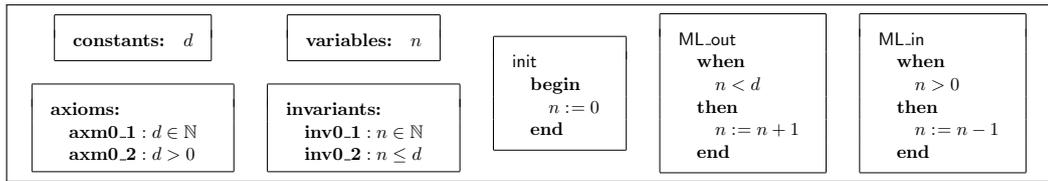
**Solution:**

- Acceptable answer 1:  $\mathbf{a} \mid\text{-}\> \mathbf{b(a)} + \mathbf{v} \quad (\{\mathbf{a}\} \ll\mid \mathbf{b})$
- Acceptable answer 2:  $\mathbf{b} \text{ <+ } \{\mathbf{a} \mid\text{-}\> \mathbf{b(a)} + \mathbf{v}\}$

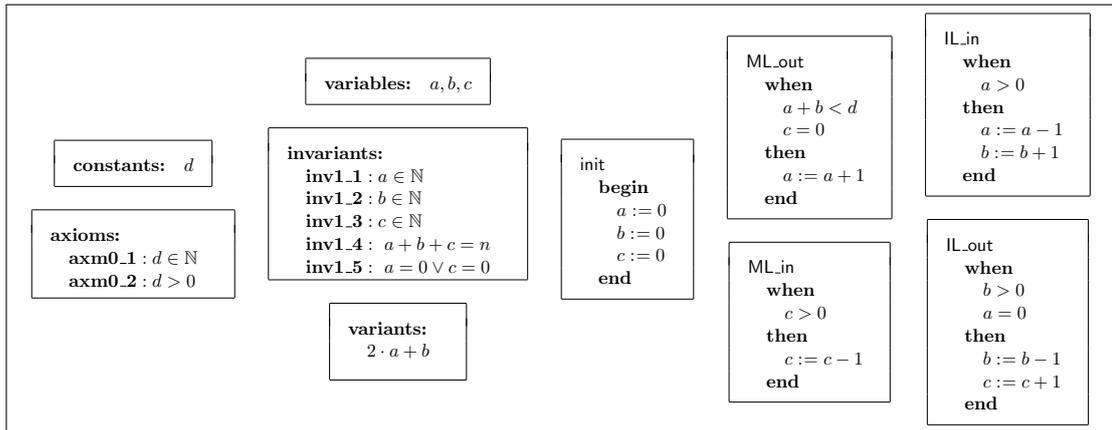
[      of 10 marks]

5. Consider the following models of the bridge controller system:

**m0: Initial Model**



**m1: First Refinement**



Formulate and prove **ML\_in/GRD**.

**Solution:**

Proof tree for **ML\_in/GRD**:

- Root node (Axioms):
  - $d \in \mathbb{N}$
  - $d > 0$
  - $n \in \mathbb{N}$
  - $n \leq d$
  - $a \in \mathbb{N}$
  - $b \in \mathbb{N}$
  - $c \in \mathbb{N}$
  - $a + b + c = n$
  - $a = 0 \vee c = 0$
  - $c > 0$
  - $\top$
  - $n > 0$
- Rule **MON** applied to root node yields:
  - $b \in \mathbb{N}$
  - $a + b + c = n$
  - $a = 0 \vee c = 0$
  - $c > 0$
  - $\top$
  - $n > 0$
- Rule **OR.L** applied to the previous node yields two branches:
  - Top branch:
    - $b \in \mathbb{N}$
    - $a + b + c = n$
    - $a = 0$
    - $c > 0$
    - $\top$
    - $n > 0$
  - Bottom branch:
    - $b \in \mathbb{N}$
    - $a + b + c = n$
    - $c = 0$
    - $c > 0$
    - $\top$
    - $n > 0$
- From the top branch, rule **EQ\_LR, MON** yields:
  - $b \in \mathbb{N}$
  - $0 + b + c = n$
  - $c > 0$
  - $\top$
  - $n > 0$
- From the bottom branch, rule **EQ\_LR** yields:
  - $b \in \mathbb{N}$
  - $a + b + 0 = n$
  - $c = 0$
  - $0 > 0$
  - $\top$
  - $n > 0$
- From the top node of the EQ\_LR, MON branch, rule **ARI** yields:
  - $b \in \mathbb{N}$
  - $b + c = n$
  - $c > 0$
  - $\top$
  - $n > 0$
- From the top node of the ARI branch, rule **ARI** yields:
  - $c \leq n$
  - $c > 0$
  - $n > 0$
  - $\top$
- From the top node of the second ARI branch, rule **ARI** yields:
  - $n > 0$
  - $\top$
  - $n > 0$
- From the top node of the ARI branch, rule **HYP** yields:
  - $n > 0$
  - $\top$
  - $n > 0$
- From the bottom node of the EQ\_LR branch, rule **EQ\_LR, MON** yields:
  - $0 > 0$
  - $\top$
  - $n > 0$
- From the top node of the EQ\_LR, MON branch, rule **ARI** yields:
  - $\perp$
  - $\top$
  - $n > 0$
- From the top node of the ARI branch, rule **FALSE.L** yields:
  - $\perp$
  - $\top$
  - $n > 0$

It is also expected that each application of **ARI** is justified.

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