Overview of Compilation

Readings: EAC2 Chapter 1

EECS4302 A:
Compilers and Interpreters
Fall 2022

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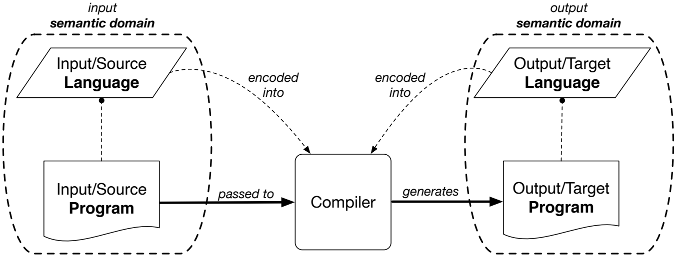
What is a Compiler? (1)

A **software system** that **automatically** ***translates/transforms*** ***input/source*** programs (written in one language) to ***output/target*** programs (written in another language).



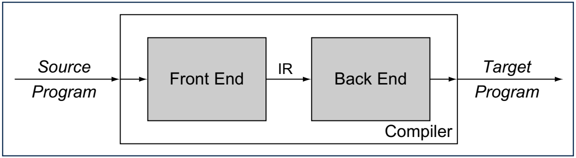
- *Semantic Domain* : Context with its own vocabulary & meanings
  e.g., OO (EECS1022/2030/2011), database (3421), predicates (1090)
- ***Source*** and ***target*** may be in **different** ***semantic domains***.
  e.g., Java programs to SQL relational database schemas/queries
  e.g., C procedural programs to MISP assembly instructions

What is a Compiler? (2)

- The idea about a ***compiler*** is extremely powerful:
  You can turn anything to anything else,
  as long as the following are ***clear*** about these two things:
  - SYNTAX [***specifiable*** as CFGs]
  - SEMANTICS [***programmable*** as mapping functions]

  Mental Exercise. Let's consider an A+ challenge.
- A compiler should be constructed with good *SE principles*.
  - ***Modularity***
    [interacting components]
  - ***Information Hiding***
    [hiding unstable, revealing stable]
  - ***Single Choice Principle***
    [a change only causing minimum impact]
  - ***Design Patterns***
    [polymorphism & dynamic binding]
  - ***Regression Testing***
    [e.g., unit-level, acceptance-level]

Compiler: Typical Infrastructure (1)



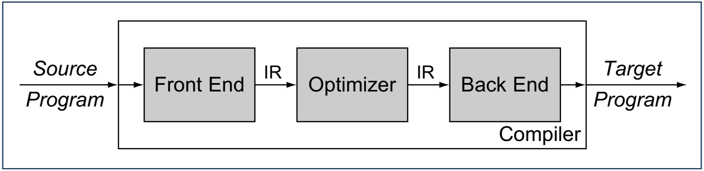
- **FRON END**:
  - Encodes: knowledge of the **source** language
  - Transforms: from the **source** to some ***IR*** (*intermediate representation*)
  - Principle: ***meaning*** of the source must be *preserved* in the ***IR***.
- **BACK END**:
  - Encodes knowledge of the **target** language
  - Transforms: from the ***IR*** to the **target**
  - Principle: ***meaning*** of the ***IR*** must be *reflected* in the **target**.

Q. How many ***IRs*** needed for building a number of compilers:
JAVA-TO-C, C#-TO-C, JAVA-TO-PYTHON, C#-TO-PYTHON?

A. Two ***IRs*** suffice: One for ***OO***; one for ***procedural***.

⇒ IR should be as *language-independent* as possible.

Compiler: Typical Infrastructure (2)



OPTIMIZER:

- An ***IR**-to-**IR*** transformer that aims at "improving" the **output** of front end, before passing it as **input** of the back end.
- Think of this transformer as attempting to discover an "***optimal***" solution to some computational problem.
  e.g., runtime performance, static design

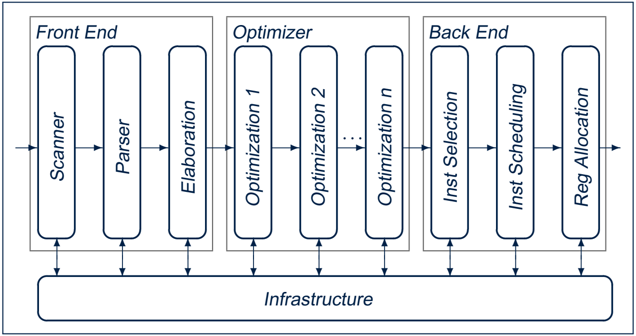
Q. Behaviour of the **target** program depends upon?

1. ***Meaning*** of the **source** preserved in ***IR***?
2. ***IR**-to-**IR*** transformation of the optimizer ***semantics-preserving***?
3. ***Meaning*** of ***IR*** preserved in the generated **target**?

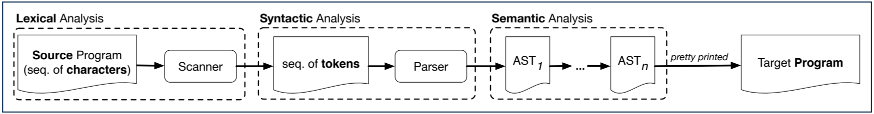
   (1) – (3) necessary & sufficient for the *soundness* of a compiler.

Example Compiler 1

- Consider a <u>conventional</u> compiler which turns a ***C-like program*** into executable ***machine instructions***.
- The ***source*** and ***target*** are at different levels of *abstractions*:
  - C-like program is like "high-level" ***specification***.
  - Macine instructions are the low-level, efficient ***implementation***.



Compiler Infrastructure: Scanner vs. Parser vs. Optimizer



- The same input program may be perceived differently:
  1. As a ***character sequence*** [subject to ***lexical*** analysis]
  2. As a ***token sequence*** [subject to ***syntactic*** analysis]
  3. As a *abstract syntax tree (AST)* [subject to *semantic* analysis]
- *(1)* & *(2)* are routine tasks of lexical/grammar rule specification.
- *(3)* is where the most creativity is used to a compiler:

  A series of ***semantics-preserving*** ***AST***-to-***AST*** transformations.

Compiler Infrastructure: Scanner

- The source program is perceived as a sequence of ***characters***.
- A scanner performs *lexical analysis* on the input character sequence and produces a sequence of ***tokens***.
- ANALOGY: Tokens are like individual ***words*** in an essay.

  $\Rightarrow$ Invalid tokens $\approx$ Misspelt words

  e.g., a token for a useless delimiter: e.g., space, tab, new line

  e.g., a token for a useful delimiter: e.g., `(`, `)`, `{`, `}`, `,`

  e.g., a token for an identifier (for e.g., a variable, a function)

  e.g., a token for a keyword (e.g,. `int`, `char`, `if`, `for`, `while`)

  e.g., a token for a number (for e.g., `1.23`, `2.46`)

  Q. How to specify such ***pattern of characters***?

  A. ***Regular Expressions*** (***REs***)

  e.g., RE for keyword `while` [`while`]

  e.g., RE for an identifier [`[a-zA-Z][a-zA-Z0-9_]*`]

  e.g., RE for a white space [`[ \t\r]+`]

Compiler Infrastructure: Parser

- A parser's input is a sequence of ***tokens*** (by some scanner).
- A parser performs *syntactic analysis* on the input token sequence and produces an ***abstract syntax tree (AST)***.
- ANALOGY: ASTs are like individual ***sentences*** in an essay.

  ⇒ Tokens not ***parseable*** into a valid AST ≈ Grammatical errors

  Q. An essay with no speling and grammatical errors good enough?
  A. No, it may talk about non-sense (sentences in wrong contexts).

  ⇒ An input program with no lexical/syntactic errors should still be subject to *semantic analysis* (e.g., type checking, code optimization).

Q.: How to specify such ***pattern of tokens***?
A.: ***Context-Free Grammars*** (***CFGs***)

e.g., CFG (with **terminals** and **non-terminals**) for a while-loop:

WhileLoop	::=	`WHILE` `LPAREN` *BoolExpr* `RPAREN` `LCBRAC` *Impl* `RCBRAC`
Impl	::=	
	|	*Instruction* `SEMICOL` *Impl*

Compiler Infrastructure: Optimizer (1)

- Consider an input *AST* which has the pretty printing:

```
b := ... ; c := ... ; a := ...
across i |..| n is i
  loop
    read d
    a := a * 2 * b * c * d
  end
```

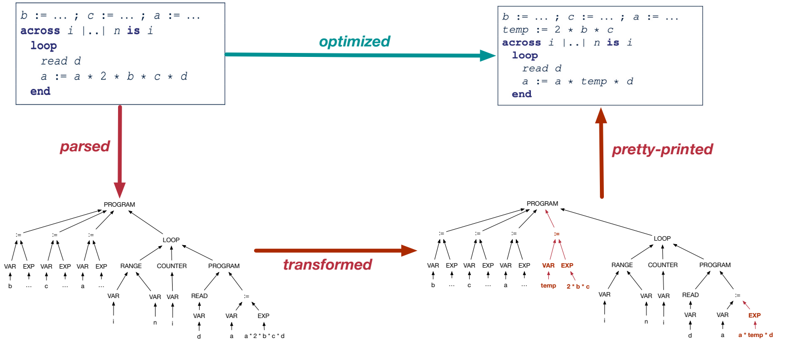
  Q. AST of above program ***optimized*** for performance?
  A. No ∵ values of `2`, `b`, `c` stay invariant within the loop.

- An ***optimizer*** may *transform* AST like above into:

```
b := ... ; c := ... ; a := ...
temp := 2 * b * c
across i |..| n is i
  loop
    read d
    a := a * temp * d
  end
```

Compiler Infrastructure: Optimizer (2)

Problem: Given a user-written program, ***optimize*** it for best runtime performance.



Example Compiler 2

- Consider a compiler which turns an object-based ***Domain-Specific Language (DSL)*** into a **SQL *database***.
- Why is an ***object-to-relational compiler*** valuable?

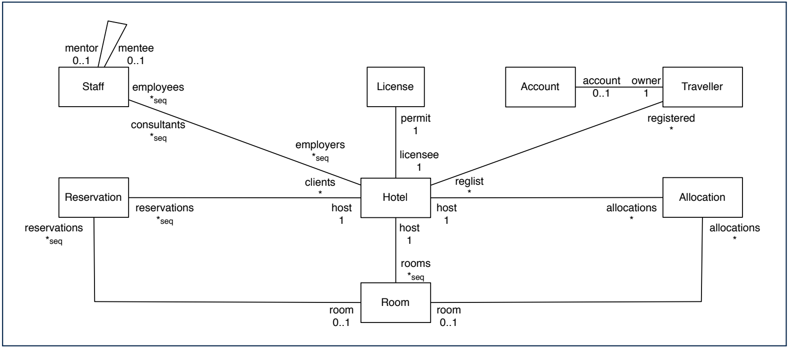
  Hint. Which semantic domain is better for high-level specification?
  Hint. Which semantic domain is better for data management?

	managing big data	specifying data & updates
object-oriented environment	×	✓
relational database	✓	×

- *Challenge* : ***Object-Relational Impedance Mismatch***

Example Compiler 2

- The input/source contains 2 parts:
  - **DATA MODEL**: ***classes*** & ***associations***
    e.g., data model of a Hotel Reservation System:



  - **BEHAVIOURAL MODEL**: update methods specified as ***predicates***

Example Compiler 2: Transforming Data

```
class A {
  attributes
    s: string
    bs: set(B . a) [*] }
```

```
class B {
  attributes
    is: set (int)
    a: A . bs }
```

- Each class is turned into a ***class table***:
  - Column `oid` stores the object reference. [PRIMARY KEY]
  - Implementation strategy for attributes:

	SINGLE-VALUED	MULTI-VALUED
PRIMITIVE-TYPED	column in ***class table***	***collection table***
REFERENCE-TYPED	***association table***	

- Each ***collection table***:
  - Column `oid` stores the context object.
  - 1 column stores the corresponding primitive value or `oid`.
- Each ***association table***:
  - Column `oid` stores the association reference.
  - 2 columns store `oid`'s of both association ends. [FOREIGN KEY]

Example Compiler 2: Input/Source

- Consider a **valid** input/source program:

```
class Account {
  attributes
    owner: Traveller . account
    balance: int
}
```

```
class Traveller {
  attributes
    name: string
    reglist: set(Hotel . registered)[*]
}
```

```
class Hotel {
  attributes
    name: string
    registered: set(Traveller . reglist)[*]
  methods
    register {
        t? : extent(Traveller)
      & t? /: registered
      ==>
          registered := registered \/ {t?}
      || t?.reglist := t?.reglist \/ {this}
    }
}
```

- How do you specify the ***scanner*** and ***parser*** accordingly?

Example Compiler 2: Output/Target

- Class ***associations*** are transformed to ***database schemas***.

```
CREATE TABLE `Account`(
  `oid` INTEGER AUTO_INCREMENT,`balance` INTEGER,
  PRIMARY KEY (`oid`));
CREATE TABLE `Traveller`(
  `oid` INTEGER AUTO_INCREMENT,`name` CHAR(30),
  PRIMARY KEY (`oid`));
CREATE TABLE `Hotel`(
  `oid` INTEGER AUTO_INCREMENT,`name` CHAR(30),
  PRIMARY KEY (`oid`));
CREATE TABLE `Account_owner_Traveller_account`(
  `oid` INTEGER AUTO_INCREMENT, `owner` INTEGER, `account` INTEGER,
  PRIMARY KEY (`oid`));
CREATE TABLE `Traveller_reglist_Hotel_registered`(
  `oid` INTEGER AUTO_INCREMENT, `reglist` INTEGER, `registered` INTEGER,
  PRIMARY KEY (`oid`));
```

- Method ***predicates*** are compiled into ***stored procedures***.

```
CREATE PROCEDURE `Hotel_register`(IN `this?` INTEGER, IN `t?` INTEGER)
  BEGIN
    ...
  END
```

Example Compiler 2: Transforming Updates

Challenge: Transform ***dot notations*** into ***relational queries***.

e.g., The AST corresponding to the following dot notation
(in the context of class `Account`,
retrieving the owner's list of registrations)

```
this.owner.reglist
```

may be transformed into the following (nested) table lookups:

```
SELECT (VAR `reglist`)
       (TABLE `Hotel_registered_Traveller_reglist`)
       (VAR `registered` = (SELECT (VAR `owner`)
                                  (TABLE `Account_owner_Traveller_account`)
                                  (VAR `owner` = VAR `this`)))
```

Beyond this lecture . . .

- Read Chapter 1 of EAC2 to find out more about Example Compiler 1
- Read this paper to find out more about Example Compiler 2:

  http://dx.doi.org/10.4204/EPTCS.105.8

Index (1)

Index (2)

Scanner: Lexical Analysis

Readings: EAC2 Chapter 2

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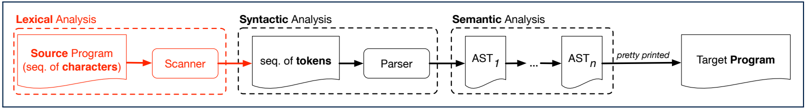
Scanner in Context

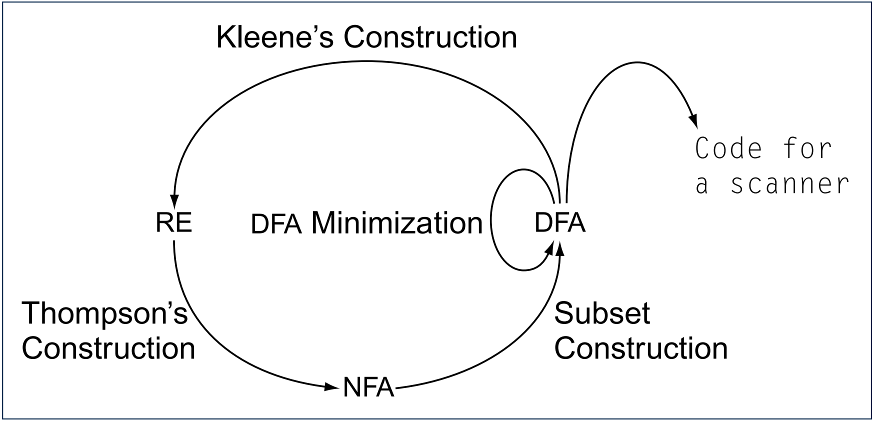
- Recall:



- Treats the input programas as a ***a sequence of characters***
- Applies rules ***recognizing*** character sequences as ***tokens***

  [***lexical*** analysis]
- Upon termination:
  - Reports character sequences not recognizable as tokens
  - Produces a ***a sequence of tokens***
- Only part of compiler touching ***every character*** in input program.
- Tokens *recognizable* by scanner constitute a *regular language*.

Scanner: Formulation & Implementation



Alphabets

An *alphabet* is a ***finite***, ***nonempty*** set of symbols.

- The convention is to write Σ, possibly with a informative subscript, to denote the alphabet in question.
- Use either a ***set enumeration*** or a ***set comprehension*** to define your own alphabet.

  e.g., $\Sigma_{eng} = \{a, b, \ldots, z, A, B, \ldots, Z\}$ [the English alphabet]

  e.g., $\Sigma_{bin} = \{0, 1\}$ [the binary alphabet]

  e.g., $\Sigma_{dec} = \{d \mid 0 \leq d \leq 9\}$ [the decimal alphabet]

  e.g., Σ_{key} [the keyboard alphabet]

Strings (1)

- A *string* or a *word* is ***finite*** sequence of symbols chosen from some ***alphabet***.

  e.g., Oxford is a string over the English alphabet Σ_{eng}

  e.g., 01010 is a string over the binary alphabet Σ_{bin}

  e.g., 01010.01 is **not** a string over Σ_{bin}

  e.g., 57 is a string over the decimal alphabet Σ_{dec}
- It is **not** correct to say, e.g., $01010 \in \Sigma_{bin}$ [Why?]
- The *length* of a string w, denoted as $|w|$, is the number of characters it contains.
  - e.g., $|Oxford| = 6$
  - ϵ is the ***empty string*** ($|\epsilon| = 0$) that may be from any alphabet.
- Given two strings x and y, their *concatenation*, denoted as xy, is a new string formed by a copy of x followed by a copy of y.
  - e.g., Let $x = 01101$ and $y = 110$, then $xy = 01101110$
  - The empty string ϵ is the *identity for concatenation*:

    $\epsilon w = w = w\epsilon$ for any string w

Strings (2)

- Given an ***alphabet*** Σ, we write Σ^k, where $k \in \mathbb{N}$, to denote the ***set of strings of length k from*** Σ

$$\Sigma^k = \{w \mid \underbrace{w \text{ is a string over } \Sigma}_{\textit{more formal?}} \wedge |w| = k\}$$

  - e.g., $\{0,1\}^2 = \{00, 01, 10, 11\}$
  - Given Σ, Σ^0 is $\{\epsilon\}$
- Given Σ, Σ^+ is the ***set of nonempty strings***.

$$\Sigma^+ = \Sigma^1 \cup \Sigma^2 \cup \Sigma^3 \cup \ldots = \{w \mid w \in \Sigma^k \wedge k > 0\} = \bigcup_{k>0} \Sigma^k$$

- Given Σ, Σ^* is the ***set of strings of all possible lengths***.

$$\Sigma^* = \Sigma^+ \cup \{\epsilon\}$$

Review Exercises: Strings

1. What is $|\{a, b, \ldots, z\}^5|$?
2. Enumerate, in a systematic manner, the set $\{a, b, c\}^4$.
3. Explain the difference between Σ and Σ^1.
4. Prove or disprove: $\Sigma_1 \subseteq \Sigma_2 \Rightarrow \Sigma_1^* \subseteq \Sigma_2^*$

Languages

- *A language L over Σ* (where $|\Sigma|$ is finite) is a set of strings s.t.

$$L \subseteq \Sigma^*$$

- When useful, include an informative subscript to denote the ***language*** L in question.
  - e.g., The language of ***compilable*** Java programs

$$L_{Java} = \{prog \mid prog \in \Sigma_{key}^* \land prog \texttt{ compiles in Eclipse}\}$$

    Note. *prog* compiling means **no** ***lexical***, ***syntactical***, or ***type*** errors.

  - e.g., The language of strings with n 0's followed by n 1's ($n \geq 0$)

$$\{\epsilon, 01, 0011, 000111, \dots\} = \{0^n 1^n \mid n \geq 0\}$$

  - e.g., The language of strings with an equal number of 0's and 1's

$$\begin{array}{ll} & \{\epsilon, 01, 10, 0011, 0101, 0110, 1100, 1010, 1001, \dots\} \\ = & \{w \mid \texttt{\# of 0's in } w = \texttt{\# of 1's in } w\} \end{array}$$

Review Exercises: Languages

1. Use ***set comprehensions*** to define the following ***languages***. Be as ***formal*** as possible.
   - A language over $\{0, 1\}$ consisting of strings beginning with some 0's (possibly none) followed by at least as many 1's.
   - A language over $\{a, b, c\}$ consisting of strings beginning with some a's (possibly none), followed by some b's and then some c's, s.t. the # of a's is at least as many as the sum of #'s of b's and c's.
2. Explain the difference between the two languages $\{\epsilon\}$ and $\varnothing$.
3. Justify that Σ^*, $\varnothing$, and $\{\epsilon\}$ are all languages over Σ.
4. Prove or disprove: If L is a language over Σ, and $\Sigma_2 \supseteq \Sigma$, then L is also a language over Σ_2.
   Hint: Prove that $\Sigma \subseteq \Sigma_2 \land L \subseteq \Sigma^* \Rightarrow L \subseteq \Sigma_2^*$
5. Prove or disprove: If L is a language over Σ, and $\Sigma_2 \subseteq \Sigma$, then L is also a language over Σ_2.
   Hint: Prove that $\Sigma_2 \subseteq \Sigma \land L \subseteq \Sigma^* \Rightarrow L \subseteq \Sigma_2^*$

Problems

- Given a ***language*** L over some ***alphabet*** Σ, a *problem* is the ***decision*** on whether or not a given ***string*** w is a member of L.

$$w \in L$$

  Is this equivalent to deciding $w \in \Sigma^*$? [***No***]

  $w \in \Sigma^* \Rightarrow w \in L$ is **not** necessarily true.

- e.g., The Java compiler solves the problem of ***deciding*** if a user-supplied ***string of symbols*** is a **member** of L_{Java}.

Regular Expressions (RE): Introduction

- *Regular expressions* (RegExp's) are:
  - A type of language-defining notation
    - This is ***similar*** to the equally-expressive ***DFA***, ***NFA***, and ϵ-***NFA***.
  - *Textual* and look just like a programming language
    - e.g., Set of strings denoted by $\texttt{01}^* + \texttt{10}^*$? [specify formally]
      $L = \{0x \mid x \in \{1\}^*\} \cup \{1x \mid x \in \{0\}^*\}$
    - e.g., Set of strings denoted by $\texttt{(0}^*\texttt{10}^*\texttt{10}^*\texttt{)}^*\texttt{10}^*$?
      $L = \{w \mid w \texttt{ has odd \# of } 1\texttt{'s}\}$
    - This is ***dissimilar*** to the diagrammatic ***DFA***, ***NFA***, and ϵ-***NFA***.
    - RegExp's can be considered as a "user-friendly" alternative to ***NFA*** for describing software components. [e.g., text search]
    - Writing a RegExp is like writing an algebraic expression, using the defined operators, e.g., `((4 + 3) * 5) % 6`
- Despite the programming convenience they provide, ***RegExp's***, ***DFA***, ***NFA***, and ϵ-***NFA*** are all *provably equivalent*.
  - They are capable of defining **all** and **only** regular languages.

RE: Language Operations (1)

- Given Σ of input alphabets, the simplest RegExp is? [$s \in \Sigma^1$]
  - e.g., Given $\Sigma = \{a, b, c\}$, expression a denotes the language $\{ a \}$ consisting of a single string a.
- Given two languages $L, M \in \Sigma^*$, there are 3 operators for building a *larger language* out of them:

1. ***Union***

$$L \cup M = \{w \mid w \in L \vee w \in M\}$$

   In the textual form, we write $+$ for union.

2. ***Concatenation***

$$LM = \{xy \mid x \in L \wedge y \in M\}$$

   In the textual form, we write either . or nothing at all for concatenation.

RE: Language Operations (2)

3. ***Kleene Closure*** (or ***Kleene Star***)

$$L^* = \bigcup_{i \geq 0} L^i$$

where

$$\begin{array}{lcl}
L^0 & = & \{\epsilon\} \\
L^1 & = & L \\
L^2 & = & \{x_1 x_2 \mid x_1 \in L \wedge x_2 \in L\} \\
\ldots & & \\
L^i & = & \{ \underbrace{x_1 x_2 \ldots x_i}_{i \texttt{ concatenations}} \mid x_j \in L \wedge 1 \leq j \leq i\} \\
\ldots & &
\end{array}$$

In the textual form, we write $\star$ for closure.

Question: What is $|L^i|$ ($i \in \mathbb{N}$)? [$|L|^i$]

Question: Given that $L = \{0\}^*$, what is L^*? [L]

RE: Construction (1)

We may build *regular expressions* ***recursively***:

- Each (***basic*** or ***recursive***) form of regular expressions denotes a ***language*** (i.e., a set of strings that it accepts).
- ***Base Case***:
  - Constants ϵ and $\varnothing$ are regular expressions.

$$L(\ \epsilon\ ) = \{\epsilon\}$$
$$L(\ \varnothing\ ) = \varnothing$$

  - An input symbol $a \in \Sigma$ is a regular expression.

$$L(\ \mathbf{a}\ ) = \{a\}$$

    If we want a regular expression for the language consisting of only the string $w \in \Sigma^*$, we write w as the regular expression.
  - Variables such as **L**, **M**, *etc.*, might also denote languages.

RE: Construction (2)

- ***Recursive Case***: Given that E and F are regular expressions:
  - The union $E + F$ is a regular expression.

$$L(\ \mathbf{E+F}\ ) = L(E) \cup L(F)$$

  - The concatenation EF is a regular expression.

$$L(\ \mathbf{EF}\ ) = L(E)L(F)$$

  - Kleene closure of E is a regular expression.

$$L(\ E^*\ ) = (L(E))^*$$

  - A parenthesized E is a regular expression.

$$L(\ \mathbf{(E)}\ ) = L(E)$$

RE: Construction (3)

Exercises:

- $\varnothing + L$ $\qquad$ [$\varnothing + L = L = \varnothing + L$]
- $\varnothing L$ $\qquad$ [$\varnothing L = \varnothing = L \varnothing$]
- $\varnothing^*$

$$\begin{aligned} \varnothing^* &= \varnothing^0 \cup \varnothing^1 \cup \varnothing^2 \cup \ldots \\ &= \{\epsilon\} \cup \varnothing \cup \varnothing \cup \ldots \\ &= \{\epsilon\} \end{aligned}$$

- $\varnothing^* L$ $\qquad$ [$\varnothing^* L = L = L \varnothing^*$]

RE: Construction (4)

Write a regular expression for the following language

$$\{ w \mid w \texttt{ has alternating } 0\texttt{'s and } 1\texttt{'s} \}$$

- Would $(01)^*$ work? [alternating 10's?]
- Would $(01)^* + (10)^*$ work? [starting and ending with 1?]
- $0(10)^* + (01)^* + (10)^* + 1(01)^*$
- It seems that:
  - 1st and 3rd terms have $(10)^*$ as the common factor.
  - 2nd and 4th terms have $(01)^*$ as the common factor.
- Can we simplify the above regular expression?
- $(\epsilon + 0)(10)^* + (\epsilon + 1)(01)^*$

RE: Review Exercises

Write the regular expressions to describe the following languages:

- $\{ w \mid w$ ends with $01 \}$
- $\{ w \mid w$ contains 01 as a substring $\}$
- $\{ w \mid w$ contains no more than three consecutive 1's $\}$
- $\{ w \mid w$ ends with $01 \lor w$ has an odd # of 0's $\}$
- $$\left\{ sx.y \;\middle|\; \begin{array}{ll} & s \in \{+, -, \epsilon\} \\ \land & x \in \Sigma_{dec}^{*} \\ \land & y \in \Sigma_{dec}^{*} \\ \land & \neg(x = \epsilon \land y = \epsilon) \end{array} \right\}$$
- $$\left\{ xy \;\middle|\; \begin{array}{ll} & x \in \{0,1\}^{*} \land y \in \{0,1\}^{*} \\ \land & x \text{ has alternating } 0\text{'s and } 1\text{'s} \\ \land & y \text{ has an odd \# } 0\text{'s and an odd \# } 1\text{'s} \end{array} \right\}$$

RE: Operator Precedence

- In an order of ***decreasing precedence***:
  - Kleene star operator
  - Concatenation operator
  - Union operator
- When necessary, use ***parentheses*** to force the intended order of evaluation.
- e.g.,
  - 10^* vs. $(10)^*$ [10^* is equivalent to $1(0^*)$]
  - $01^* + 1$ vs. $0(1^* + 1)$ [$01^* + 1$ is equivalent to $(0(1^*)) + (1)$]
  - $0 + 1^*$ vs. $(0 + 1)^*$ [$0 + 1^*$ is equivalent to $(0) + (1^*)$]

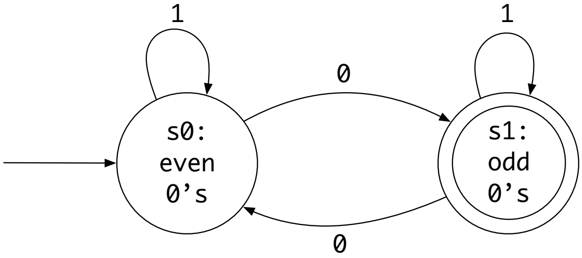
DFA: Deterministic Finite Automata (1.1)

- A *deterministic finite automata (DFA)* is a ***finite state machine*** (***FSM***) that ***accepts*** (or ***recognizes***) a pattern of behaviour.
  - For ***lexical*** analysis, we study patterns of ***strings*** (i.e., how ***alphabet*** symbols are ordered).
  - Unless otherwise specified, we consider strings in $\{0, 1\}^*$
  - Each pattern contains the set of satisfying strings.
  - We describe the patterns of strings using set comprehensions:
    - $\{ w \mid w \texttt{ has an odd number of 0's} \}$
    - $\{ w \mid w \texttt{ has an even number of 1's} \}$
    - $\left\{ w \mid \begin{array}{ll} & w \neq \epsilon \\ \land & w \texttt{ has equal \# of alternating 0's and 1's} \end{array} \right\}$
    - $\{ w \mid w \texttt{ contains } 01 \texttt{ as a substring} \}$
    - $\left\{ w \mid \begin{array}{ll} & w \texttt{ has an even number of 0's} \\ \land & w \texttt{ has an odd number of 1's} \end{array} \right\}$
- Given a pattern description, we design a ***DFA*** that accepts it.
  - The resulting ***DFA*** can be transformed into an executable program.

DFA: Deterministic Finite Automata (1.2)

- The ***transition diagram*** below defines a DFA which ***accepts***/***recognizes*** exactly the language

$\{ w \mid w$ `has an odd number of 0's` $\}$

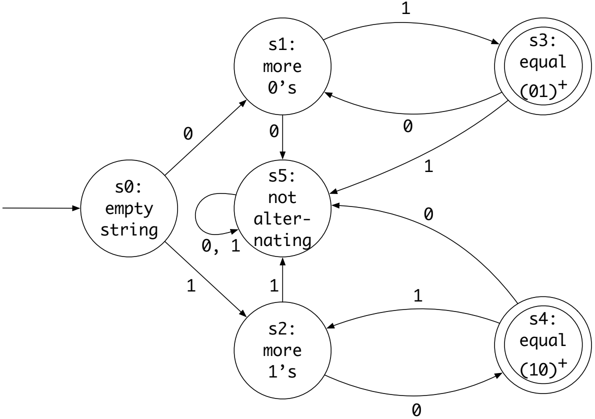


- Each **incoming** or **outgoing** arc (called a ***transition***) corresponds to an input alphabet symbol.
- s_0 with an unlabelled **incoming** transition is the ***start state***.
- s_3 drawn as a double circle is a ***final state***.
- All states have **outgoing** transitions covering $\{0, 1\}$.

DFA: Deterministic Finite Automata (1.3)

The ***transition diagram*** below defines a DFA which ***accepts***/***recognizes*** exactly the language

$$\left\{ w \;\middle|\; \begin{array}{ll} & w \neq \epsilon \\ \wedge & w \texttt{ has equal \# of alternating 0's and 1's} \end{array} \right\}$$



Review Exercises: Drawing DFAs

Draw the transition diagrams for DFAs which accept other example string patterns:

- $\{ w \mid w \texttt{ has an even number of 1's} \}$
- $\{ w \mid w \texttt{ contains } 01 \texttt{ as a substring} \}$
- $\left\{ w \;\middle|\; \begin{array}{l} \quad w \texttt{ has an even number of 0's} \\ \wedge \;\; w \texttt{ has an odd number of 1's} \end{array} \right\}$

DFA: Deterministic Finite Automata (2.1)

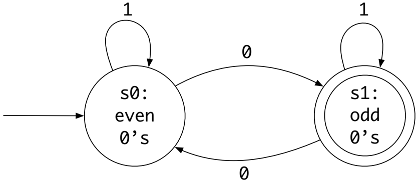
A *deterministic finite automata (DFA)* is a 5-tuple

$$M = (Q,\ \Sigma,\ \delta,\ q_0,\ F)$$

- Q is a finite set of *states*.
- Σ is a finite set of *input symbols* (i.e., the *alphabet*).
- $\delta : (Q \times \Sigma) \rightarrow Q$ is a *transition function*

  δ takes as arguments a state and an input symbol and returns a state.
- $q_0 \in Q$ is the *start state*.
- $F \subseteq Q$ is a set of *final* or *accepting states*.

DFA: Deterministic Finite Automata (2.2)



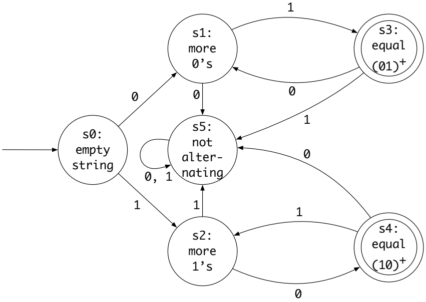
We formalize the above DFA as $M = (Q, \Sigma, \delta, q_0, F)$, where

- $Q = \{s_0, s_1\}$
- $\Sigma = \{0, 1\}$
- $\delta = \{((s_0, 0), s_1), ((s_0, 1), s_0), ((s_1, 0), s_0), ((s_1, 1), s_1)\}$

state \ input	0	1
s_0	s_1	s_0
s_1	s_0	s_1

- $q_0 = s_0$
- $F = \{s_1\}$

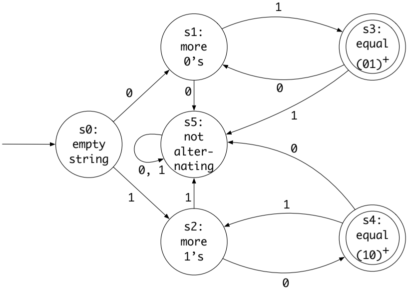
DFA: Deterministic Finite Automata (2.3.1)



We formalize the above DFA as $M = (Q, \Sigma, \delta, q_0, F)$, where

- $Q = \{s_0, s_1, s_2, s_3, s_4, s_5\}$
- $\Sigma = \{0, 1\}$
- $q_0 = s_0$
- $F = \{s_3, s_4\}$

DFA: Deterministic Finite Automata (2.3.2)



- $\delta =$

state \ input	0	1
s_0	s_1	s_2
s_1	s_5	s_3
s_2	s_4	s_5
s_3	s_1	s_5
s_4	s_5	s_2
s_5	s_5	s_5

DFA: Deterministic Finite Automata (2.4)

- Given a DFA $M = (Q, \Sigma, \delta, q_0, F)$:
  - We write *L(M)* to denote the *language of M*: the set of strings that M ***accepts***.
  - A string is ***accepted*** if it results in a sequence of transitions: beginning from the ***start*** state and ending in a ***final*** state.

$$L(M) = \left\{ \begin{array}{l} a_1 a_2 \dots a_n \mid \\ \quad 1 \le i \le n \ \wedge \ a_i \in \Sigma \ \wedge \ \delta(q_{i-1}, a_i) = q_i \ \wedge \ q_n \in F \end{array} \right\}$$

  - M ***rejects*** any string $w \notin L(M)$.
- We may also consider *L(M)* as concatenations of labels from the set of all valid ***paths*** of *M*'s transition diagram; each such path starts with q_0 and ends in a state in F.

DFA: Deterministic Finite Automata (2.5)

- Given a ***DFA*** $M = (Q, \Sigma, \delta, q_0, F)$, we may simplify the definition of *L(M)* by extending δ (which takes an input symbol) to $\hat{\delta}$ (which takes an input string).

$$\hat{\delta} : (Q \times \Sigma^*) \rightarrow Q$$

  We may define $\hat{\delta}$ recursively, using δ!

$$\begin{array}{lcl} \hat{\delta}(q, \epsilon) & = & q \\ \hat{\delta}(q, xa) & = & \delta(\ \hat{\delta}(q, x), a\ ) \end{array}$$

  where $q \in Q$, $x \in \Sigma^*$, and $a \in \Sigma$

- A neater definition of $L(M)$: the set of strings $w \in \Sigma^*$ such that $\hat{\delta}(q_0, w)$ is an ***accepting state***.

$$L(M) = \{w \mid w \in \Sigma^* \wedge \hat{\delta}(q_0, w) \in F\}$$

- A language L is said to be a *regular language*, if there is some ***DFA M*** such that $L = L(M)$.

Review Exercises: Formalizing DFAs

Formalize DFAs (as 5-tuples) for the other example string patterns mentioned:

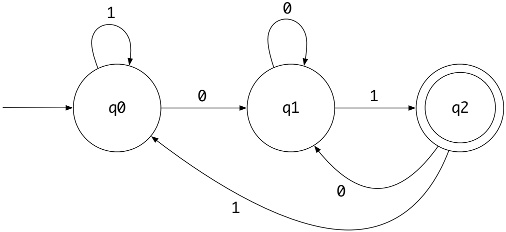
- $\{ w \mid w \texttt{ has an even number of 0's} \}$
- $\{ w \mid w \texttt{ contains } 01 \texttt{ as a substring} \}$
- $\left\{ w \mid \begin{array}{ll} & w \texttt{ has an even number of 0's} \\ \wedge & w \texttt{ has an odd number of 1's} \end{array} \right\}$

NFA: Nondeterministic Finite Automata (1.1)

Problem: Design a DFA that accepts the following language:

$$L = \{ \ x01 \mid x \in \{0,1\}^* \ \}$$

That is, L is the set of strings of 0s and 1s ending with 01.



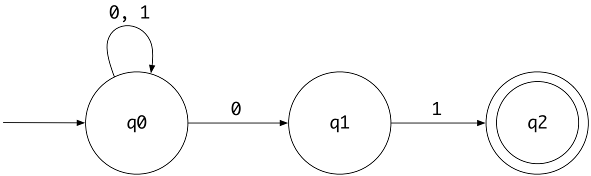
Given an input string w, we may simplify the above DFA by:

- *nondeterministically* treating state q_0 as both:
  - a state *ready* to read the last two input symbols from w
  - a state *not yet ready* to read the last two input symbols from w
- substantially reducing the outgoing transitions from q_1 and q_2

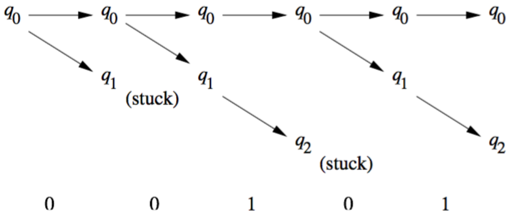
Compare the above DFA with the DFA in slide 39.

NFA: Nondeterministic Finite Automata (1.2)

- A *non-deterministic finite automata (NFA)* that accepts the same language:



- How an NFA determines if an input *00101* should be processed:



NFA: Nondeterministic Finite Automata (2)

- A *nondeterministic finite automata (NFA)*, like a **DFA**, is a ***FSM*** that ***accepts*** (or ***recognizes***) a pattern of behaviour.
- An ***NFA*** being *nondeterministic* means that from a given state, the **same input label** might corresponds to **multiple transitions** that lead to **distinct states**.
  - Each such transition offers an ***alternative path***.
  - Each alternative path is explored in parallel.
  - If **there exists** an alternative path that ***succeeds*** in processing the input string, then we say the ***NFA accepts*** that input string.
  - If **all** alternative paths get stuck at some point and ***fail*** to process the input string, then we say the ***NFA rejects*** that input string.
- ***NFAs*** are often more succinct (i.e., fewer states) and easier to design than **DFAs**.
- However, ***NFAs*** are just as ***expressive*** as are **DFAs**.
  - We can **always** convert an ***NFA*** to a **DFA**.

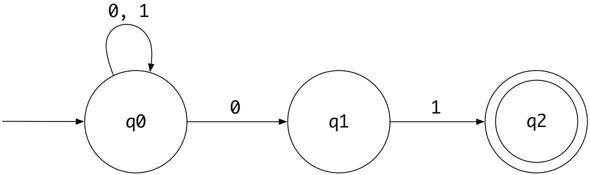
NFA: Nondeterministic Finite Automata (3.1)

- A *nondeterministic finite automata (NFA)* is a 5-tuple

$$M = (Q,\ \Sigma,\ \delta,\ q_0,\ F)$$

  - Q is a finite set of *states*.
  - Σ is a finite set of *input symbols* (i.e., the *alphabet*).
  - $\delta : (Q \times \Sigma) \rightarrow \mathbb{P}(Q)$ is a *transition function*
    - Given a state and an input symbol, δ returns a set of states.
    - Equivalently, we can write: $\delta : (Q \times \Sigma) \nrightarrow Q$ [a partial function]
  - $q_0 \in Q$ is the *start state*.
  - $F \subseteq Q$ is a set of *final* or *accepting states*.
- What is the difference between a ***DFA*** and an ***NFA***?
  - δ of a ***DFA*** returns a single state.
  - δ of an ***NFA*** returns a (possibly empty) set of states.

NFA: Nondeterministic Finite Automata (3.2)



Given an input string 00101:

- **Read 0**: $\delta(q_0, 0) = \{ q_0, q_1 \}$
- **Read 0**: $\delta(q_0, 0) \cup \delta(q_1, 0) = \{ q_0, q_1 \} \cup \emptyset = \{ q_0, q_1 \}$
- **Read 1**: $\delta(q_0, 1) \cup \delta(q_1, 1) = \{ q_0 \} \cup \{ q_2 \} = \{ q_0, q_2 \}$
- **Read 0**: $\delta(q_0, 0) \cup \delta(q_2, 0) = \{ q_0, q_1 \} \cup \emptyset = \{ q_0, q_1 \}$
- **Read 1**: $\delta(q_0, 1) \cup \delta(q_1, 1) = \{ q_0, q_1 \} \cup \{ q_2 \} = \{ q_0, q_1, q_2 \}$

$\because \{ q_0, q_1, q_2 \} \cap \{ q_2 \} \neq \emptyset \therefore$ `00101 is accepted`

NFA: Nondeterministic Finite Automata (3.3)

- Given a *NFA* $M = (Q, \Sigma, \delta, q_0, F)$, we may simplify the definition of *L(M)* by extending δ (which takes an input symbol) to $\hat{\delta}$ (which takes an input string).

$$\hat{\delta} : (Q \times \Sigma^*) \to \mathbb{P}(Q)$$

We may define $\hat{\delta}$ recursively, using δ!

$$\begin{array}{rcl} \hat{\delta}(q, \epsilon) & = & \{q\} \\ \hat{\delta}(q, xa) & = & \bigcup\{\delta(q', a) \mid q' \in \hat{\delta}(q, x)\} \end{array}$$

where $q \in Q$, $x \in \Sigma^*$, and $a \in \Sigma$

- A neater definition of $L(M)$: the set of strings $w \in \Sigma^*$ such that $\hat{\delta}(q_0, w)$ contains **at least one** *accepting state*.

$$L(M) = \{w \mid w \in \Sigma^* \land \hat{\delta}(q_0, w) \cap F \neq \varnothing\}$$

DFA $\equiv$ NFA (1)

- For many languages, constructing an accepting ***NFA*** is easier than a ***DFA***.
- From each state of an ***NFA***:
  - Outgoing transitions need **not** cover the entire Σ.
  - From a given state, the same symbol may ***non-deterministically*** lead to multiple states.
- In practice:
  - An ***NFA*** has just as many states as its equivalent DFA does.
  - An ***NFA*** often has fewer transitions than its equivalent ***DFA*** does.
- In the **worst** case:
  - While an ***NFA*** has n states, its equivalent ***DFA*** has 2^n states.
- Nonetheless, an ***NFA*** is still just as ***expressive*** as a ***DFA***.
  - A ***language*** accepted by some ***NFA*** is accepted by some ***DFA***:

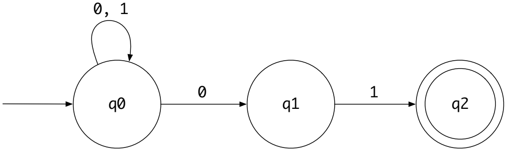
$$\forall N \bullet N \in NFA \Rightarrow (\exists D \bullet D \in DFA \land L(D) = L(N))$$

  - And vice versa, trivially?

$$\forall D \bullet D \in DFA \Rightarrow (\exists N \bullet N \in NFA \land L(D) = L(N))$$

DFA ≡ NFA (2.2): Lazy Evaluation (1)

Given an ***NFA***:



Subset construction (with ***lazy evaluation***) produces a ***DFA*** with δ as:

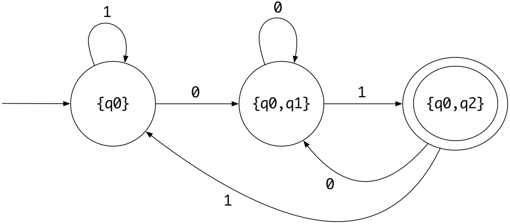
state \ input	0	1
$\{q_0\}$	$\delta(q_0, 0)$ $= \{q_0, q_1\}$	$\delta(q_0, 1)$ $= \{q_0\}$
$\{q_0, q_1\}$	$\delta(q_0, 0) \cup \delta(q_1, 0)$ $= \{q_0, q_1\} \cup \varnothing$ $= \{q_0, q_1\}$	$\delta(q_0, 1) \cup \delta(q_1, 1)$ $= \{q_0\} \cup \{q_2\}$ $= \{q_0, q_2\}$
$\{q_0, q_2\}$	$\delta(q_0, 0) \cup \delta(q_2, 0)$ $= \{q_0, q_1\} \cup \varnothing$ $= \{q_0, q_1\}$	$\delta(q_0, 1) \cup \delta(q_2, 1)$ $= \{q_0\} \cup \varnothing$ $= \{q_0\}$

DFA ≡ NFA (2.2): Lazy Evaluation (2)

Applying ***subset construction*** (with ***lazy evaluation***), we arrive in a ***DFA*** transition table:

state \ input	0	1
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1\}$	$\{q_0\}$

We then draw the ***DFA*** accordingly:



Compare the above DFA with the DFA in slide 31.

DFA ≡ NFA (2.2): Lazy Evaluation (3)

- Given an ***NFA*** $N = (Q_N, \Sigma_N, \delta_N, q_0, F_N)$:

```
ALGORITHM: ReachableSubsetStates
  INPUT: q0 : Q_N    ;    OUTPUT: Reachable ⊆ ℙ(Q_N)
PROCEDURE:
  Reachable := { {q0} }
  ToDiscover := { {q0} }
  while (ToDiscover ≠ ∅) {
    choose S : ℙ(Q_N) such that S ∈ ToDiscover
    remove S from ToDiscover
    NotYetDiscovered :=
        ( { {δ_N(s,0) | s ∈ S} } ∪ { {δ_N(s,1) | s ∈ S} } ) \ Reachable
    Reachable := Reachable ∪ NotYetDiscovered
    ToDiscover := ToDiscover ∪ NotYetDiscovered
  }
  return Reachable
```

- RT of *ReachableSubsetStates*? [$O(2^{|Q_N|})$]
- Often only a small portion of the $|\mathbb{P}(Q_N)|$ ***subset states*** is ***reachable*** from $\{q_0\}$ ⇒ ***Lazy Evaluation*** efficient in practice!

ϵ-NFA: Examples (1)

Draw the NFA for the following two languages:

1.

$$\left\{ xy \;\middle|\; \begin{array}{ll} & x \in \{0,1\}^* \\ \wedge & y \in \{0,1\}^* \\ \wedge & x \texttt{ has alternating } 0\texttt{'s and } 1\texttt{'s} \\ \wedge & y \texttt{ has an odd \# } 0\texttt{'s and an odd \# } 1\texttt{'s} \end{array} \right\}$$

2.

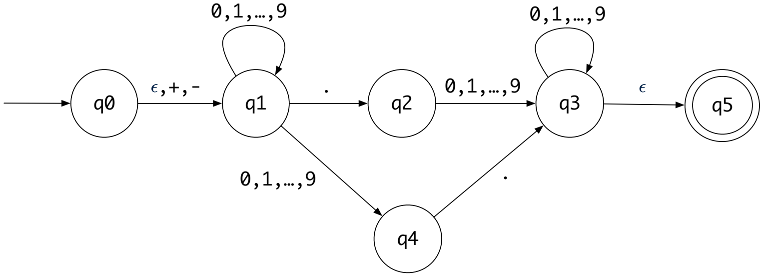
$$\left\{ w : \{0,1\}^* \;\middle|\; \begin{array}{ll} & w \texttt{ has alternating } 0\texttt{'s and } 1\texttt{'s} \\ \vee & w \texttt{ has an odd \# } 0\texttt{'s and an odd \# } 1\texttt{'s} \end{array} \right\}$$

3.

$$\left\{ sx.y \;\middle|\; \begin{array}{ll} & s \in \{+,-,\epsilon\} \\ \wedge & x \in \Sigma_{dec}^* \\ \wedge & y \in \Sigma_{dec}^* \\ \wedge & \neg(x = \epsilon \wedge y = \epsilon) \end{array} \right\}$$

ϵ-NFA: Examples (2)

$$\left\{ sx.y \;\middle|\; \begin{array}{ll} & s \in \{+, -, \epsilon\} \\ \wedge & x \in \Sigma_{dec}^{*} \\ \wedge & y \in \Sigma_{dec}^{*} \\ \wedge & \neg(x = \epsilon \wedge y = \epsilon) \end{array} \right\}$$



From q_0 to q_1, reading a sign is **optional**: a *plus* or a *minus*, or *nothing at all* (i.e., ϵ).

ϵ-NFA: Formalization (1)

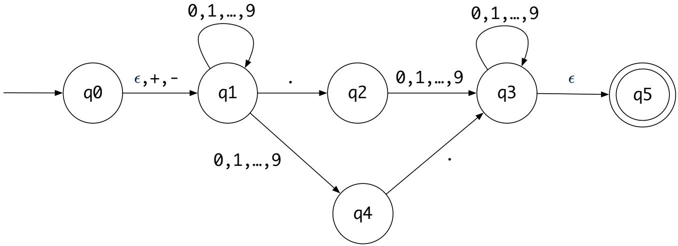
An *ϵ-NFA* is a 5-tuple

$$M = (Q,\ \Sigma,\ \delta,\ q_0,\ F)$$

- Q is a finite set of *states*.
- Σ is a finite set of *input symbols* (i.e., the *alphabet*).
- $\delta : (Q \times (\Sigma \cup \{\epsilon\})) \to \mathbb{P}(Q)$ is a *transition function*

  δ takes as arguments a state and an input symbol, or *an empty string* ϵ, and returns a set of states.
- $q_0 \in Q$ is the *start state*.
- $F \subseteq Q$ is a set of *final* or *accepting states*.

ϵ-NFA: Formalization (2)



Draw a transition table for the above NFA's δ function:

	ϵ	+, -	.	0 .. 9
q_0	$\{q_1\}$	$\{q_1\}$	$\varnothing$	$\varnothing$
q_1	$\varnothing$	$\varnothing$	$\{q_2\}$	$\{q_1, q_4\}$
q_2	$\varnothing$	$\varnothing$	$\varnothing$	$\{q_3\}$
q_3	$\{q_5\}$	$\varnothing$	$\varnothing$	$\{q_3\}$
q_4	$\varnothing$	$\varnothing$	$\{q_3\}$	$\varnothing$
q_5	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$

ϵ-NFA: Epsilon-Closures (1)

- Given *ϵ-NFA N*

$$N = (Q,\ \Sigma,\ \delta,\ q_0,\ F)$$

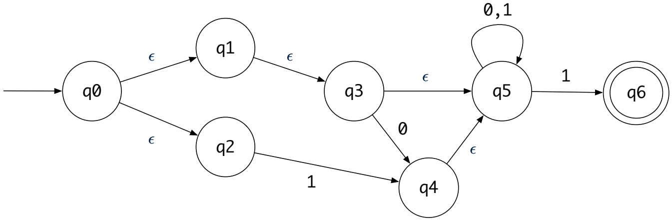
we define the *epsilon closure* (or *ϵ-closure*) as a function

$$\texttt{ECLOSE} : Q \to \mathbb{P}(Q)$$

- For any state $q \in Q$

$$\texttt{ECLOSE}(q) = \{q\} \cup \bigcup_{p \in \delta(q,\epsilon)} \texttt{ECLOSE}(p)$$

ϵ-NFA: Epsilon-Closures (2)



$$
\begin{aligned}
& \text{ECLOSE}(q_0) \\
= & \quad \{\ \delta(q_0, \epsilon) = \{q_1, q_2\}\ \} \\
& \{q_0\} \cup \text{ECLOSE}(q_1) \cup \text{ECLOSE}(q_2) \\
= & \quad \{\ \mathit{ECLOSE}(q_1),\ \delta(q_1, \epsilon) = \{q_3\},\ \mathit{ECLOSE}(q_2),\ \delta(q_2, \epsilon) = \varnothing\ \} \\
& \{q_0\} \cup (\ \{q_1\} \cup \mathit{ECLOSE}(q_3)\ ) \cup (\ \{q_2\} \cup \varnothing\ ) \\
= & \quad \{\ \mathit{ECLOSE}(q_3),\ \delta(q_3, \epsilon) = \{q_5\}\ \} \\
& \{q_0\} \cup (\ \{q_1\} \cup (\ \{q_3\} \cup \mathit{ECLOSE}(q_5)\ )\ ) \cup (\ \{q_2\} \cup \varnothing\ ) \\
= & \quad \{\ \mathit{ECLOSE}(q_5),\ \delta(q_5, \epsilon) = \varnothing\ \} \\
& \{q_0\} \cup (\ \{q_1\} \cup (\ \{q_3\} \cup (\ \{q_5\} \cup \varnothing\ )\ )\ ) \cup (\ \{q_2\} \cup \varnothing\ )
\end{aligned}
$$

ϵ-NFA: Formalization (3)

- Given a ϵ-*NFA* $M = (Q, \Sigma, \delta, q_0, F)$, we may simplify the definition of *L(M)* by extending δ (which takes an input symbol) to $\hat{\delta}$ (which takes an input string).

$$\hat{\delta} : (Q \times \Sigma^*) \to \mathbb{P}(Q)$$

We may define $\hat{\delta}$ recursively, using δ!

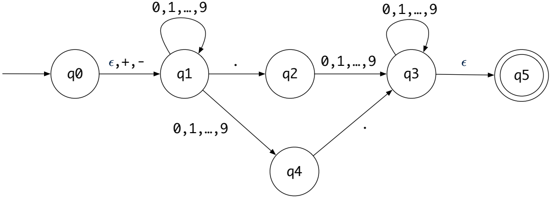
$$\begin{array}{lcl} \hat{\delta}(q, \epsilon) & = & \texttt{ECLOSE}(q) \\ \hat{\delta}(q, xa) & = & \bigcup\{ \texttt{ECLOSE}(q'') \mid q'' \in \delta(q', a) \land q' \in \hat{\delta}(q, x) \} \end{array}$$

where $q \in Q$, $x \in \Sigma^*$, and $a \in \Sigma$

- Then we define $L(M)$ as the set of strings $w \in \Sigma^*$ such that $\hat{\delta}(q_0, w)$ contains **at least one** *accepting state*.

$$L(M) = \{ w \mid w \in \Sigma^* \land \hat{\delta}(q_0, w) \cap F \neq \varnothing \}$$

ϵ-NFA: Formalization (4)



Given an input string 5.6:

$\hat{\delta}(q_0, \epsilon) = \text{ECLOSE}(q_0) = \{q_0, q_1\}$

- **Read 5**: $\delta(q_0, 5) \cup \delta(q_1, 5) = \varnothing \cup \{q_1, q_4\} = \{\ q_1, q_4\ \}$

  $\hat{\delta}(q_0, 5) = \text{ECLOSE}(q_1) \cup \text{ECLOSE}(q_4) = \{q_1\} \cup \{q_4\} = \{q_1, q_4\}$
- **Read .**: $\delta(q_1, .) \cup \delta(q_4, .) = \{q_2\} \cup \{q_3\} = \{\ q_2, q_3\ \}$

  $\hat{\delta}(q_0, 5.) = \text{ECLOSE}(q_2) \cup \text{ECLOSE}(q_3) = \{q_2\} \cup \{q_3, q_5\} = \{q_2, q_3, q_5\}$
- **Read 6**: $\delta(q_2, 6) \cup \delta(q_3, 6) \cup \delta(q_5, 6) = \{q_3\} \cup \{q_3\} \cup \varnothing = \{\ q_3\ \}$

  $\hat{\delta}(q_0, 5.6) = \text{ECLOSE}(q_3) = \{q_3, q_5\}$ [5.6 is *accepted*]

DFA $\equiv$ ϵ-NFA: Extended Subset Const. (1)

Subset construction (with ***lazy evaluation*** and *epsilon closures*) produces a ***DFA*** transition table.

	$d \in 0..9$	$s \in \{+,-\}$	.
$\{q_0, q_1\}$	$\{q_1, q_4\}$	$\{q_1\}$	$\{q_2\}$
$\{q_1, q_4\}$	$\{q_1, q_4\}$	$\varnothing$	$\{q_2, q_3, q_5\}$
$\{q_1\}$	$\{q_1, q_4\}$	$\varnothing$	$\{q_2\}$
$\{q_2\}$	$\{q_3, q_5\}$	$\varnothing$	$\varnothing$
$\{q_2, q_3, q_5\}$	$\{q_3, q_5\}$	$\varnothing$	$\varnothing$
$\{q_3, q_5\}$	$\{q_3, q_5\}$	$\varnothing$	$\varnothing$

For example, $\delta(\{q_0, q_1\}, d)$ is calculated as follows: $[d \in 0..9]$

$$
\begin{aligned}
& \bigcup\{\text{ECLOSE}(q) \mid q \in \delta(q_0, d) \cup \delta(q_1, d)\} \\
= \; & \bigcup\{\text{ECLOSE}(q) \mid q \in \varnothing \cup \{q_1, q_4\}\} \\
= \; & \bigcup\{\text{ECLOSE}(q) \mid q \in \{q_1, q_4\}\} \\
= \; & \text{ECLOSE}(q_1) \cup \text{ECLOSE}(q_4) \\
= \; & \{q_1\} \cup \{q_4\} \\
= \; & \{q_1, q_4\}
\end{aligned}
$$

DFA $\equiv$ ϵ-NFA: Extended Subset Const. (2)

Given an ϵ=*NFA* $N = (Q_N, \Sigma_N, \delta_N, q_0, F_N)$, by applying the ***extended subset construction*** to it, the resulting *DFA* $D = (Q_D, \Sigma_D, \delta_D, q_{D_{start}}, F_D)$ is such that:

$$
\begin{array}{lcl}
\Sigma_D & = & \Sigma_N \\
q_{D_{start}} & = & \text{ECLOSE}(q_0) \\
F_D & = & \{\, S \mid S \subseteq Q_N \wedge S \cap F_N \neq \varnothing \,\} \\
Q_D & = & \{\, S \mid S \subseteq Q_N \wedge (\exists w \bullet w \in \Sigma^* \Rightarrow S = \hat{\delta}_N(q_0, w)) \,\} \\
\delta_D(S, a) & = & \bigcup \{\, \text{ECLOSE}(s') \mid s \in S \wedge s' \in \delta_N(s, a) \,\}
\end{array}
$$

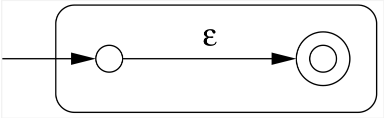
Regular Expression to ϵ-NFA

- Just as we construct each complex *regular expression* recursively, we define its equivalent *ϵ-NFA* *recursively* .
- Given a regular expression R, we construct an ϵ-NFA E, such that $L(R) = L(E)$, with
  - Exactly **one** accept state.
  - No incoming arc to the start state.
  - No outgoing arc from the accept state.

Regular Expression to ϵ-NFA

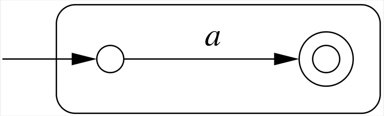
Base Cases:

- ϵ



- $\varnothing$

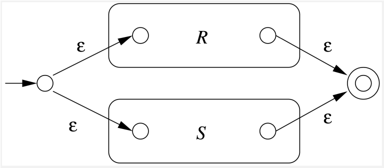
- a $[a \in \Sigma]$



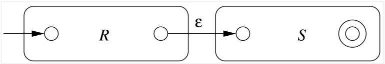
Regular Expression to ϵ-NFA

Recursive Cases: [*R* and *S* are RE's]

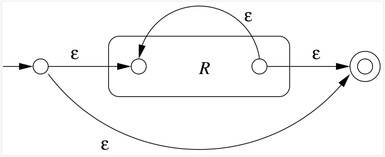
- $R + S$



- RS

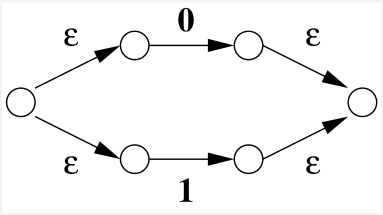


- R^*

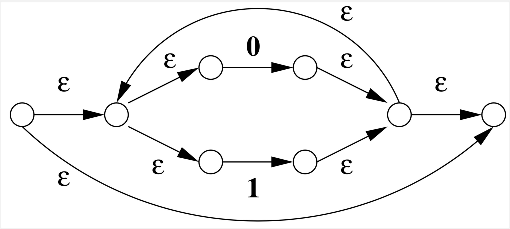


Regular Expression to ϵ-NFA: Examples (1.1)

- $0 + 1$

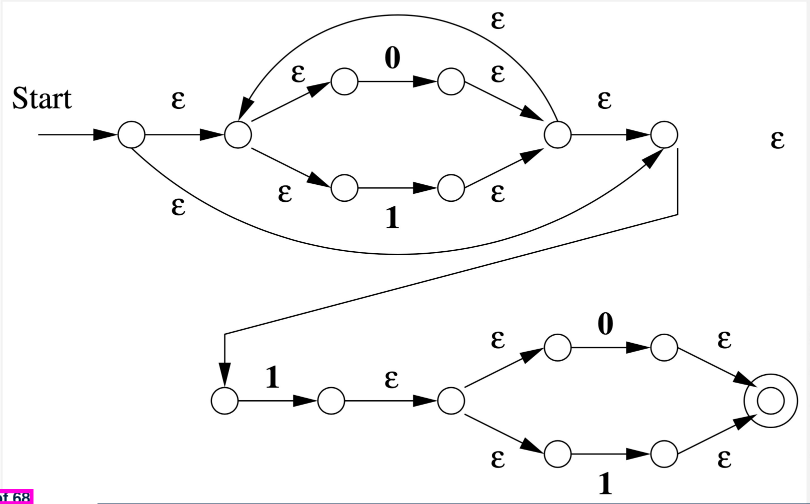


- $(0 + 1)^*$



Regular Expression to ϵ-NFA: Examples (1.2)

- $(0+1)^*1(0+1)$



Minimizing DFA: Motivation

- Recall: Regular Expresion $\longrightarrow$ ϵ-NFA $\longrightarrow$ DFA
- DFA produced by the ***extended subset construction*** (with ***lazy evaluation***) may **not** be ***minimum*** on its size of state.
- When the required size of memory is sensitive

  (e.g., processor's cache memory),

  the fewer number of DFA states, the better.

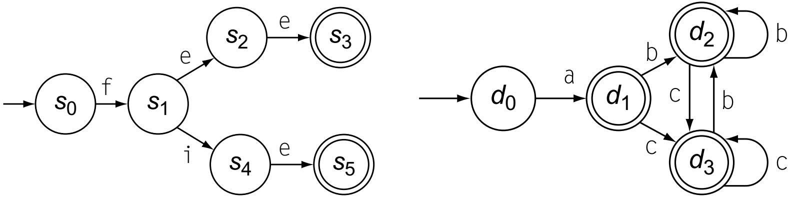
Minimizing DFA: Algorithm

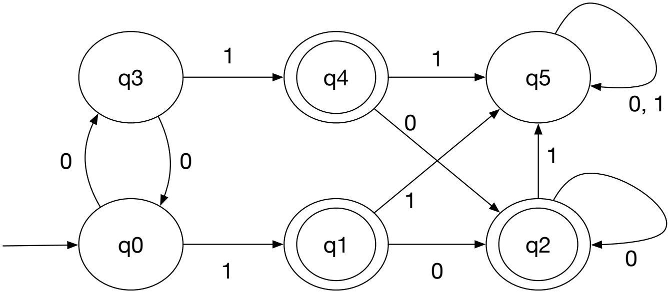
```
ALGORITHM: MinimizeDFAStates
  INPUT: DFA M = (Q, Σ, δ, q0, F)
  OUTPUT: M′ s.t. minimum |Q| and equivalent behaviour as M
PROCEDURE:
  P := ∅ /* refined partition so far */
  T := { F, Q − F } /* last refined partition */
  while (P ≠ T):
    P := T
    T := ∅
    for(p ∈ P):
      find the maximal S ⊂ p s.t. splittable(p, S)
      if S ≠ ∅ then
        T := T ∪ {S, p − S}
      else
        T := T ∪ {p}
      end
```

splittable(p, S) holds iff there is $c \in \Sigma$ s.t.

1. $S \subset p$ (or equivalently: $p - S \neq \varnothing$)
2. Transitions via c lead all $s \in S$ to states in ***same partition*** $p1$ ($p1 \neq p$).

Minimizing DFA: Examples





Exercises: Minimize the DFA from **here**; Q1 & Q2, p59, EAC2.

Exercise:
Regular Expression to Minimized DFA

Given regular expression `r[0..9]+` which specifies the pattern of a register name, derive the equivalent DFA with the minimum number of states. Show all steps.

Implementing DFA as Scanner

- The source language has a list of ***syntactic categories***:

  e.g., keyword `while` [`while`]

  e.g., identifiers [`[a-zA-Z][a-zA-Z0-9_]*`]

  e.g., white spaces [`[ \t\r]+`]

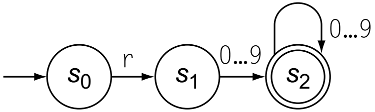
- A compiler's *scanner* must recognize ***words*** from **all** syntactic categories of the source language.
  - Each syntactic category is specified via a ***regular expression***.

$$\underbrace{r_1}_{\texttt{syn. cat. 1}} + \underbrace{r_1}_{\texttt{syn. cat. 2}} + \ldots + \underbrace{r_n}_{\texttt{syn. cat. } n}$$

  - Overall, a scanner should be implemented based on the ***minimized DFA*** accommodating all syntactic categories.
- Principles of a scanner:
  - Returns one ***word*** at a time
  - Each returned word is the ***longest possible*** that matches a ***pattern***
  - A **priority** may be specified among patterns (e.g., `new` is a keyword, not identifier)

Implementing DFA: Table-Driven Scanner (1)

- Consider the ***syntactic category*** of `register` names.
- Specified as a *regular expression* : `r[0..9]+`
- Afer conversion to ϵ-NFA, then to DFA, then to ***minimized DFA***:



- The following tables encode knowledge about the above DFA:

Classifier (`CharCat`)

`r`	`0,1,2,...,9`	`EOF`	**Other**
Register	*Digit*	*Other*	*Other*

Transition (δ)

	Register	***Digit***	***Other***
s_0	s_1	s_e	s_e
s_1	s_e	s_2	s_e
s_2	s_e	s_2	s_e
s_e	s_e	s_e	s_e

Token Type (`Type`)

s_0	s_1	s_2	s_e
invalid	*invalid*	*register*	*invalid*

Implementing DFA: Table-Driven Scanner (2)

The scanner then is implemented via a 4-stage skeleton:

```
NextWord()
  -- Stage 1:  Initialization
  state := s0 ; word := ε
  initialize an empty stack s ; s.push(bad)
  -- Stage 2:  Scanning Loop
  while (state ≠ se)
    NextChar(char) ; word := word + char
    if state ∈ F then reset stack s end
    s.push(state)
    cat := CharCat[char]
    state := δ[state, cat]
  -- Stage 3:  Rollback Loop
  while (state ∉ F ∧ state ≠ bad)
    state := s.pop()
    truncate word
  -- Stage 4:  Interpret and Report
  if state ∈ F then return Type[state]
  else return invalid
  end
```

Index (1)

Index (2)

Index (3)

Index (4)

Index (5)

Index (6)

Parser: Syntactic Analysis

Readings: EAC2 Chapter 3

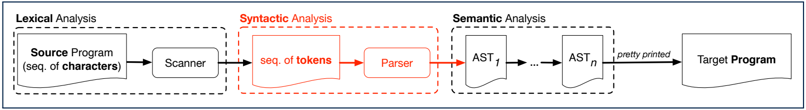
YORK UNIVERSITÉ UNIVERSITY

EECS4302 A:
Compilers and Interpreters
Fall 2022

CHEN-WEI WANG

Parser in Context

- Recall:



- Treats the input programas as a ***a sequence of classified tokens/words***
- Applies rules ***parsing*** token sequences as ***abstract syntax trees (ASTs)*** [***syntactic*** analysis]
- Upon termination:
  - Reports token sequences not derivable as ASTs
  - Produces an ***AST***
- No longer considers ***every character*** in input program.
- *Derivable* token sequences constitute a *context-free language (CFL)*.

Context-Free Languages: Introduction

- We have seen ***regular languages***:
  - Can be described using ***finite automata*** or ***regular expressions***.
  - Satisfy the ***pumping lemma***.
- Language with ***recursive*** structures are provably ***non-regular***.
  e.g., $\{0^n 1^n \mid n \geq 0\}$
- *Context-Free Grammars (CFG's)* are used to describe strings that can be generated in a ***recursive*** fashion.
- *Context-Free Languages (CFL's)* are:
  - Languages that can be described using CFG's.
  - A proper superset of the set of regular languages.

CFG: Example (1.1)

- The following language that is ***non-regular***

$$\{0^n\#1^n \mid n \geq 0\}$$

can be described using a ***context-free grammar (CFG)***:

$$\begin{array}{lcl} A & \rightarrow & 0A1 \\ A & \rightarrow & B \\ B & \rightarrow & \# \end{array}$$

- A grammar contains a collection of ***substitution*** or ***production*** rules, where:
  - A **terminal** is a word $w \in \Sigma^*$ (e.g., 0, 1, *etc.*).
  - A ***variable*** or ***non-terminal*** is a word $w \notin \Sigma^*$ (e.g., A, B, *etc.*).
  - A ***start variable*** occurs on the LHS of the topmost rule (e.g., A).

CFG: Example (1.2)

- Given a grammar, generate a string by:
  1. Write down the ***start variable***.
  2. Choose a production rule where the ***start variable*** appears on the LHS of the arrow, and ***substitute*** it by the RHS.
  3. There are two cases of the re-written string:
     - **3.1** It contains **no** variables, then you are done.
     - **3.2** It contains **some** variables, then ***substitute*** each variable using the relevant ***production rules***.
  4. Repeat Step 3.
- e.g., We can generate an infinite number of strings from

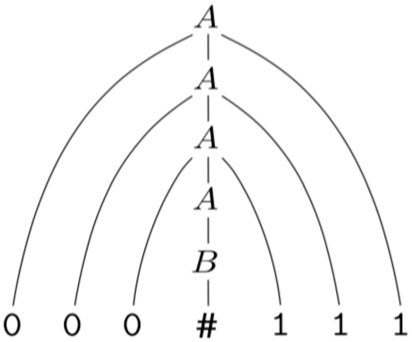
$$\begin{array}{lcl} A & \rightarrow & 0A1 \\ A & \rightarrow & B \\ B & \rightarrow & \# \end{array}$$

  - $A \Rightarrow B \Rightarrow \#$
  - $A \Rightarrow 0A1 \Rightarrow 0B1 \Rightarrow 0\#1$
  - $A \Rightarrow 0A1 \Rightarrow 00A11 \Rightarrow 00B11 \Rightarrow 00\#11$
  - ...

CFG: Example (1.2)

Given a CFG, a string's ***derivation*** can be shown as a ***parse tree***.

e.g., The derivation of 000#111 has the parse tree



CFG: Example (2)

Design a CFG for the following language:

$$\{w \mid w \in \{0,1\}^* \land w \texttt{ is a palidrome}\}$$

e.g., 00, 11, 0110, 1001, *etc.*

$$\begin{array}{lcl} P & \rightarrow & \epsilon \\ P & \rightarrow & 0 \\ P & \rightarrow & 1 \\ P & \rightarrow & 0P0 \\ P & \rightarrow & 1P1 \end{array}$$

CFG: Example (3)

Design a CFG for the following language:

$$\{ww^R \mid w \in \{0,1\}^*\}$$

e.g., 00, 11, 0110, *etc.*

$$\begin{array}{lcl} P & \rightarrow & \epsilon \\ P & \rightarrow & 0P0 \\ P & \rightarrow & 1P1 \end{array}$$

CFG: Example (4)

Design a CFG for the set of binary strings, where each block of 0's followed by at least as many 1's.

e.g., 000111, 0001111, *etc.*

- We use S to represent one such string, and A to represent each such block in S.

$$
\begin{array}{lcll}
S & \rightarrow & \epsilon & \{\texttt{BC of } S\} \\
S & \rightarrow & AS & \{\texttt{RC of } S\} \\
A & \rightarrow & \epsilon & \{\texttt{BC of } A\} \\
A & \rightarrow & 01 & \{\texttt{BC of } A\} \\
A & \rightarrow & 0A1 & \{\texttt{RC of } A\texttt{: equal 0's and 1's}\} \\
A & \rightarrow & A1 & \{\texttt{RC of } A\texttt{: more 1's}\}
\end{array}
$$

CFG: Example (5.1) Version 1

Design the grammar for the following small expression language, which supports:

- Arithmetic operations: `+`, `−`, `*`, `/`
- Relational operations: `>`, `<`, `>=`, `<=`, `==`, `/=`
- Logical operations: `true`, `false`, `!`, `&&`, `||`, `=>`

  Start with the variable ***Expression***.
- There are two possible versions:
  1. All operations are mixed together.
  2. Relevant operations are grouped together.
     Try both!

CFG: Example (5.2) Version 1

$$
\begin{array}{lcl}
\textit{Expression} & \rightarrow & \textit{IntegerConstant} \\
 & | & -\textit{IntegerConstant} \\
 & | & \textit{BooleanConstant} \\
 & | & \textit{BinaryOp} \\
 & | & \textit{UnaryOp} \\
 & | & (\ \textit{Expression}\ ) \\
 & & \\
\textit{IntegerConstant} & \rightarrow & \textit{Digit} \\
 & | & \textit{Digit IntegerConstant} \\
 & & \\
\textit{Digit} & \rightarrow & 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9 \\
 & & \\
\textit{BooleanConstant} & \rightarrow & \texttt{TRUE} \\
 & | & \texttt{FALSE}
\end{array}
$$

CFG: Example (5.3) Version 1

$$
\begin{array}{lcl}
\mathit{BinaryOp} & \rightarrow & \mathit{Expression} + \mathit{Expression} \\
& | & \mathit{Expression} - \mathit{Expression} \\
& | & \mathit{Expression} * \mathit{Expression} \\
& | & \mathit{Expression} \,/\, \mathit{Expression} \\
& | & \mathit{Expression} \;\&\& \; \mathit{Expression} \\
& | & \mathit{Expression} \;||\; \mathit{Expression} \\
& | & \mathit{Expression} => \mathit{Expression} \\
& | & \mathit{Expression} == \mathit{Expression} \\
& | & \mathit{Expression} \;/=\; \mathit{Expression} \\
& | & \mathit{Expression} > \mathit{Expression} \\
& | & \mathit{Expression} < \mathit{Expression} \\
\\
\mathit{UnaryOp} & \rightarrow & !\ \mathit{Expression}
\end{array}
$$

CFG: Example (5.4) Version 1

However, Version 1 of CFG:

- ***Parses*** string that requires further ***semantic analysis*** (e.g., type checking):
  e.g., `3 => 6`
- Is ***ambiguous***, meaning?
  - Some string may have more than one ways to interpreting it.
  - An interpretation is either visualized as a ***parse tree***, or written as a sequence of ***derivations***.

  e.g., Draw the parse tree(s) for `3 * 5 + 4`

CFG: Example (5.5) Version 2

$$
\begin{array}{lcl}
\mathit{Expression} & \rightarrow & \mathit{ArithmeticOp} \\
 & | & \mathit{RelationalOp} \\
 & | & \mathit{LogicalOp} \\
 & | & (\ \mathit{Expression}\ ) \\
\\
\mathit{IntegerConstant} & \rightarrow & \mathit{Digit} \\
 & | & \mathit{Digit}\ \mathit{IntegerConstant} \\
\\
\mathit{Digit} & \rightarrow & 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9 \\
\\
\mathit{BooleanConstant} & \rightarrow & \texttt{TRUE} \\
 & | & \texttt{FALSE}
\end{array}
$$

CFG: Example (5.6) Version 2

$$
\begin{array}{lcl}
\mathit{ArithmeticOp} & \rightarrow & \mathit{ArithmeticOp} + \mathit{ArithmeticOp} \\
& | & \mathit{ArithmeticOp} - \mathit{ArithmeticOp} \\
& | & \mathit{ArithmeticOp} * \mathit{ArithmeticOp} \\
& | & \mathit{ArithmeticOp} \; / \; \mathit{ArithmeticOp} \\
& | & (\mathit{ArithmeticOp}) \\
& | & \mathit{IntegerConstant} \\
& | & -\mathit{IntegerConstant} \\
\mathit{RelationalOp} & \rightarrow & \mathit{ArithmeticOp} == \mathit{ArithmeticOp} \\
& | & \mathit{ArithmeticOp} \; /= \; \mathit{ArithmeticOp} \\
& | & \mathit{ArithmeticOp} > \mathit{ArithmeticOp} \\
& | & \mathit{ArithmeticOp} < \mathit{ArithmeticOp} \\
\mathit{LogicalOp} & \rightarrow & \mathit{LogicalOp} \; \&\& \; \mathit{LogicalOp} \\
& | & \mathit{LogicalOp} \; || \; \mathit{LogicalOp} \\
& | & \mathit{LogicalOp} => \mathit{LogicalOp} \\
& | & ! \; \mathit{LogicalOp} \\
& | & (\mathit{LogicalOp}) \\
& | & \mathit{RelationalOp} \\
& | & \mathit{BooleanConstant}
\end{array}
$$

CFG: Example (5.7) Version 2

However, Version 2 of CFG:

- Eliminates some cases for further semantic analysis:
  e.g., `(1 + 2) => (5 / 4)` [no parse tree]
- Still ***parses*** strings that might require further ***semantic analysis***:
  e.g., `(1 + 2) / (5 - (2 + 3))`
- Still is ***ambiguous***.
  e.g., Draw the parse tree(s) for `3 * 5 + 4`

CFG: Formal Definition (1)

- A ***context-free grammar (CFG)*** is a 4-tuple (V, Σ, R, S):
  - V is a finite set of **variables**.
  - Σ is a finite set of ***terminals***. $[V \cap \Sigma = \varnothing]$
  - R is a finite set of ***rules*** s.t.

$$R \subseteq \{v \rightarrow s \mid v \in V \land s \in (V \cup \Sigma)^*\}$$

  - $S \in V$ is is the **start variable**.
- Given strings $u, v, w \in (V \cup \Sigma)^*$, variable $A \in V$, a rule $A \rightarrow w$:
  - $uAv \Rightarrow uwv$ menas that uAv ***yields*** uwv.
  - $u \overset{*}{\Rightarrow} v$ means that u ***derives*** v, if:
    - $u = v$; or
    - $u \Rightarrow u_1 \Rightarrow u_2 \Rightarrow \cdots \Rightarrow u_k \Rightarrow v$ [a ***yield sequence***]
- Given a CFG $G = (V, \Sigma, R, S)$, the language of G

$$L(G) = \{w \in \Sigma^* \mid S \overset{*}{\Rightarrow} w\}$$

CFG: Formal Definition (2): Example

- Design the ***CFG*** for strings of properly-nested parentheses.
  e.g., ( ), ( ) ( ), ((( ) ( ))) ( ), *etc.*
  Present your answer in a ***formal*** manner.
- $G = (\{S\}, \{ (,) \}, R, S)$, where R is

$$S \rightarrow (\ S \ ) \mid SS \mid \epsilon$$

- Draw ***parse trees*** for the above three strings that G generates.

CFG: Formal Definition (3): Example

- Consider the grammar $G = (V, \Sigma, R, S)$:
  - R is

$$
\begin{array}{lcl}
Expr & \rightarrow & Expr + Term \\
 & | & Term \\
Term & \rightarrow & Term \star Factor \\
 & | & Factor \\
Factor & \rightarrow & (\,Expr\,) \\
 & | & \mathtt{a}
\end{array}
$$

  - $V = \{Expr, Term, Factor\}$
  - $\Sigma = \{\mathtt{a}, +, \star, (,)\}$
  - $S = Expr$
- ***Precedence*** of operators $+$, $\star$ is embedded in the grammar.
  - "Plus" is specified at a **higher** level (*Expr*) than is "times" (*Term*).
  - Both operands of a multiplication (*Factor*) may be **parenthesized**.

Regular Expressions to CFG's

- Recall the semantics of regular expressions (assuming that we do not consider $\varnothing$):

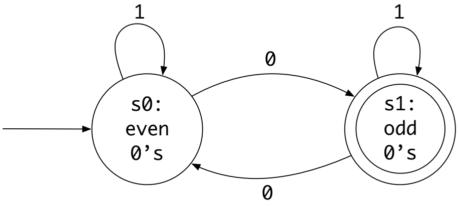
$$
\begin{array}{lcl}
L(\ \epsilon\ ) & = & \{\epsilon\} \\
L(\ a\ ) & = & \{a\} \\
L(\ E + F\ ) & = & L(E) \cup L(F) \\
L(\ EF\ ) & = & L(E)L(F) \\
L(\ E^*\ ) & = & (L(E))^* \\
L(\ (E)\ ) & = & L(E)
\end{array}
$$

- e.g., Grammar for $(00 + 1)^* + (11 + 0)^*$

$$
\begin{array}{lcl}
S & \rightarrow & A \mid B \\
A & \rightarrow & \epsilon \mid AC \\
C & \rightarrow & 00 \mid 1 \\
B & \rightarrow & \epsilon \mid BD \\
D & \rightarrow & 11 \mid 0
\end{array}
$$

DFA to CFG's

- Given a DFA $M = (Q, \Sigma, \delta, q_0, F)$:
  - Make a ***variable*** R_i for each ***state*** $q_i \in Q$.
  - Make R_0 the ***start variable***, where q_0 is the ***start state*** of M.
  - Add a rule $R_i \rightarrow aR_j$ to the grammar if $\delta(q_i, a) = q_j$.
  - Add a rule $R_i \rightarrow \epsilon$ if $q_i \in F$.
- e.g., Grammar for



$$
\begin{array}{lll}
R_0 & \rightarrow & 1R_0 \mid 0R_1 \\
R_1 & \rightarrow & 0R_0 \mid 1R_1 \mid \epsilon
\end{array}
$$

CFG: Leftmost Derivations (1)

$$\begin{array}{lcl} Expr & \rightarrow & Expr + Term \mid Term \\ Term & \rightarrow & Term \star Factor \mid Factor \\ Factor & \rightarrow & (Expr) \mid a \end{array}$$

- Given a string ($\in (V \cup \Sigma)^*$), a ***left-most derivation (LMD)*** keeps substituting the leftmost non-terminal ($\in V$).
- ***Unique LMD*** for the string `a + a * a`:

$$\begin{array}{lcl} Expr & \Rightarrow & Expr + Term \\ & \Rightarrow & Term + Term \\ & \Rightarrow & Factor + Term \\ & \Rightarrow & a + Term \\ & \Rightarrow & a + Term \star Factor \\ & \Rightarrow & a + Factor \star Factor \\ & \Rightarrow & a + a \star Factor \\ & \Rightarrow & a + a \star a \end{array}$$

- This ***LMD*** suggests that `a * a` is the right operand of $+$.

CFG: Rightmost Derivations (1)

$$\begin{array}{lcl} Expr & \rightarrow & Expr \; + \; Term \mid Term \\ Term & \rightarrow & Term \; * \; Factor \mid Factor \\ Factor & \rightarrow & (\, Expr \,) \mid a \end{array}$$

- Given a string ($\in (V \cup \Sigma)^*$), a ***right-most derivation (RMD)*** keeps substituting the rightmost non-terminal ($\in V$).
- ***Unique RMD*** for the string `a + a * a`:

$$\begin{array}{lcl} Expr & \Rightarrow & Expr \; + \; Term \\ & \Rightarrow & Expr \; + \; Term \; * \; Factor \\ & \Rightarrow & Expr \; + \; Term \; * \; a \\ & \Rightarrow & Expr \; + \; Factor \; * \; a \\ & \Rightarrow & Expr \; + \; a \; * \; a \\ & \Rightarrow & Term \; + \; a \; * \; a \\ & \Rightarrow & Factor \; + \; a \; * \; a \\ & \Rightarrow & a \; + \; a \; * \; a \end{array}$$

- This ***RMD*** suggests that `a * a` is the right operand of `+`.

CFG: Leftmost Derivations (2)

$$\begin{array}{lcl} \mathit{Expr} & \rightarrow & \mathit{Expr} \ + \ \mathit{Term} \mid \mathit{Term} \\ \mathit{Term} & \rightarrow & \mathit{Term} \ \star \ \mathit{Factor} \mid \mathit{Factor} \\ \mathit{Factor} & \rightarrow & (\ \mathit{Expr}\ ) \mid a \end{array}$$

- ***Unique LMD*** for the string `(a + a) * a`:

$$\begin{array}{lcl} \mathit{Expr} & \Rightarrow & \mathit{Term} \\ & \Rightarrow & \mathit{Term} \ \star \ \mathit{Factor} \\ & \Rightarrow & \mathit{Factor} \ \star \ \mathit{Factor} \\ & \Rightarrow & (\ \mathit{Expr}\ ) \ \star \ \mathit{Factor} \\ & \Rightarrow & (\ \mathit{Expr} + \mathit{Term}\ ) \ \star \ \mathit{Factor} \\ & \Rightarrow & (\ \mathit{Term} + \mathit{Term}\ ) \ \star \ \mathit{Factor} \\ & \Rightarrow & (\ \mathit{Factor} + \mathit{Term}\ ) \ \star \ \mathit{Factor} \\ & \Rightarrow & (\ a + \mathit{Term}\ ) \ \star \ \mathit{Factor} \\ & \Rightarrow & (\ a + \mathit{Factor}\ ) \ \star \ \mathit{Factor} \\ & \Rightarrow & (\ a + a\ ) \ \star \ \mathit{Factor} \\ & \Rightarrow & (\ a + a\ ) \ \star \ a \end{array}$$

- This ***LMD*** suggests that `(a + a)` is the left operand of $\star$.

CFG: Rightmost Derivations (2)

$$
\begin{array}{lcl}
Expr & \rightarrow & Expr + Term \mid Term \\
Term & \rightarrow & Term \star Factor \mid Factor \\
Factor & \rightarrow & (Expr) \mid a
\end{array}
$$

- ***Unique RMD*** for the string `(a + a) * a`:

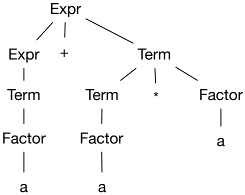
$$
\begin{array}{lcl}
Expr & \Rightarrow & Term \\
& \Rightarrow & Term \star Factor \\
& \Rightarrow & Term \star a \\
& \Rightarrow & Factor \star a \\
& \Rightarrow & (\; Expr \;) \star a \\
& \Rightarrow & (\; Expr + Term \;) \star a \\
& \Rightarrow & (\; Expr + Factor \;) \star a \\
& \Rightarrow & (\; Expr + a \;) \star a \\
& \Rightarrow & (\; Term + a \;) \star a \\
& \Rightarrow & (\; Factor + a \;) \star a \\
& \Rightarrow & (\; a + a \;) \star a
\end{array}
$$

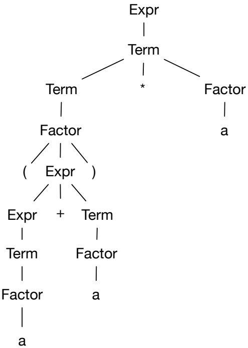
- This ***RMD*** suggests that `(a + a)` is the left operand of $\star$.

CFG: Parse Trees vs. Derivations (1)

- *Parse trees* for (leftmost & rightmost) *derivations* of expressions:

a + a * a	(a + a) * a





- Orders in which ***derivations*** are performed are ***not*** reflected on parse trees.

CFG: Parse Trees vs. Derivations (2)

- A string $w \in \Sigma^*$ may have <u>more than one</u> ***derivations***.

  Q: distinct ***derivations*** for $w \in \Sigma^* \Rightarrow$ distinct ***parse trees*** for w?

  A: Not in general $\because$ Derivations with ***distinct orders*** of variable substitutions may still result in the ***same parse tree***.
- For example:

$$
\begin{array}{lcl}
Expr & \rightarrow & Expr \ \texttt{+} \ Term \mid Term \\
Term & \rightarrow & Term \ \texttt{*} \ Factor \mid Factor \\
Factor & \rightarrow & \texttt{(} \ Expr \ \texttt{)} \mid a
\end{array}
$$

For string `a + a * a`, the ***LMD*** and ***RMD*** have ***distinct orders*** of variable substitutions, but their corresponding ***parse trees are the <u>same</u>***.

CFG: Ambiguity: Definition

Given a grammar $G = (V, \Sigma, R, S)$:

- A string $w \in \Sigma^*$ is derived *ambiguously* in G if there exist two or more ***distinct*** ***parse trees*** or, equally, two or more ***distinct*** ***LMDs*** or, equally, two or more ***distinct*** ***RMDs***.

  We require that all such derivations are completed by following a consisten order (**leftmost** or **rightmost**) to avoid ***false positive***.

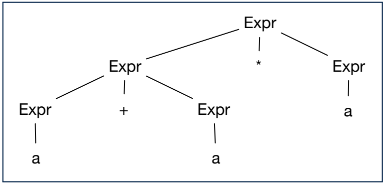
- G is *ambiguous* if it generates some string ambiguously.

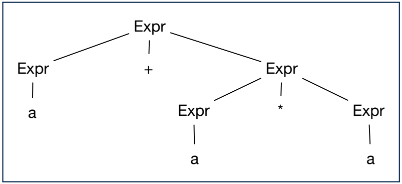
CFG: Ambiguity: Exercise (1)

- Is the following grammar *ambiguous*?

$$Expr \rightarrow Expr \; \texttt{+} \; Expr \mid Expr \; \texttt{*} \; Expr \mid \texttt{(} \; Expr \; \texttt{)} \mid a$$

- Yes ∵ it generates the string `a + a * a` *ambiguously*:





- ***Distinct ASTs*** (for the ***same input***) imply ***distinct semantic interpretations***: e.g., a pre-order traversal for evaluation
- **Exercise**: Show ***LMDs*** for the two parse trees.

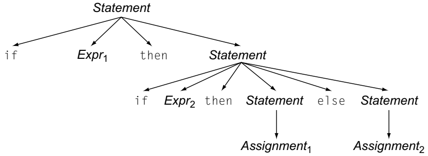
CFG: Ambiguity: Exercise (2.1)

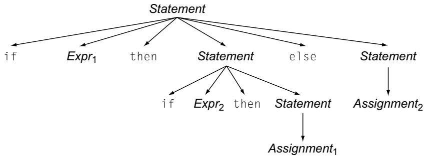
- Is the following grammar *ambiguous*?

$$
\begin{array}{rcl}
\textit{Statement} & \rightarrow & \texttt{if}\ \textit{Expr}\ \texttt{then}\ \textit{Statement} \\
 & | & \texttt{if}\ \textit{Expr}\ \texttt{then}\ \textit{Statement}\ \texttt{else}\ \textit{Statement} \\
 & | & \textit{Assignment} \\
 & \ldots &
\end{array}
$$

- Yes $\because$ it derives the following string *ambiguously*:

$\texttt{if}\ \textit{Expr}_1\ \texttt{then}\ \texttt{if}\ \textit{Expr}_2\ \texttt{then}\ \textit{Assignment}_1\ \texttt{else}\ \textit{Assignment}_2$

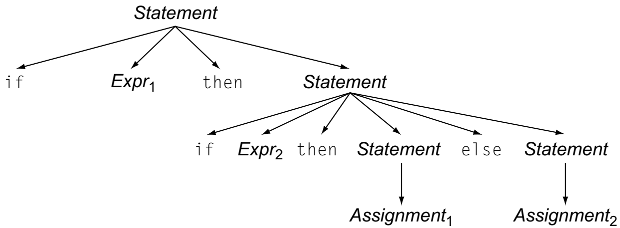




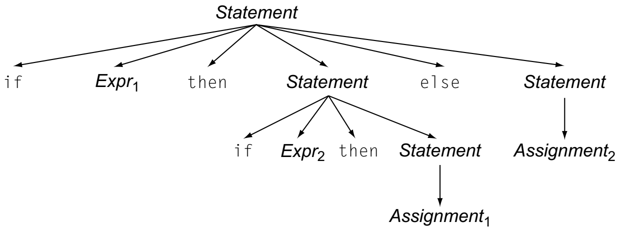
- This is called the ***dangling else*** problem.
- **Exercise:** Show ***LMDs*** for the two parse trees.

CFG: Ambiguity: Exercise (2.2)

(***Meaning 1***) *Assignment*$_2$ may be associated with the <u>inner</u> `if`:



(***Meaning 2***) *Assignment*$_2$ may be associated with the <u>outer</u> `if`:



CFG: Ambiguity: Exercise (2.3)

- We may remove the ***ambiguity*** by specifying that the *dangling* `else` is associated with the **nearest** **`if`**:

$$
\begin{array}{lcl}
\mathit{Statement} & \rightarrow & \texttt{if}\ \mathit{Expr}\ \texttt{then}\ \mathit{Statement} \\
 & | & \texttt{if}\ \mathit{Expr}\ \texttt{then}\ \mathit{WithElse}\ \texttt{else}\ \mathit{Statement} \\
 & | & \mathit{Assignment} \\
\mathit{WithElse} & \rightarrow & \texttt{if}\ \mathit{Expr}\ \texttt{then}\ \mathit{WithElse}\ \texttt{else}\ \mathit{WithElse} \\
 & | & \mathit{Assignment}
\end{array}
$$

- When applying `if ... then` *WithElse* `else` *Statement*:
  - The ***true*** branch will be produced via *WithElse*.
  - The ***false*** branch will be produced via *Statement*.

  There is ***no circularity*** between the two non-terminals.

Discovering Derivations

- Given a CFG $G = (V, \Sigma, R, S)$ and an input program $p \in \Sigma^*$:
  - So far we **manually** come up a valid ***derivation*** s.t. $S \stackrel{*}{\Rightarrow} p$.
  - A *parser* is supposed to **automate** this ***derivation*** process.
    - Input: **A sequence of (t, c) pairs**, where each ***token*** t (e.g., `r241`) belongs to a ***syntactic category*** c (e.g., `register`); and a ***CFG*** G.
    - Output: A ***valid derivation*** (as an ***AST***); or A ***parse error***.
- In the process of constructing an ***AST*** for the input program:
  - ***Root*** of AST: The ***start symbol*** S of G
  - ***Internal nodes***: A ***subset of variables*** V of G
  - ***Leaves*** of AST: A ***token/terminal*** sequence

  $\Rightarrow$ Discovering the *grammatical connections* (w.r.t. R of G) between the ***root***, ***internal nodes***, and ***leaves*** is the hard part!
- Approaches to Parsing: $[\, w \in (V \cup \Sigma)^*,\ A \in V,\ \boxed{A \rightarrow w} \in R \,]$
  - **Top-down** parsing

    For a node representing ***A***, extend it with a subtree representing ***w***.
  - **Bottom-up** parsing

    For a substring matching ***w***, build a node representing ***A*** accordingly.

TDP: Discovering Leftmost Derivation

```
ALGORITHM: TDParse
  INPUT: CFG G = (V, Σ, R, S)
  OUTPUT: Root of a Parse Tree or Syntax Error
PROCEDURE:
  root := a new node for the start symbol S
  focus := root
  initialize an empty stack trace
  trace.push(null)
  word := NextWord()
  while (true):
    if focus ∈ V then
      if ∃ unvisited rule focus → β1β2...βn ∈ R then
        create β1, β2...βn as children of focus
        trace.push(βnβn−1...β2)
        focus := β1
      else
        if focus = S then report syntax error
        else backtrack
    elseif word matches focus then
      word := NextWord()
      focus := trace.pop()
    elseif word = EOF ∧ focus = null then return root
    else backtrack
```

backtrack ≜ pop *focus*.siblings; *focus* := *focus*.parent; *focus*.resetChildren

TDP: Exercise (1)

- Given the following CFG **G**:

$$
\begin{array}{lcl}
\mathit{Expr} & \rightarrow & \mathit{Expr} + \mathit{Term} \\
 & | & \mathit{Term} \\
\mathit{Term} & \rightarrow & \mathit{Term} \star \mathit{Factor} \\
 & | & \mathit{Factor} \\
\mathit{Factor} & \rightarrow & (\,\mathit{Expr}\,) \\
 & | & \mathtt{a}
\end{array}
$$

  Trace *TDParse* on how to build an AST for input `a + a * a`.
- Running *TDParse* with **G** results an ***infinite loop***!!!
  - *TDParse* focuses on the ***leftmost*** non-terminal.
  - The grammar **G** contains ***left-recursions***.
- We must first convert left-recursions in **G** to ***right-recursions***.

TDP: Exercise (2)

- Given the following CFG **G**:

$$
\begin{array}{lcl}
Expr & \rightarrow & Term \quad Expr' \\
Expr' & \rightarrow & + \; Term \quad Expr' \\
 & | & \epsilon \\
Term & \rightarrow & Factor \quad Term' \\
Term' & \rightarrow & \star \; Factor \quad Term' \\
 & | & \epsilon \\
Factor & \rightarrow & (\, Expr \,) \\
 & | & \mathtt{a}
\end{array}
$$

Exercise. Trace *TDParse* on building AST for `a + a * a`.
Exercise. Trace *TDParse* on building AST for `(a + a) * a`.
Q: How to handle ϵ-productions (e.g., $Expr \rightarrow \epsilon$)?
A: Execute $focus := trace.\mathtt{pop()}$ to advance to next node.

- Running *TDParse* will ***terminate*** $\because$ **G** is ***right-recursive***.
- We will learn about a systematic approach to converting left-recursions in a given grammar to ***right-recursions***.

Left-Recursions (LR): Direct vs. Indirect

Given CFG $G = (V, \Sigma, R, S)$, $\alpha, \beta, \gamma \in (V \cup \Sigma)^*$, G contains:

- A *cycle* if $\exists A \in V \bullet A \overset{*}{\Rightarrow} A$
- A ***direct*** LR if $A \rightarrow A\alpha \in R$ for non-terminal $A \in V$

e.g.,

$$\begin{array}{lcl} Expr & \rightarrow & Expr + Term \\ & \mid & Term \\ Term & \rightarrow & Term * Factor \\ & \mid & Factor \\ Factor & \rightarrow & (Expr) \\ & \mid & \texttt{a} \end{array}$$

e.g.,

$$\begin{array}{lcl} Expr & \rightarrow & Expr + Term \\ & \mid & Expr - Term \\ & \mid & Term \\ Term & \rightarrow & Term * Factor \\ & \mid & Term \ / \ Factor \\ & \mid & Factor \end{array}$$

- An ***indirect*** LR if $A \rightarrow B\beta \in R$ for non-terminals $A, B \in V$, $B \overset{*}{\Rightarrow} A\gamma$

$$\begin{array}{lcl} A & \rightarrow & Br \\ B & \rightarrow & Cd \\ C & \rightarrow & At \end{array}$$

$$A \rightarrow Br, B \overset{*}{\Rightarrow} Atd$$

$$\begin{array}{lcllll} A & \rightarrow & Ba & \mid & b & \\ B & \rightarrow & Cd & \mid & e & \\ C & \rightarrow & Df & \mid & g & \\ D & \rightarrow & f & \mid & Aa & \mid Cg \end{array}$$

$$A \rightarrow Ba, B \overset{*}{\Rightarrow} Aafd$$

TDP: (Preventively) Eliminating LRs

1 **ALGORITHM:** *RemoveLR*
2   **INPUT:** *CFG* $G = (V,\ \Sigma,\ R,\ S)$
3   **ASSUME:** G *has no* ϵ*-productions*
4   **OUTPUT:** G' *s.t.* $G' \equiv G$, G' *has no*
5   ***indirect*** & ***direct*** *left-recursions*
6 **PROCEDURE:**
7   *impose an order on* V: $\langle\langle A_1, A_2, \dots, A_n \rangle\rangle$
8   **for** i: $1 \ ..\ n$:
9   **for** j: $1 \ ..\ i-1$:
10   **if** $\exists\ A_i \rightarrow A_j\gamma \in R \ \wedge\ A_j \rightarrow \delta_1 \mid \delta_2 \mid \dots \mid \delta_m \in R$ **then**
11   ***replace*** $A_i \rightarrow A_j\gamma$ ***with*** $A_i \rightarrow \delta_1\gamma \mid \delta_2\gamma \mid \dots \mid \delta_m\gamma$
12   **end**
13   **for** $A_i \rightarrow A_i\alpha \mid \beta \in R$:
14   ***replace it with:*** $A_i \rightarrow \beta A_i',\ A_i' \rightarrow \alpha A_i' \mid \epsilon$

- **L9** to **L12**: Remove ***indirect*** left-recursions from A_1 to A_{i-1}.
- **L13** to **L14**: Remove ***direct*** left-recursions from A_1 to A_{i-1}.
- ***Loop Invariant*** **(outer for-loop)**? At the start of i^{th} iteration:
  - No ***direct*** or ***indirect*** left-recursions for $A_1, A_2, \dots, A_{i-1}$.
  - More precisely: $\forall j : j < i \bullet \neg(\exists k \bullet k \leq j \wedge A_j \rightarrow A_k \cdots \in R)$

CFG: Eliminating ϵ-Productions (1)

- Motivations:
  - ***TDParse*** handles each ϵ-production as a special case.
  - ***RemoveLR*** produces CFG which may contain ϵ-productions.
- $\epsilon \notin L \Rightarrow \exists$ CFG $G = (V, \Sigma, R, S)$ s.t. G has no ϵ-productions.

  An *ϵ-production* has the form $A \rightarrow \epsilon$.
- A variable A is ***nullable*** if $A \stackrel{*}{\Rightarrow} \epsilon$.
  - Each terminal symbol is ***not nullable***.
  - Variable A is ***nullable*** if either:
    - $A \rightarrow \epsilon \in R$; or
    - $A \rightarrow B_1 B_2 \dots B_k \in R$, where each variable B_i ($1 \leq i \leq k$) is a ***nullable***.
- Given a production $B \rightarrow CAD$, if only variable A is ***nullable***, then there are 2 versions of B: $B \rightarrow CAD \mid CD$
- In general, given a production $A \rightarrow X_1 X_2 \dots X_k$ with k symbols, if m of the k symbols are ***nullable***:
  - $m < k$: There are 2^m versions of A.
  - $m = k$: There are $2^m - 1$ versions of A. [excluding $A \rightarrow \epsilon$]

CFG: Eliminating ϵ-Productions (2)

- Eliminate ϵ-productions from the following grammar:

$$
\begin{array}{lcl}
S & \rightarrow & AB \\
A & \rightarrow & aAA \mid \epsilon \\
B & \rightarrow & bBB \mid \epsilon
\end{array}
$$

- Which are the ***nullable*** variables? [S, A, B]

$$
\begin{array}{lcll}
S & \rightarrow & A \mid B \mid AB & \{S \rightarrow \epsilon \texttt{ not included}\} \\
A & \rightarrow & aAA \mid aA \mid a & \{A \rightarrow aA \texttt{ duplicated}\} \\
B & \rightarrow & bBB \mid bB \mid b & \{B \rightarrow bB \texttt{ duplicated}\}
\end{array}
$$

Backtrack-Free Parsing (1)

- `TDParse` automates the ***top-down***, ***leftmost*** derivation process by consistently choosing production rules (e.g., in order of their appearance in CFG).
  - This ***inflexibility*** may lead to ***inefficient*** runtime performance due to the need to *backtrack*.
  - e.g., It may take the ***construction of a giant subtree*** to find out a ***mismatch*** with the input tokens, which end up requiring it to *backtrack* all the way back to the ***root*** (start symbol).
- We may avoid backtracking with a modification to the parser:
  - When deciding which production rule to choose, consider:
    (1) the ***current*** input symbol
    (2) the consequential ***first*** symbol if a rule was applied for `focus`
    [*lookahead* symbol]
  - Using a ***one symbol lookhead***, w.r.t. a ***right-recursive*** CFG, each alternative for the ***leftmost nonterminal*** leads to a ***unique terminal***, allowing the parser to decide on a choice that prevents *backtracking*.
  - Such CFG is ***backtrack free*** with the ***lookhead*** of one symbol.
  - We also call such backtrack-free CFG a ***predictive grammar***.

The FIRST Set: Definition

- Say we write $T \subset \mathbb{P}(\Sigma^*)$ to denote the set of valid tokens recognizable by the scanner.
- **FIRST** $(\alpha) \triangleq$ set of symbols that can appear as the ***first word*** in some string derived from α.
- More precisely:

$$\textbf{FIRST}(\alpha) = \begin{cases} \{\alpha\} & \text{if } \alpha \in T \\ \{w \mid w \in \Sigma^* \wedge \alpha \overset{*}{\Rightarrow} w\beta \wedge \beta \in (V \cup \Sigma)^*\} & \text{if } \alpha \in V \end{cases}$$

The FIRST Set: Examples

- Consider this ***right***-recursive CFG:

0	$Goal \rightarrow Expr$			6	$Term' \rightarrow$	x	$Factor\ Term'$
1	$Expr \rightarrow Term\ Expr'$			7	$\mid$	÷	$Factor\ Term'$
2	$Expr' \rightarrow$	+	$Term\ Expr'$	8	$\mid$	ϵ	
3	$\mid$	-	$Term\ Expr'$	9	$Factor \rightarrow$	(	$Expr$)
4	$\mid$	ϵ		10	$\mid$	num	
5	$Term \rightarrow Factor\ Term'$			11	$\mid$	name	

- Compute **FIRST** for each terminal (e.g., num, +, ( ):

	num	name	+	-	×	÷	(	)	eof	ϵ
FIRST	num	name	+	-	x	÷	(	)	eof	ϵ

- Compute **FIRST** for each non-terminal (e.g., *Expr*, *Term'*):

	Expr	***Expr'***	***Term***	***Term'***	***Factor***
FIRST	(, name, num	+, -, ϵ	(, name, num	x, ÷, ϵ	(, name, num

Computing the FIRST Set

$$\text{FIRST}(\alpha) = \begin{cases} \{\alpha\} & \text{if } \alpha \in T \\ \{w \mid w \in \Sigma^* \land \alpha \overset{*}{\Rightarrow} w\beta \land \beta \in (V \cup \Sigma)^*\} & \text{if } \alpha \in V \end{cases}$$

```
ALGORITHM: GetFirst
  INPUT: CFG G = (V, Σ, R, S)
  T ⊂ Σ* denotes valid terminals
  OUTPUT: FIRST : V ∪ T ∪ {ϵ, eof} ⟶ ℙ(T ∪ {ϵ, eof})
PROCEDURE:
  for α ∈ (T ∪ {eof, ϵ}): FIRST(α) := {α}
  for A ∈ V: FIRST(A) := ∅
  lastFirst := ∅
  while (lastFirst ≠ FIRST):
    lastFirst := FIRST
    for A → β1β2...βk ∈ R s.t. ∀βj : βj ∈ (T ∪ V):
      rhs := FIRST(β1) − {ϵ}
      for (i := 1; ϵ ∈ FIRST(βi) ∧ i < k; i++):
        rhs := rhs ∪ (FIRST(βi+1) − {ϵ})
      if i = k ∧ ϵ ∈ FIRST(βk) then
        rhs := rhs ∪ {ϵ}
      end
      FIRST(A) := FIRST(A) ∪ rhs
```

Computing the FIRST Set: Extension

- Recall: **FIRST** takes as input a token or a variable.

$$\textbf{FIRST} : V \cup T \cup \{\epsilon, eof\} \longrightarrow \mathbb{P}(T \cup \{\epsilon, eof\})$$

- The computation of variable ***rhs*** in algoritm `GetFirst` actually suggests an extended, overloaded version:

$$\textbf{FIRST} : (V \cup T \cup \{\epsilon, eof\})^* \longrightarrow \mathbb{P}(T \cup \{\epsilon, eof\})$$

FIRST may also take as input a string $\beta_1 \beta_2 \dots \beta_n$ (RHS of rules).

- More precisely:

$$\textbf{FIRST}(\beta_1 \beta_2 \dots \beta_n) = \left\{ \textbf{FIRST}(\beta_1) \cup \textbf{FIRST}(\beta_2) \cup \dots \cup \textbf{FIRST}(\beta_{k-1}) \cup \textbf{FIRST}(\beta_k) \;\middle|\; \begin{array}{l} \forall i : 1 \leq i < k \bullet \epsilon \in \textbf{FIRST}(\beta_i) \\ \wedge \\ \epsilon \notin \textbf{FIRST}(\beta_k) \end{array} \right.$$

Note. β_k is the first symbol whose **FIRST** set does not contain ϵ.

Extended FIRST Set: Examples

Consider this ***right***-recursive CFG:

0	*Goal*	$\rightarrow$	*Expr*			6	*Term'*	$\rightarrow$	x	*Factor Term'*
1	*Expr*	$\rightarrow$	*Term Expr'*			7		|	÷	*Factor Term'*
2	*Expr'*	$\rightarrow$	+	*Term Expr'*		8		|	ϵ	
3		|	-	*Term Expr'*		9	*Factor*	$\rightarrow$	(	*Expr*)
4		|	ϵ			10		|	num	
5	*Term*	$\rightarrow$	*Factor Term'*			11		|	name	

e.g., FIRST(*Term Expr'*) = FIRST(*Term*) = { (, name, num }

e.g., FIRST(+ *Term* *Expr'*) = FIRST(+) = {+}

e.g., FIRST(− *Term* *Expr'*) = FIRST(−) = {−}

e.g., FIRST(ϵ) = {ϵ}

Is the FIRST Set Sufficient

- Consider the following three productions:

$$\begin{array}{lllll} Expr' & \rightarrow & + & Term\ Term' & (1) \\ & \mid & - & Term\ Term' & (2) \\ & \mid & \epsilon & & (3) \end{array}$$

  In TDP, when the parser attempts to expand an *Expr′* node, it ***looks ahead with one symbol*** to decide on the choice of rule: **FIRST**$(+) = \{+\}$, **FIRST**$(-) = \{-\}$, and **FIRST**$(\epsilon) = \{\epsilon\}$.

  Q. When to choose rule (3) (causing ***focus := trace.pop()***)?

  A?. Choose rule (3) when *focus* $\neq$ **FIRST**$(+) \wedge$ *focus* $\neq$ **FIRST**$(-)$?

  - ***Correct*** but ***inefficient*** in case of illegal input string: syntax error is only reported after possibly a long series of ***backtrack***.
  - Useful if parser knows which words can appear, after an application of the ϵ-production (rule (3)), as leadling symbols.

- **FOLLOW** $(v : V) \triangleq$ set of symbols that can appear to the immediate right of a string derived from v.

$$\text{FOLLOW}(v) = \{\mathbf{w} \mid w, x, y \in \Sigma^* \wedge v \overset{*}{\Rightarrow} \mathbf{x} \wedge S \overset{*}{\Rightarrow} \mathbf{xwy}\}$$

The FOLLOW Set: Examples

- Consider this ***right***-recursive CFG:

0	*Goal*	$\rightarrow$	*Expr*	6	*Term′*	$\rightarrow$	x *Factor Term′*
1	*Expr*	$\rightarrow$	*Term Expr′*	7		|	÷ *Factor Term′*
2	*Expr′*	$\rightarrow$	+ *Term Expr′*	8		|	ϵ
3		|	- *Term Expr′*	9	*Factor*	$\rightarrow$	(*Expr*)
4		|	ϵ	10		|	num
5	*Term*	$\rightarrow$	*Factor Term′*	11		|	name

- Compute **FOLLOW** for each non-terminal (e.g., *Expr*, *Term′*):

	Expr	***Expr′***	***Term***	***Term′***	***Factor***
FOLLOW	eof,)	eof,)	eof, +, -,)	eof, +, -,)	eof, +, -, x, ÷,)

Computing the FOLLOW Set

$$\text{FOLLOW}(v) = \{ w \mid w, x, y \in \Sigma^* \wedge v \overset{*}{\Rightarrow} x \wedge S \overset{*}{\Rightarrow} xwy \}$$

```
ALGORITHM: GetFollow
  INPUT: CFG G = (V, Σ, R, S)
  OUTPUT: FOLLOW : V ⟶ ℙ(T ∪ {eof})
PROCEDURE:
  for A ∈ V: FOLLOW(A) := ∅
  FOLLOW(S) := {eof}
  lastFollow := ∅
  while (lastFollow ≠ FOLLOW):
    lastFollow := FOLLOW
    for A → β1β2...βk ∈ R:
      trailer := FOLLOW(A)
      for i: k .. 1:
        if βi ∈ V then
          FOLLOW(βi) := FOLLOW(βi) ∪ trailer
          if ϵ ∈ FIRST(βi)
            then trailer := trailer ∪ (FIRST(βi) − ϵ)
            else trailer := FIRST(βi)
        else
          trailer := FIRST(βi)
```

Backtrack-Free Grammar

- A ***backtrack-free grammar*** (for a **top-down parser**), when expanding the `focus` ***internal node***, is always able to choose a unique rule with the ***one-symbol lookahead*** (or report a ***syntax error*** when no rule applies).
- To formulate this, we first define:

$$\textsc{Start}(A \rightarrow \beta) = \begin{cases} \textsc{First}(\beta) & \text{if } \epsilon \notin \textsc{First}(\beta) \\ \textsc{First}(\beta) \cup \textsc{Follow}(A) & \text{otherwise} \end{cases}$$

$\textsc{First}(\beta)$ is the extended version where β may be $\beta_1 \beta_2 \dots \beta_n$

- A ***backtrack-free grammar*** has each of its productions $A \rightarrow \gamma_1 \mid \gamma_2 \mid \dots \mid \gamma_n$ satisfying:

$$\forall i, j : 1 \leq i, j \leq n \wedge i \neq j \bullet \textsc{Start}(\gamma_i) \cap \textsc{Start}(\gamma_j) = \varnothing$$

TDP: Lookahead with One Symbol

```
ALGORITHM: TDParse
  INPUT: CFG G = (V, Σ, R, S)
  OUTPUT: Root of a Parse Tree or Syntax Error
PROCEDURE:
  root := a new node for the start symbol S
  focus := root
  initialize an empty stack trace
  trace.push(null)
  word := NextWord()
  while (true):
    if focus ∈ V then
      if ∃ unvisited rule focus → β1 β2 ... βn ∈ R ∧ word ∈ START(β) then
        create β1, β2 ... βn as children of focus
        trace.push(βn βn−1 ... β2)
        focus := β1
      else
        if focus = S then report syntax error
        else backtrack
    elseif word matches focus then
      word := NextWord()
      focus := trace.pop()
    elseif word = EOF ∧ focus = null then return root
    else backtrack
```

backtrack ≙ pop *focus*.siblings; *focus* := *focus*.parent; *focus*.resetChildren

Backtrack-Free Grammar: Exercise

Is the following CFG ***backtrack free***?

11	*Factor*	$\rightarrow$	`name`
12		|	`name` `[` *ArgList* `]`
13		|	`name` `(` *ArgList* `)`
15	*ArgList*	$\rightarrow$	*Expr MoreArgs*
16	*MoreArgs*	$\rightarrow$	`,` *Expr MoreArgs*
17		|	ϵ

- $\epsilon \notin$ **FIRST**(*Factor*) $\Rightarrow$ **START**(*Factor*) = **FIRST**(*Factor*)
- **FIRST**(*Factor* → `name`) = {`name`}
- **FIRST**(*Factor* → `name` `[`*ArgList*`]`) = {`name`}
- **FIRST**(*Factor* → `name` `(`*ArgList*`)`) = {`name`}

$\therefore$ The above grammar is ***not*** backtrack free.
$\Rightarrow$ To expand an AST node of *Factor*, with a ***lookahead*** of `name`, the parser has no basis to choose among rules 11, 12, and 13.

Backtrack-Free Grammar: Left-Factoring

- A CFG is not backtrack free if there exists a ***common prefix*** (name) among the RHS of ***multiple*** production rules.
- To make such a CFG ***backtrack-free***, we may transform it using *left factoring*: a process of extracting and isolating ***common prefixes*** in a set of production rules.
  - Identify a common prefix α:

$$A \rightarrow \alpha\beta_1 \mid \alpha\beta_2 \mid \dots \mid \alpha\beta_n \mid \gamma_1 \mid \gamma_2 \mid \dots \mid \gamma_j$$

[each of $\gamma_1, \gamma_2, \dots, \gamma_j$ does not begin with α]

  - Rewrite that production rule as:

$$\begin{array}{lcl} A & \rightarrow & \alpha B \mid \gamma_1 \mid \gamma_2 \mid \dots \mid \gamma_j \\ B & \rightarrow & \beta_1 \mid \beta_2 \mid \dots \mid \beta_n \end{array}$$

  - New rule $B \rightarrow \beta_1 \mid \beta_2 \mid \dots \mid \beta_n$ may also contain ***common prefixes***.
  - Rewriting continues until no common prefixes are identified.

Left-Factoring: Exercise

- Use *left-factoring* to remove all ***common prefixes*** from the following grammar.

11	*Factor*	$\rightarrow$	`name`
12		|	`name` [*ArgList*]
13		|	`name` (*ArgList*)
15	*ArgList*	$\rightarrow$	*Expr MoreArgs*
16	*MoreArgs*	$\rightarrow$	, *Expr MoreArgs*
17		|	ϵ

- Identify common prefix `name` and rewrite rules 11, 12, and 13:

Factor	$\rightarrow$	`name` *Arguments*
Arguments	$\rightarrow$	[*ArgList*]
	|	(*ArgList*)
	|	ϵ

Any more ***common prefixes***? [No]

TDP: Terminating and Backtrack-Free

- Given an arbitrary CFG as input to a ***top-down parser***:
  - **Q.** How do we avoid a ***non-terminating*** parsing process?
    A. Convert left-recursions to right-recursion.
  - **Q.** How do we minimize the need of ***backtracking***?
    A. left-factoring & one-symbol lookahead using **START**
- ***Not*** every context-free language has a corresponding ***backtrack-free*** context-free grammar.

  Given a CFL l, the following is ***undecidable***:

$$\exists cfg \mid L(cfg) = l \wedge isBacktrackFree(cfg)$$

- Given a CFG $g = (V, \Sigma, R, S)$, whether or not g is ***backtrack-free*** is ***decidable***:

  For each $A \rightarrow \gamma_1 \mid \gamma_2 \mid \ldots \mid \gamma_n \in R$:

$$\forall i, j : 1 \leq i, j \leq n \wedge i \neq j \bullet \textbf{START}(\gamma_i) \cap \textbf{START}(\gamma_j) = \emptyset$$

Backtrack-Free Parsing (2.1)

- A *recursive-descent* parser is:
  - A top-down parser
  - Structured as a set of *mutually recursive* procedures

    Each procedure corresponds to a ***non-terminal*** in the grammar.

  See an example.
- Given a ***backtrack-free*** grammar, a tool (a.k.a. *parser generator*) can automatically generate:
  - **FIRST**, **FOLLOW**, and **START** sets
  - An efficient ***recursive-descent*** parser

  This generated parser is called an *LL(1) parser*, which:
    - Processes input from **L**eft to right
    - Constructs a **L**eftmost derivation
    - Uses a lookahead of **1** symbol
- *LL(1) grammars* are those working in an *LL(1)* scheme.

  LL(1) grammars are ***backtrack-free*** by definition.

Backtrack-Free Parsing (2.2)

- Consider this CFG with **START** sets of the RHSs:

	Production			FIRST$^+$
2	$Expr'$	$\rightarrow$	$+$ $Term$ $Expr'$	{ + }
3		|	- $Term$ $Expr'$	{ - }
4		|	ϵ	{ ϵ, eof,) }

- The corresponding ***recursive-descent*** parser is structured as:

```
ExprPrim()
  if word = + ∨ word = − then /* Rules 2, 3 */
    word := NextWord()
    if(Term())
      then return ExprPrim()
      else return false
  elseif word = ) ∨ word = eof then /* Rule 4 */
    return true
  else
    report a syntax error
    return false
  end

Term()
  ...
```

See: parser generator

LL(1) Parser: Exercise

Consider the following grammar:

$$L \rightarrow R\ \texttt{a} \mid Q\ \texttt{ba} \qquad R \rightarrow \texttt{aba} \mid \texttt{caba} \mid R\ \texttt{bc} \qquad Q \rightarrow \texttt{bbc} \mid \texttt{bc}$$

Q. Is it suitable for a ***top-down predictive*** parser?

- If so, show that it satisfies the *LL(1)* condition.
- If not, identify the problem(s) and correct it (them). Also show that the revised grammar satisfies the *LL(1)* condition.

BUP: Discovering Rightmost Derivation

- In TDP, we build the start variable as the ***root node***, and then work towards the ***leaves***. [**leftmost** derivation]
- In Bottom-Up Parsing (BUP):
  - Words (terminals) are still returned from **left** to **right** by the scanner.
  - As terminals, or a mix of terminals and variables, are identified as *reducible* to some variable *A* (i.e., matching the RHS of some production rule for *A*), then a layer is added.
  - Eventually:
    - ***accept***:
      The ***start variable*** is reduced and **all** words have been consumed.
    - ***reject***:
      The next word is not `eof`, but no further *reduction* can be identified.

Q. Why can BUP find the ***rightmost*** derivation (RMD), if any?

A. BUP discovers steps in a ***RMD*** in its *reverse* order.

BUP: Discovering Rightmost Derivation (1)

- ***table***-driven *LR(1)* parser: an implementation for BUP, which
  - Processes input from **L**eft to right
  - Constructs a **R**ightmost derivation
  - Uses a lookahead of **1** symbol
- A language has the *LR(1)* property if it:
  - Can be parsed in a single **L**eft to right scan,
  - To build a *reversed* **R**ightmost derivation,
  - Using a lookahead of **1** symbol to determine parsing actions.
- Critical step in a ***bottom-up parser*** is to find the ***next*** *handle*.

BUP: Discovering Rightmost Derivation (2)

```
ALGORITHM: BUParse
  INPUT: CFG G = (V, Σ, R, S), Action & Goto Tables
  OUTPUT: Report Parse Success or Syntax Error
PROCEDURE:
  initialize an empty stack trace
  trace.push(0) /* start state */
  word := NextWord()
  while(true)
    state := trace.top()
    act := Action[state, word]
    if act = ``accept'' then
      succeed()
    elseif act = ``reduce based on A → β'' then
      trace.pop() 2 × |β| times /* word + state */
      state := trace.top()
      trace.push(A)
      next := Goto[state, A]
      trace.push(next)
    elseif act = ``shift to s_i'' then
      trace.push(word)
      trace.push(i)
      word := NextWord()
    else
      fail()
```

BUP: Example Tracing (1)

- Consider the following grammar for parentheses:

1	*Goal*	$\rightarrow$	*List*
2	*List*	$\rightarrow$	*List Pair*
3		|	*Pair*
4	*Pair*	$\rightarrow$	(*Pair*)
5		|	( )

- Assume: tables ***Action*** and ***Goto*** constructed accordingly:

State	Action Table			Goto Table	
	eof	(	)	***List***	***Pair***
0		s 3		1	2
1	acc	s 3			4
2	r 3	r 3			
3		s 6	s 7		5
4	r 2	r 2			
5			s 8		
6		s 6	s 10		9
7	r 5	r 5			
8	r 4	r 4			
9			s 11		
10			r 5		
11			r 4		

In ***Action*** table:

- s_i: shift to state i
- r_j: reduce to the LHS of production #j

BUP: Example Tracing (2.1)

Consider the steps of performing BUP on input `()`:

Iteration	State	word	Stack	Handle	Action
initial	—	(	$ 0	— *none* —	—
1	0	(	$ 0	— *none* —	*shift 3*
2	3	)	$ 0 (3	— *none* —	*shift 7*
3	7	eof	$ 0 (3) 7	( )	*reduce 5*
4	2	eof	$ 0 *Pair* 2	*Pair*	*reduce 3*
5	1	eof	$ 0 *List* 1	*List*	*accept*

BUP: Example Tracing (2.2)

Consider the steps of performing BUP on input (( )) ( ):

Iteration	State	word	Stack	Handle	Action
initial	—	(	$ 0	— *none* —	—
1	0	(	$ 0	— *none* —	*shift 3*
2	3	(	$ 0 (3	— *none* —	*shift 6*
3	6	)	$ 0 (3 (6	— *none* —	*shift 10*
4	10	)	$ 0 (3 (6) 10	( )	*reduce 5*
5	5	)	$ 0 (3 *Pair* 5	— *none* —	*shift 8*
6	8	(	$ 0 (3 *Pair* 5) 8	(*Pair*)	*reduce 4*
7	2	(	$ 0 *Pair* 2	*Pair*	*reduce 3*
8	1	(	$ 0 *List* 1	— *none* —	*shift 3*
9	3	)	$ 0 *List* 1 (3	— *none* —	*shift 7*
10	7	eof	$ 0 *List* 1 (3) 7	( )	*reduce 5*
11	4	eof	$ 0 *List* 1 *Pair* 4	*List Pair*	*reduce 2*
12	1	eof	$ 0 *List* 1	*List*	*accept*

BUP: Example Tracing (2.3)

Consider the steps of performing BUP on input `( ) )`:

Iteration	State	word	Stack	Handle	Action
initial	—	(	$ 0	— *none* —	—
1	0	(	$ 0	— *none* —	*shift 3*
2	3	)	$ 0 (3	— *none* —	*shift 7*
3	7	)	$ 0 (3) 7	— *none* —	*error*

LR(1) Items: Definition

- In **LR(1)** parsing, ***Action*** and ***Goto*** tabeles encode legitimate ways (w.r.t. a CFG) for finding ***handles*** (for ***reductions***).
- In a ***table***-driven **LR(1)** parser, the table-construction algorithm represents each potential ***handle*** (for a ***reduction***) with an **LR(1)** item e.g.,

$$[A \rightarrow \beta \bullet \gamma,\ \mathtt{a}]$$

where:

  - A ***production rule*** $\boxed{A \rightarrow \beta\gamma}$ is currently being applied.
  - A ***terminal symbol*** $\boxed{\mathtt{a}}$ servers as a ***lookahead symbol***.
  - A ***placeholder*** $\boxed{\bullet}$ indicates the parser's ***stack top***.
    - ✓ The parser's ***stack*** contains β ("left context").
    - ✓ γ is yet to be matched.
    - Upon matching $\beta\gamma$, if $\mathtt{a}$ matches the current word, then we "replace" $\beta\gamma$ (and their associated states) with A (and its associated state).

LR(1) Items: Scenarios

An *LR(1) item* can denote:

1. **POSSIBILITY** $[A \to \bullet\beta\gamma,\ \mathtt{a}]$
   - In the current parsing context, an A would be valid.
   - $\bullet$ represents the position of the parser's ***stack top***
   - Recognizing a β next would be one step towards discovering an A.
2. **PARTIAL COMPLETION** $[A \to \beta \bullet \gamma,\ \mathtt{a}]$
   - The parser has progressed from $[A \to \bullet\beta\gamma,\ \mathtt{a}]$ by recognizing β.
   - Recognizing a γ next would be one step towards discovering an A.
3. **COMPLETION** $[A \to \beta\gamma\bullet,\ \mathtt{a}]$
   - Parser has progressed from $[A \to \bullet\beta\gamma,\ \mathtt{a}]$ by recognizing $\beta\gamma$.
   - $\beta\gamma$ found in a context where an A followed by $\mathtt{a}$ would be valid.
   - If the current input word matches $\mathtt{a}$, then:
     - Current ***complet item*** is a *handle*.
     - Parser can ***reduce*** $\beta\gamma$ to A
     - Accordingly, in the ***stack***, $\beta\gamma$ (and their associated states) are replaced with A (and its associated state).

LR(1) Items: Example (1.1)

Consider the following grammar for parentheses:

1	*Goal*	$\rightarrow$	*List*
2	*List*	$\rightarrow$	*List Pair*
3		|	*Pair*
4	*Pair*	$\rightarrow$	$\underline{(}$ *Pair* $\underline{)}$
5		|	$\underline{(}$ $\underline{)}$

Initial State: $[Goal \rightarrow \bullet List, \texttt{eof}]$

Desired Final State: $[Goal \rightarrow List \bullet, \texttt{eof}]$

Intermediate States: Subset Construction

Q. Derive all *LR(1) items* for the above grammar.

- FOLLOW(*List*) = $\{\texttt{eof}, \texttt{(}\}$ FOLLOW(*Pair*) = $\{\texttt{eof}, \texttt{(}, \texttt{)}\}$
- For each production $A \rightarrow \beta$, given FOLLOW(A), *LR(1) items* are:

$$\{\, [A \rightarrow \bullet\beta\gamma,\ \texttt{a}] \mid a \in \text{FOLLOW}(A) \,\}$$
$$\cup$$
$$\{\, [A \rightarrow \beta \bullet \gamma,\ \texttt{a}] \mid a \in \text{FOLLOW}(A) \,\}$$
$$\cup$$
$$\{\, [A \rightarrow \beta\gamma\bullet,\ \texttt{a}] \mid a \in \text{FOLLOW}(A) \,\}$$

LR(1) Items: Example (1.2)

Q. Given production $A \rightarrow \beta$ (e.g., $Pair \rightarrow$ `(` $Pair$ `)`), how many *LR(1) items* can be generated?

- The current parsing progress (on matching the RHS) can be:
  1. $\bullet$ `(` *Pair* `)`
  2. `(` $\bullet$*Pair* `)`
  3. `(` *Pair*$\bullet$ `)`
  4. `(` *Pair* `)`$\bullet$
- Lookahead symbol following *Pair*? **FOLLOW**$(Pair) = \{$`eof`, `(`, `)`$\}$
- All possible *LR(1) items* related to $Pair \rightarrow$ `(` $Pair$ `)`?

✓ [$\bullet$(*Pair*), eof]	[$\bullet$(*Pair*), (]	[$\bullet$(*Pair*),)]
✓ [($\bullet$*Pair*), eof]	[($\bullet$*Pair*), (]	[($\bullet$*Pair*),)]
✓ [(*Pair*$\bullet$), eof]	[(*Pair*$\bullet$), (]	[(*Pair*$\bullet$),)]
✓ [(*Pair*)$\bullet$, eof]	[(*Pair*)$\bullet$, (]	[(*Pair*)$\bullet$,)]

A. How many in general (in terms of A and β)?

$$\underbrace{|\beta| + 1}_{\text{possible positions of } \bullet} \times \underbrace{|\textbf{FOLLOW}(A)|}_{\text{possible lookahead symbols}}$$

LR(1) Items: Example (1.3)

A. There are 33 *LR(1) items* in the parentheses grammar.

$[Goal \rightarrow \bullet\ List, \texttt{eof}]$		
$[Goal \rightarrow List\ \bullet, \texttt{eof}]$		
$[List \rightarrow \bullet\ List\ \ Pair, \texttt{eof}]$	$[List \rightarrow \bullet\ List\ \ Pair, \underline{(}\,]$	
$[List \rightarrow List\ \bullet\ Pair, \texttt{eof}]$	$[List \rightarrow List\ \bullet\ Pair, \underline{(}\,]$	
$[List \rightarrow List\ \ Pair\ \bullet, \texttt{eof}]$	$[List \rightarrow List\ \ Pair\ \bullet, \underline{(}\,]$	
$[List \rightarrow \bullet\ Pair, \texttt{eof}]$	$[List \rightarrow \bullet\ Pair, \underline{(}\,]$	
$[List \rightarrow Pair\ \bullet, \texttt{eof}]$	$[List \rightarrow Pair\ \bullet, \underline{(}\,]$	
$[Pair \rightarrow \bullet\ \underline{(}\ Pair\ \underline{)}, \texttt{eof}]$	$[Pair \rightarrow \bullet\ \underline{(}\ Pair\ \underline{)}, \underline{)}]$	$[Pair \rightarrow \bullet\ \underline{(}\ Pair\ \underline{)}, \underline{(}]$
$[Pair \rightarrow \underline{(}\ \bullet\ Pair\ \underline{)}, \texttt{eof}]$	$[Pair \rightarrow \underline{(}\ \bullet\ Pair\ \underline{)}, \underline{)}]$	$[Pair \rightarrow \underline{(}\ \bullet\ Pair\ \underline{)}, \underline{(}]$
$[Pair \rightarrow \underline{(}\ Pair\ \bullet\ \underline{)}, \texttt{eof}]$	$[Pair \rightarrow \underline{(}\ Pair\ \bullet\ \underline{)}, \underline{)}]$	$[Pair \rightarrow \underline{(}\ Pair\ \bullet\ \underline{)}, \underline{(}]$
$[Pair \rightarrow \underline{(}\ Pair\ \underline{)}\ \bullet, \texttt{eof}]$	$[Pair \rightarrow \underline{(}\ Pair\ \underline{)}\ \bullet, \underline{)}]$	$[Pair \rightarrow \underline{(}\ Pair\ \underline{)}\ \bullet, \underline{(}]$
$[Pair \rightarrow \bullet\ \underline{(}\ \ \underline{)}, \texttt{eof}]$	$[Pair \rightarrow \bullet\ \underline{(}\ \ \underline{)}, \underline{(}]$	$[Pair \rightarrow \bullet\ \underline{(}\ \ \underline{)}, \underline{)}]$
$[Pair \rightarrow \underline{(}\ \bullet\ \underline{)}, \texttt{eof}]$	$[Pair \rightarrow \underline{(}\ \bullet\ \underline{)}, \underline{(}]$	$[Pair \rightarrow \underline{(}\ \bullet\ \underline{)}, \underline{)}]$
$[Pair \rightarrow \underline{(}\ \ \underline{)}\ \bullet, \texttt{eof}]$	$[Pair \rightarrow \underline{(}\ \ \underline{)}\ \bullet, \underline{(}]$	$[Pair \rightarrow \underline{(}\ \ \underline{)}\ \bullet, \underline{)}]$

LR(1) Items: Example (2)

Consider the following grammar for expressions:

0	$Goal \rightarrow$	$Expr$		6	$Term' \rightarrow$	x $Factor\ Term'$
1	$Expr \rightarrow$	$Term\ Expr'$		7	$\mid$	$\div$ $Factor\ Term'$
2	$Expr' \rightarrow$	$+$ $Term\ Expr'$		8	$\mid$	ϵ
3	$\mid$	- $Term\ Expr'$		9	$Factor \rightarrow$	($Expr$)
4	$\mid$	ϵ		10	$\mid$	num
5	$Term \rightarrow$	$Factor\ Term'$		11	$\mid$	name

Q. Derive all *LR(1) items* for the above grammar.

Hints. First compute **FOLLOW** for each non-terminal:

	Expr	*Expr'*	*Term*	*Term'*	*Factor*
FOLLOW	eof,)	eof,)	eof, +, -,)	eof, +, -,)	eof, +, -, x, ÷,)

Tips. Ignore ϵ ***production*** such as $Expr' \rightarrow \epsilon$

since the **FOLLOW** sets already take them into consideration.

Canonical Collection ($\mathcal{CC}$) vs. LR(1) items

1	*Goal*	$\rightarrow$	*List*
2	*List*	$\rightarrow$	*List Pair*
3		|	*Pair*
4	*Pair*	$\rightarrow$	(*Pair*)
5		|	( )

Recall:

LR(1) Items: 33 items

Initial State: $[Goal \rightarrow \bullet List, \texttt{eof}]$

Desired Final State: $[Goal \rightarrow List\bullet, \texttt{eof}]$

- The *canonical collection* [Example of $\mathcal{CC}$]

$$\mathcal{CC} = \{cc_0, cc_1, cc_2, \ldots, cc_n\}$$

  denotes the set of **valid subset states** of a **LR(1) parser**.
  - Each $cc_i \in \mathcal{CC}$ $(0 \leq i \leq n)$ is a set of ***LR(1) items***.
  - $\mathcal{CC} \subseteq \mathbb{P}($***LR(1) items***$)$ $|\mathcal{CC}|$? [$|\mathcal{CC}| \leq 2^{|\textit{LR(1) items}|}$]
- To model a **LR(1) parser**, we use techniques analogous to how an ϵ-NFA is converted into a DFA (subset construction and ϵ-closure).
- **Analogies.**
  - ✓ ***LR(1) items*** $\approx$ states of source *NFA*
  - ✓ $\mathcal{CC}$ $\approx$ subset states of target *DFA*

Constructing $\mathcal{CC}$: The *closure* Procedure (1)

```
1   ALGORITHM: closure
2     INPUT: CFG G = (V, Σ, R, S), a set s of LR(1) items
3     OUTPUT: a set of LR(1) items
4   PROCEDURE:
5     lastS := ∅
6     while (lastS ≠ s):
7       lastS := s
8       for [A → ··· • C δ, a] ∈ s:
9         for C → γ ∈ R:
10          for b ∈ FIRST(δa):
11            s := s ∪ { [C → •γ, b] }
12    return s
```

- **Line 8**: $[A \rightarrow \cdots \bullet\ C\ \delta,\ a] \in s$ indicates that the parser's next task is to match $C\ \delta$ with a lookahead symbol a.
- **Line 9**: Given: matching γ can reduce to C
- **Line 10**: Given: $\mathrm{b} \in \textsc{First}(\delta a)$ is a valid lookahead symbol after reducing γ to C
- **Line 11**: Add a new item $[C \rightarrow \bullet\gamma,\ \mathrm{b}]$ into *s*.
- **Line 6**: Termination is guaranteed.

  ∵ Each iteration adds ≥ 1 item to *s* (otherwise $lastS \neq s$ is *false*).

Constructing $\mathcal{CC}$: The *closure* Procedure (2.1)

1	*Goal*	$\rightarrow$	*List*
2	*List*	$\rightarrow$	*List Pair*
3		$\mid$	*Pair*
4	*Pair*	$\rightarrow$	$\underline{(}$ *Pair* $\underline{)}$
5		$\mid$	$\underline{(}$ $\underline{)}$

Initial State: $[Goal \rightarrow \bullet List,\ \texttt{eof}]$

Calculate cc_0 = ***closure***$(\{\ [Goal \rightarrow \bullet List,\ \texttt{eof}]\ \})$.

Constructing $\mathcal{CC}$: The *goto* Procedure (1)

```
1  ALGORITHM: goto
2    INPUT: a set s of LR(1) items, a symbol x
3    OUTPUT: a set of LR(1) items
4  PROCEDURE:
5    moved := ∅
6    for item ∈ s:
7      if item = [α → β • xδ, a] then
8        moved := moved ∪ { [α → βx • δ, a] }
9      end
10   return closure(moved)
```

Line 7: Given: item $[\alpha \rightarrow \beta \bullet x\delta,\ \mathtt{a}]$ (where x is the next to match)
Line 8: Add $[\alpha \rightarrow \beta x \bullet \delta,\ \mathtt{a}]$ (indicating $\mathtt{x}$ is matched) to *moved*
Line 10: Calculate and return ***closure***(*moved*) as the "***next subset state***" from *s* with a "transition" $\mathtt{x}$.

Constructing $\mathcal{CC}$: The *goto* Procedure (2)

1	*Goal*	$\rightarrow$	*List*
2	*List*	$\rightarrow$	*List Pair*
3		|	*Pair*
4	*Pair*	$\rightarrow$	$\underline{(}$ *Pair* $\underline{)}$
5		|	$\underline{(}$ $\underline{)}$

$$cc_0 = \left\{ \begin{array}{ccc} [Goal \rightarrow \bullet\, List, \texttt{eof}] & [List \rightarrow \bullet\, List\ Pair, \texttt{eof}] & [List \rightarrow \bullet\, List\ Pair, \underline{(}] \\ [List \rightarrow \bullet\, Pair, \texttt{eof}] & [List \rightarrow \bullet\, Pair, \underline{(}] & [Pair \rightarrow \bullet\, \underline{(}\ Pair\ \underline{)}, \texttt{eof}] \\ [Pair \rightarrow \bullet\, \underline{(}\ Pair\ \underline{)}, \underline{(}] & [Pair \rightarrow \bullet\, \underline{(}\ \underline{)}, \texttt{eof}] & [Pair \rightarrow \bullet\, \underline{(}\ \underline{)}, \underline{(}] \end{array} \right\}$$

Calculate *goto*(cc_0, ( ). [“next state” from cc_0 taking (]

Constructing $\mathcal{CC}$: The Algorithm (1)

```
ALGORITHM: BuildCC
  INPUT: a grammar G = (V, Σ, R, S),  goal production S → S′
  OUTPUT:
    (1) a set CC = {cc0, cc1, ..., ccn} where cci ⊆ G′s LR(1) items
    (2) a transition function
PROCEDURE:
  cc0 := closure({[S → •S′, eof]})
  CC := {cc0}
  processed := {cc0}
  lastCC := ∅
  while(lastCC ≠ CC):
    lastCC := CC
    for cci s.t. cci ∈ CC ∧ cci ∉ processed:
      processed := processed ∪ {cci}
      for x s.t. [··· → ··· • x ...] ∈ cci
        temp := goto(cci, x)
        if temp ∉ CC then
          CC := CC ∪ {temp}
        end
        δ := δ ∪ (cci, x, temp)
```

Constructing $\mathcal{CC}$: The Algorithm (2.1)

1	*Goal*	$\rightarrow$	*List*
2	*List*	$\rightarrow$	*List Pair*
3		$\mid$	*Pair*
4	*Pair*	$\rightarrow$	$\underline{(}$ *Pair* $\underline{)}$
5		$\mid$	$\underline{(}$ $\underline{)}$

- Calculate $\mathcal{CC} = \{cc_0, cc_1, \ldots, cc_{11}\}$
- Calculate the transition function $\delta : \mathcal{CC} \times (\Sigma \cup V) \rightarrow \mathcal{CC}$

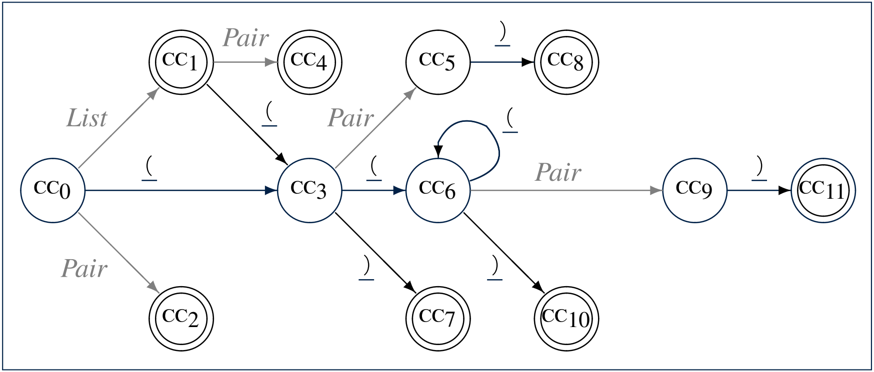
Constructing $\mathcal{CC}$: The Algorithm (2.2)

Resulting transition table:

Iteration	Item	*Goal*	*List*	*Pair*	(	)	eof
0	cc_0	∅	cc_1	cc_2	cc_3	∅	∅
1	cc_1	∅	∅	cc_4	cc_3	∅	∅
	cc_2	∅	∅	∅	∅	∅	∅
	cc_3	∅	∅	cc_5	cc_6	cc_7	∅
2	cc_4	∅	∅	∅	∅	∅	∅
	cc_5	∅	∅	∅	∅	cc_8	∅
	cc_6	∅	∅	cc_9	cc_6	cc_{10}	∅
	cc_7	∅	∅	∅	∅	∅	∅
3	cc_8	∅	∅	∅	∅	∅	∅
	cc_9	∅	∅	∅	∅	cc_{11}	∅
	cc_{10}	∅	∅	∅	∅	∅	∅
4	cc_{11}	∅	∅	∅	∅	∅	∅

Constructing $\mathcal{CC}$: The Algorithm (2.3)

Resulting DFA for the parser:



Constructing $\mathcal{CC}$: The Algorithm (2.4.1)

Resulting canonical collection $\mathcal{CC}$: [Def. of $\mathcal{CC}$]

$$cc_0 = \left\{ \begin{array}{ccc} [\mathit{Goal} \rightarrow \bullet\, \mathit{List}, \texttt{eof}] & [\mathit{List} \rightarrow \bullet\, \mathit{List}\ \mathit{Pair}, \texttt{eof}] & [\mathit{List} \rightarrow \bullet\, \mathit{List}\ \mathit{Pair}, \underline{(}] \\ [\mathit{List} \rightarrow \bullet\, \mathit{Pair}, \texttt{eof}] & [\mathit{List} \rightarrow \bullet\, \mathit{Pair}, \underline{(}] & [\mathit{Pair} \rightarrow \bullet\, \underline{(}\ \mathit{Pair}\ \underline{)}, \texttt{eof}] \\ [\mathit{Pair} \rightarrow \bullet\, \underline{(}\ \mathit{Pair}\ \underline{)}, \underline{(}] & [\mathit{Pair} \rightarrow \bullet\, \underline{(}\ \underline{)}, \texttt{eof}] & [\mathit{Pair} \rightarrow \bullet\, \underline{(}\ \underline{)}, \underline{(}] \end{array} \right\}$$

$$cc_1 = \left\{ \begin{array}{ccc} [\mathit{Goal} \rightarrow \mathit{List}\, \bullet, \texttt{eof}] & [\mathit{List} \rightarrow \mathit{List}\, \bullet\, \mathit{Pair}, \texttt{eof}] & [\mathit{List} \rightarrow \mathit{List}\, \bullet\, \mathit{Pair}, \underline{(}] \\ [\mathit{Pair} \rightarrow \bullet\, \underline{(}\ \mathit{Pair}\ \underline{)}, \texttt{eof}] & [\mathit{Pair} \rightarrow \bullet\, \underline{(}\ \mathit{Pair}\ \underline{)}, \underline{(}] & [\mathit{Pair} \rightarrow \bullet\, \underline{(}\ \underline{)}, \texttt{eof}] \\ & [\mathit{Pair} \rightarrow \bullet\, \underline{(}\ \underline{)}, \underline{(}] & \end{array} \right\}$$

$$cc_2 = \left\{ [\mathit{List} \rightarrow \mathit{Pair}\, \bullet, \texttt{eof}] \quad [\mathit{List} \rightarrow \mathit{Pair}\, \bullet, \underline{(}] \right\}$$

$$cc_3 = \left\{ \begin{array}{ccc} [\mathit{Pair} \rightarrow \bullet\, \underline{(}\ \mathit{Pair}\ \underline{)}, \underline{)}] & [\mathit{Pair} \rightarrow \underline{(}\, \bullet\, \mathit{Pair}\ \underline{)}, \texttt{eof}] & [\mathit{Pair} \rightarrow \underline{(}\, \bullet\, \mathit{Pair}\ \underline{)}, \underline{(}] \\ [\mathit{Pair} \rightarrow \bullet\, \underline{(}\ \underline{)}, \underline{)}] & [\mathit{Pair} \rightarrow \underline{(}\, \bullet\, \underline{)}, \texttt{eof}] & [\mathit{Pair} \rightarrow \underline{(}\, \bullet\, \underline{)}, \underline{(}] \end{array} \right\}$$

$$cc_4 = \left\{ [\mathit{List} \rightarrow \mathit{List}\ \mathit{Pair}\, \bullet, \texttt{eof}] \quad [\mathit{List} \rightarrow \mathit{List}\ \mathit{Pair}\, \bullet, \underline{(}] \right\}$$

$$cc_5 = \left\{ [\mathit{Pair} \rightarrow \underline{(}\ \mathit{Pair}\, \bullet\, \underline{)}, \texttt{eof}] \quad [\mathit{Pair} \rightarrow \underline{(}\ \mathit{Pair}\, \bullet\, \underline{)}, \underline{(}] \right\}$$

$$cc_6 = \left\{ \begin{array}{cc} [\mathit{Pair} \rightarrow \bullet\, \underline{(}\ \mathit{Pair}\ \underline{)}, \underline{)}] & [\mathit{Pair} \rightarrow \underline{(}\, \bullet\, \mathit{Pair}\ \underline{)}, \underline{)}] \\ [\mathit{Pair} \rightarrow \bullet\, \underline{(}\ \underline{)}, \underline{)}] & [\mathit{Pair} \rightarrow \underline{(}\, \bullet\, \underline{)}, \underline{)}] \end{array} \right\}$$

$$cc_7 = \left\{ [\mathit{Pair} \rightarrow \underline{(}\ \underline{)}\, \bullet, \texttt{eof}] \quad [\mathit{Pair} \rightarrow \underline{(}\ \underline{)}\, \bullet, \underline{(}] \right\}$$

$$cc_8 = \left\{ [\mathit{Pair} \rightarrow \underline{(}\ \mathit{Pair}\ \underline{)}\, \bullet, \texttt{eof}] \quad [\mathit{Pair} \rightarrow \underline{(}\ \mathit{Pair}\ \underline{)}\, \bullet, \underline{(}] \right\}$$

$$cc_9 = \left\{ [\mathit{Pair} \rightarrow \underline{(}\ \mathit{Pair}\, \bullet\, \underline{)}, \underline{)}] \right\}$$

$$cc_{10} = \left\{ [\mathit{Pair} \rightarrow \underline{(}\ \underline{)}\, \bullet, \underline{)}] \right\}$$

$$cc_{11} = \left\{ [\mathit{Pair} \rightarrow \underline{(}\ \mathit{Pair}\ \underline{)}\, \bullet, \underline{)}] \right\}$$

Constructing *Action* and *Goto* Tables (1)

```
1  ALGORITHM: BuildActionGotoTables
2    INPUT:
3      (1) a grammar G = (V, Σ, R, S)
4      (2) goal production S → S′
5      (3) a canonical collection CC = {cc_0, cc_1, ..., cc_n}
6      (4) a transition function δ : CC × Σ → CC
7    OUTPUT: Action Table & Goto Table
8  PROCEDURE:
9    for cc_i ∈ CC:
10     for item ∈ cc_i:
11       if item = [A → β • xγ, a] ∧ δ(cc_i, x) = cc_j then
12         Action[i, x] := shift j
13       elseif item = [A → β•, a] then
14         Action[i, a] := reduce A → β
15       elseif item = [S → S′•, eof] then
16         Action[i, eof] := accept
17       end
18     for v ∈ V:
19       if δ(cc_i, v) = cc_j then
20         Goto[i, v] = j
21       end
```

- **L12, 13**: Next valid step in discovering A is to match terminal symbol x.
- **L14, 15**: Having recognized β, if current word matches lookahead a, reduce β to A.
- **L16, 17**: Accept if input exhausted and what's recognized reducible to start var. S.
- **L20, 21**: Record consequence of a reduction to non-terminal v from state i

Constructing *Action* and *Goto* Tables (2)

Resulting **Action** and **Goto** tables:

State	*Action* Table			*Goto* Table	
	eof	(	)	***List***	***Pair***
0		s 3		1	2
1	acc	s 3			4
2	r 3	r 3			
3		s 6	s 7		5
4	r 2	r 2			
5			s 8		
6		s 6	s 10		9
7	r 5	r 5			
8	r 4	r 4			
9			s 11		
10			r 5		
11			r 4		

BUP: Discovering Ambiguity (1)

1	*Goal*	$\rightarrow$	*Stmt*
2	*Stmt*	$\rightarrow$	`if expr then` *Stmt*
3		|	`if expr then` *Stmt* `else` *Stmt*
4		|	`assign`

- Calculate $\mathcal{CC} = \{cc_0, cc_1, \ldots,\}$
- Calculate the transition function $\delta : \mathcal{CC} \times \Sigma \rightarrow \mathcal{CC}$

BUP: Discovering Ambiguity (2.1)

Resulting transition table:

	Item	*Goal*	*Stmt*	if	expr	then	else	assign	eof
0	cc_0	∅	cc_1	cc_2	∅	∅	∅	cc_3	∅
1	cc_1	∅	∅	∅	∅	∅	∅	∅	∅
	cc_2	∅	∅	∅	cc_4	∅	∅	∅	∅
	cc_3	∅	∅	∅	∅	∅	∅	∅	∅
2	cc_4	∅	∅	∅	∅	cc_5	∅	∅	∅
3	cc_5	∅	cc_6	cc_7	∅	∅	∅	cc_8	∅
4	cc_6	∅	∅	∅	∅	∅	cc_9	∅	∅
	cc_7	∅	∅	∅	cc_{10}	∅	∅	∅	∅
	cc_8	∅	∅	∅	∅	∅	∅	∅	∅
5	cc_9	∅	cc_{11}	cc_2	∅	∅	∅	cc_3	∅
	cc_{10}	∅	∅	∅	∅	cc_{12}	∅	∅	∅
6	cc_{11}	∅	∅	∅	∅	∅	∅	∅	∅
	cc_{12}	∅	cc_{13}	cc_7	∅	∅	∅	cc_8	∅
7	cc_{13}	∅	∅	∅	∅	∅	cc_{14}	∅	∅
8	cc_{14}	∅	cc_{15}	cc_7	∅	∅	∅	cc_8	∅
9	cc_{15}	∅	∅	∅	∅	∅	∅	∅	∅

BUP: Discovering Ambiguity (2.2.1)

Resulting canonical collection $\mathcal{CC}$:

$$cc_0 = \left\{ \begin{array}{ll} [Goal \rightarrow \bullet\ Stmt, \texttt{eof}] & [Stmt \rightarrow \bullet\ \texttt{if expr then}\ Stmt, \texttt{eof}] \\ [Stmt \rightarrow \bullet\ \texttt{assign}, \texttt{eof}] & [Stmt \rightarrow \bullet\ \texttt{if expr then}\ Stmt\ \texttt{else}\ Stmt, \texttt{eof}] \end{array} \right\}$$

$$cc_1 = \left\{ [Goal \rightarrow Stmt\ \bullet, \texttt{eof}] \right\}$$

$$cc_2 = \left\{ \begin{array}{l} [Stmt \rightarrow \texttt{if}\ \bullet\ \texttt{expr then}\ Stmt, \texttt{eof}], \\ [Stmt \rightarrow \texttt{if}\ \bullet\ \texttt{expr then}\ Stmt\ \texttt{else}\ Stmt, \texttt{eof}] \end{array} \right\}$$

$$cc_3 = \left\{ [Stmt \rightarrow \texttt{assign}\ \bullet, \texttt{eof}] \right\}$$

$$cc_4 = \left\{ \begin{array}{l} [Stmt \rightarrow \texttt{if expr}\ \bullet\ \texttt{then}\ Stmt, \texttt{eof}], \\ [Stmt \rightarrow \texttt{if expr}\ \bullet\ \texttt{then}\ Stmt\ \texttt{else}\ Stmt, \texttt{eof}] \end{array} \right\}$$

$$cc_5 = \left\{ \begin{array}{l} [Stmt \rightarrow \texttt{if expr then}\ \bullet\ Stmt, \texttt{eof}], \\ [Stmt \rightarrow \texttt{if expr then}\ \bullet\ Stmt\ \texttt{else}\ Stmt, \texttt{eof}], \\ [Stmt \rightarrow \bullet\ \texttt{if expr then}\ Stmt, \{\texttt{eof}, \texttt{else}\}], \\ [Stmt \rightarrow \bullet\ \texttt{assign}, \{\texttt{eof}, \texttt{else}\}], \\ [Stmt \rightarrow \bullet\ \texttt{if expr then}\ Stmt\ \texttt{else}\ Stmt, \{\texttt{eof}, \texttt{else}\}] \end{array} \right\}$$

$$cc_6 = \left\{ \begin{array}{l} [Stmt \rightarrow \texttt{if expr then}\ Stmt\ \bullet, \texttt{eof}], \\ [Stmt \rightarrow \texttt{if expr then}\ Stmt\ \bullet\ \texttt{else}\ Stmt, \texttt{eof}] \end{array} \right\}$$

$$cc_7 = \left\{ \begin{array}{l} [Stmt \rightarrow \texttt{if}\ \bullet\ \texttt{expr then}\ Stmt, \{\texttt{eof}, \texttt{else}\}], \\ [Stmt \rightarrow \texttt{if}\ \bullet\ \texttt{expr then}\ Stmt\ \texttt{else}\ Stmt, \{\texttt{eof}, \texttt{else}\}] \end{array} \right\}$$

BUP: Discovering Ambiguity (2.2.2)

Resulting canonical collection $\mathcal{CC}$:

$$\text{CC}_8 = \{[Stmt \rightarrow \texttt{assign} \bullet, \{\texttt{eof}, \texttt{else}\}]\}$$

$$\text{CC}_9 = \left\{\begin{array}{l} [Stmt \rightarrow \texttt{if expr then } Stmt \texttt{ else } \bullet \; Stmt, \texttt{eof}], \\ [Stmt \rightarrow \bullet \; \texttt{if expr then } Stmt, \texttt{eof}], \\ [Stmt \rightarrow \bullet \; \texttt{if expr then } Stmt \texttt{ else } Stmt, \texttt{eof}], \\ [Stmt \rightarrow \bullet \; \texttt{assign}, \texttt{eof}] \end{array}\right\}$$

$$\text{CC}_{10} = \left\{\begin{array}{l} [Stmt \rightarrow \texttt{if expr} \bullet \texttt{ then } Stmt, \{\texttt{eof}, \texttt{else}\}], \\ [Stmt \rightarrow \texttt{if expr} \bullet \texttt{ then } Stmt \texttt{ else } Stmt, \{\texttt{eof}, \texttt{else}\}] \end{array}\right\}$$

$$\text{CC}_{11} = \{[Stmt \rightarrow \texttt{if expr then } Stmt \texttt{ else } Stmt \; \bullet, \texttt{eof}]\}$$

$$\text{CC}_{12} = \left\{\begin{array}{l} [Stmt \rightarrow \texttt{if expr then} \bullet \; Stmt, \{\texttt{eof}, \texttt{else}\}], \\ [Stmt \rightarrow \texttt{if expr then} \bullet \; Stmt \texttt{ else } Stmt, \{\texttt{eof}, \texttt{else}\}], \\ [Stmt \rightarrow \bullet \texttt{if expr then } Stmt, \{\texttt{eof}, \texttt{else}\}], \\ [Stmt \rightarrow \bullet \texttt{if expr then } Stmt \texttt{ else } Stmt, \{\texttt{eof}, \texttt{else}\}], \\ [Stmt \rightarrow \bullet \texttt{assign}, \{\texttt{eof}, \texttt{else}\}] \end{array}\right\}$$

$$\text{CC}_{13} = \left\{\begin{array}{l} [Stmt \rightarrow \texttt{if expr then } Stmt \bullet \;, \{\texttt{eof}, \texttt{else}\}], \\ [Stmt \rightarrow \texttt{if expr then } Stmt \bullet \texttt{ else } Stmt, \{\texttt{eof}, \texttt{else}\}] \end{array}\right\}$$

$$\text{CC}_{14} = \left\{\begin{array}{l} [Stmt \rightarrow \texttt{if expr then } Stmt \texttt{ else } \bullet \; Stmt, \{\texttt{eof}, \texttt{else}\}], \\ [Stmt \rightarrow \bullet \; \texttt{if expr then } Stmt, \{\texttt{eof}, \texttt{else}\}], \\ [Stmt \rightarrow \bullet \; \texttt{if expr then } Stmt \texttt{ else } Stmt, \{\texttt{eof}, \texttt{else}\}], \\ [Stmt \rightarrow \bullet \; \texttt{assign}, \{\texttt{eof}, \texttt{else}\}] \end{array}\right\}$$

BUP: Discovering Ambiguity (3)

- Consider cc_{13}

$$\text{CC}_{13} = \left\{ \begin{array}{l} [\mathit{Stmt} \rightarrow \texttt{if expr then}\ \mathit{Stmt} \bullet\ ,\{\texttt{eof},\texttt{else}\}], \\ [\mathit{Stmt} \rightarrow \texttt{if expr then}\ \mathit{Stmt} \bullet\ \texttt{else}\ \mathit{Stmt},\{\texttt{eof},\texttt{else}\}] \end{array} \right\}$$

Q. What does it mean if the current word to consume is `else`?
A. We can either ***shift*** (then expecting to match another *Stmt*) or ***reduce*** to a *Stmt*.
Action[13, `else`] cannot hold ***shift*** and ***reduce*** simultaneously.
⇒ This is known as the *shift-reduce conflict* .

- Consider another scenario:

$$cc_i = \left\{ \begin{array}{l} [A \rightarrow \gamma\delta\bullet,\ \texttt{a}], \\ [B \rightarrow \gamma\delta\bullet,\ \texttt{a}] \end{array} \right\}$$

Q. What does it mean if the current word to consume is `a`?
A. We can either ***reduce*** to *A* or ***reduce*** to *B*.
Action[i, a] cannot hold *A* and *B* simultaneously.
⇒ This is known as the *reduce-reduce conflict* .

Index (1)

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Composite & Visitor Design Patterns

EECS4302 A:
Compilers and Interpreters
Fall 2022

CHEN-WEI WANG

Learning Objectives

1. Motivating Problem: ***Recursive*** Systems
2. Three Design Attempts
3. Inheritance: ***Abstract Class*** vs. ***Interface***
4. Fourth Design Attempt: ***Composite Design Pattern***
5. Implementing and Testing the Composite Design Pattern

Motivating Problem (1)

- Many manufactured systems, such as computer systems or stereo systems, are composed of ***individual components*** and ***sub-systems*** that contain components.

  e.g., A computer system is composed of:

  - Base equipment (*hard drives*, *cd-rom drives*)

    e.g., Each *drive* has **properties**: e.g., power consumption and cost.
  - Composite equipment such as *cabinets*, *busses*, and *chassis*

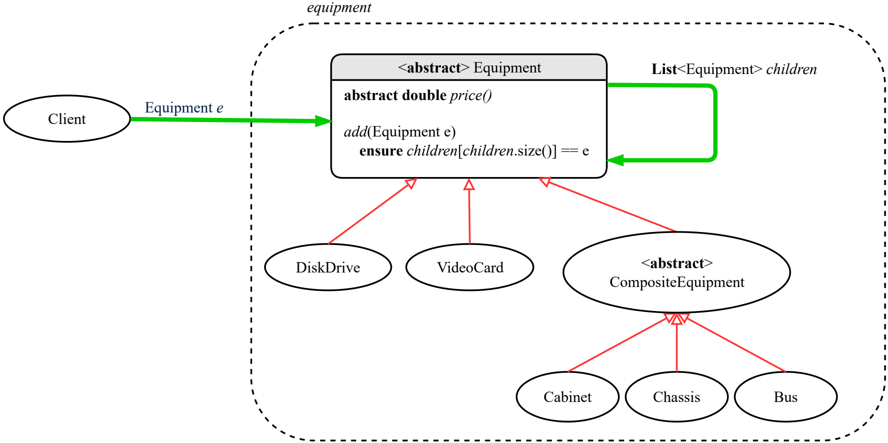
    e.g., Each *cabinet* contains various types of *chassis*, each of which containing components (*hard-drive*, *power-supply*) and *busses* that contain *cards*.
- Design a system that will allow us to easily *build* systems and *compute* their aggregate cost and power consumption.

Motivating Problem (2)

Design of ***hierarchies*** represented in ***tree structures***

Challenge : There are ***base*** and ***recursive*** modelling artifacts.

Design Attempt 1: Architecture



Java List API

Design Attempt 1: Flaw?

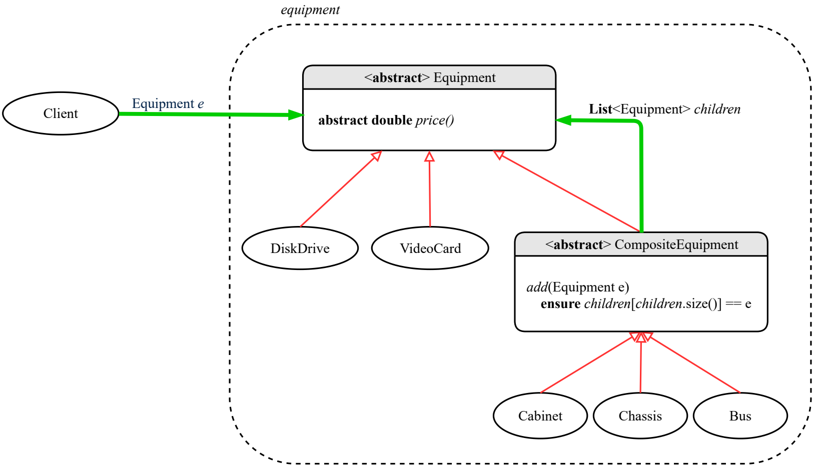
Q: Any flaw of this first design?

A: Two “composite” features defined at the `Equipment` level:

- `List<Equipment> children`
- `add(Equipment child)`

⇒ Inherited to each ***base*** equipment (e.g., `DiskDrive`), for which such features are **not** applicable.

Design Attempt 2: Architecture



Design Attempt 2: Flaw?

Q: Any flaw of this second design?

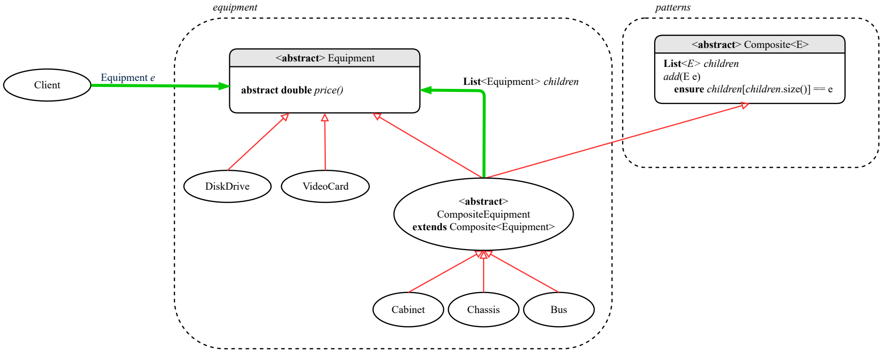
A: Two “composite” features defined at the `Composite` level:

- `List<Equipment> children`
- `add(Equipment child)`

⇒ Multiple ***types*** of the composite (e.g., equipment, furniture) cause duplicates of the `Composite` class.

⇒ Use a ***generic (type) parameter*** to ***abstract*** away the ***concrete*** type of any potential composite.

Design Attempt 3: Architecture



Design Attempt 3: Flaw?

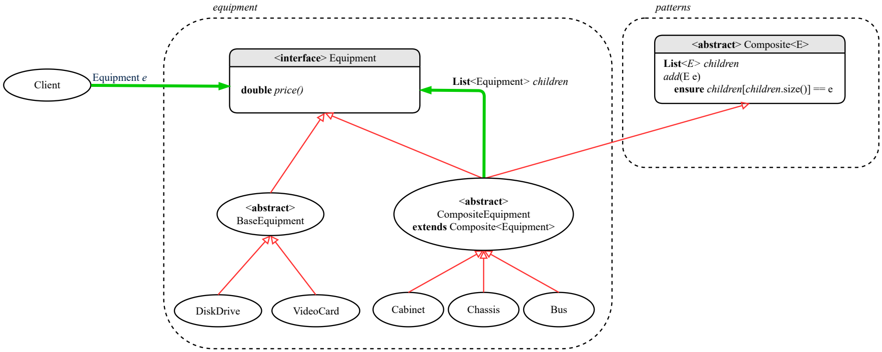
Q: Any flaw of this third design?

A: It does **not** compile:

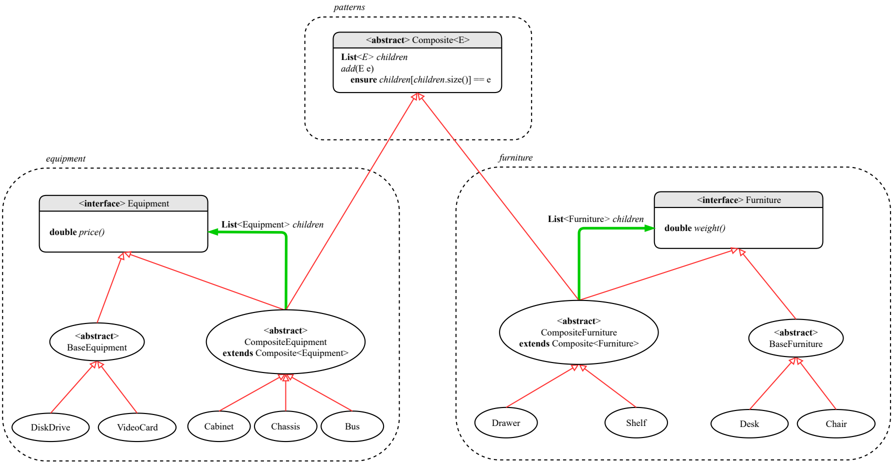
Java does not support ***multiple inheritance***!

- See: https://docs.oracle.com/javase/tutorial/java/IandI/multipleinheritance.html
- A class may inherit from at most one class (**abstract** or not).
  Rationale. ***MI*** results in name clashes
  [a.k.a. the ***Diamond Problem***].
- However, a class may implement multiple ***interfaces***.
  [workaround for implementation]

The Composite Pattern: Architecture



The Composite Pattern: Instantiations



Implementing the Composite Pattern (1)

```java
public interface Equipment {
  public String name();
  public double price(); /* uniform access */
}
```

```java
public abstract class BaseEquipment implements Equipment {
  private String name;
  private double price;
  public BaseEquipment(String name, double price) {
    this.name = name; this.price = price;
  }
  public String name() { return this.name; }
  public double price() { return this.price; }
}
```

```java
public class VideoCard extends BaseEquipment {
  public VideoCard(String name, double price) {
    super(name, price);
  }
}
```

Implementing the Composite Pattern (2.1)

```java
import java.util.List;

public abstract class Composite<E> {
  protected List<E> children;

  public void add(E child) {
    children.add(child); /* polymorphism */
  }
}
```

Implementing the Composite Pattern (2.2)

```java
import java.util.ArrayList;

public abstract class CompositeEquipment
  extends Composite<Equipment>
  implements Equipment
{
  private String name;
  public CompositeEquipment(String name) {
    this.name = name;
    this.children = new ArrayList<>();
  }
  public String name() { return this.name; }
  public double price() {
    double result = 0.0;
    for(Equipment child : this.children) {
      result = result + child.price(); /* dynamic binding */
    }
    return result;
  }
}
```

Implementing the Composite Pattern (2.2)

```
public class Chassis extends CompositeEquipment {
  public Chassis(String name) {
    super(name);
  }
}
```

Testing the Composite Pattern

```java
@Test
public void test_equipment() {
  Equipment card, drive;
  Bus bus;
  Cabinet cabinet;
  Chassis chassis;

  card = new VideoCard("16Mbs Token Ring", 200);
  drive = new DiskDrive("500 GB harddrive", 500);
  bus = new Bus("MCA Bus");
  chassis = new Chassis("PC Chassis");
  cabinet = new Cabinet("PC Cabinet");
  bus.add(card);
  chassis.add(bus);
  chassis.add(drive);
  cabinet.add(chassis);

  assertEquals(700.00, cabinet.price(), 0.1);
}
```

Summay: The Composite Pattern

- **Design**: Categorize into ***base*** artifacts or ***recursive*** artifacts.
- **Programming**:
  Build the ***tree structure*** representing some ***hierarchy***.
- **Runtime**:
  Allow clients to treat ***base*** objects (leafs) and ***recursive*** compositions (nodes) *uniformly* (e.g., `price()`).

  ⇒ *Polymorphism*: ***leafs*** and ***nodes*** are "substitutable".

  ⇒ *Dynamic Binding*: Different versions of the same operation is applied on ***base objects*** and ***composite objects***.

  e.g., Given ***Equipment*** e:

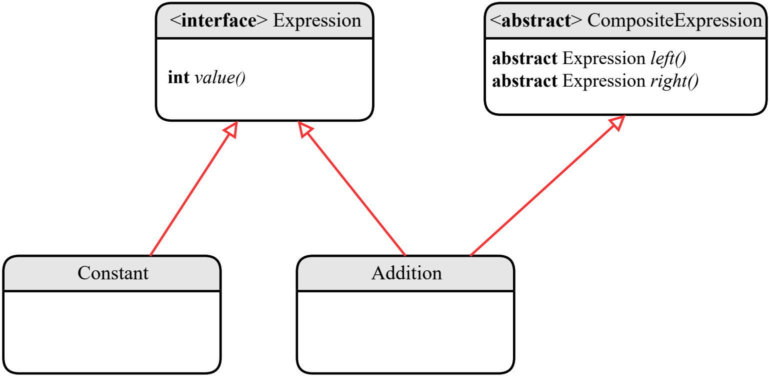
  - `e.price()` may return the unit price, e.g., of a ***`DiskDrive`***.
  - `e.price()` may sum prices, e.g., of a ***`Chassis`***' containing equipment.

Learning Objectives

1. Motivating Problem: ***Processing*** Recursive Systems
2. First Design Attempt: Cohesion & Single-Choice Principle?
3. Design Principles:
   - *Cohesion*
   - *Single Choice* Principle
   - *Open-Closed* Principle
4. Second Design Attempt: ***Visitor Design Pattern***
5. Implementing and Testing the Visitor Design Pattern

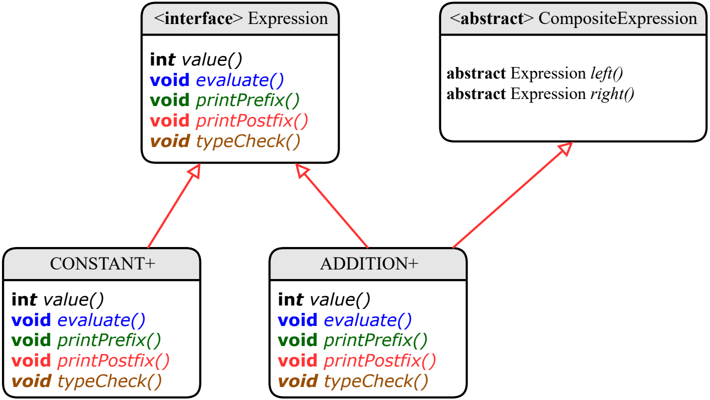
Motivating Problem (1)

Based on the *composite pattern* you learned, design classes to model ***structures*** of arithmetic expressions (e.g., `341`, `2`, `341 + 2`).



Motivating Problem (2)

Extend the *composite pattern* to support ***operations*** such as `evaluate`, pretty printing (`print_prefix`, `print_postfix`), and `type_check`.



Design Principles: Information Hiding & Single Choice

- ***Cohesion***:
  - A class/module groups ***relevant*** features (data & operations).
- ***Single Choice Principle*** (SCP):
  - When a ***change*** is needed, there should be ***a single place*** (or ***a minimal number of places***) where you need to make that change.
  - Violation of SCP means that your design contains ***redundancies***.

Problems of Extended Composite Pattern

- Distributing **unrelated** *operations* across nodes of the ***abstract syntax tree*** violates the ***single-choice principle***:

  To add/delete/modify an operation

  ⇒ Change of all descendants of `Expression`

- Each node class lacks in ***cohesion***:

  A **class** should group ***relevant*** concepts in a **single** place.

  ⇒ Confusing to mix codes for evaluation, pretty printing, type checking.

  ⇒ Avoid "polluting" the classes with these **unrelated** operations.

Open/Closed Principle

- Software entities (classes, features, etc.) should be ***open*** for ***extension***, but ***closed*** for ***modification***.

  ⇒ As a system evolves, we:
  - May add/modify the ***open*** (unstable) part of system.
  - May **not** add/modify the ***closed*** (stable) part of system.
- e.g., In designing the application of an expression language:
  - **ALTERNATIVE 1**:
    Syntactic constructs of the language may be ***open***, whereas operations on the language may be ***closed***.
  - **ALTERNATIVE 2**:
    Syntactic constructs of the language may be ***closed***, whereas operations on the language may be ***open***.

Visitor Pattern

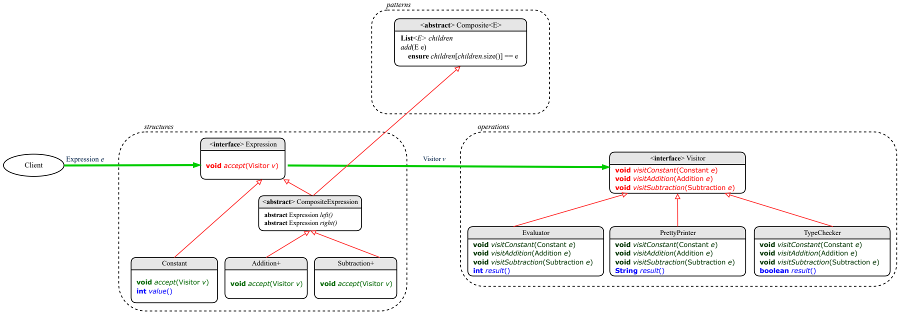
- ***Separation of concerns***:
  - Set of language (syntactic) constructs
  - Set of operations

  ⇒ Classes from these two sets are *decoupled* and organized into two separate packages.
- ***Open-Closed Principle*** (OCP): [**ALTERNATIVE 2**]
  - ***Closed***, staple part of system: set of language constructs
  - ***Open***, unstable part of system: set of operations

  ⇒ ***OCP*** helps us determine if the ***Visitor Pattern*** is applicable.

  ⇒ If it is determined that language constructs are ***open*** and operations are ***closed***, then do **not** use the Visitor Pattern.

Visitor Pattern: Architecture



Visitor Pattern Implementation: Structures

Package ***structures***

- Declare `void accept(Visitor v)` in <u>abstract</u> class `Expression`.
- Implement `accept` in each of `Expression`'s <u>descendant</u> classes.

```
public class Constant implements Expression {
  ...
  public void accept(Visitor v) {
    v.visitConstant(this);
  }
}
```

```
public class Addition extends CompositeExpression {
  ...
  public void accept(Visitor v) {
    v.visitAddition(this);
  }
}
```

Visitor Pattern Implementation: Operations

Package ***operations***

- For each <u>descendant</u> class `C` of `Expression`, declare a method header `void visitC (e: C)` in the ***interface*** `Visitor`.

```
public interface Visitor {
  public void visitConstant(Constant e);
  public void visitAddition(Addition e);
  public void visitSubtraction(Subtraction e);
}
```

- Each descendant of `VISITOR` denotes a kind of operation.

```
public class Evaluator implements Visitor {
  private int result;
  ...
  public void visitConstant(Constant e) {
    this.result = e.value();
  }
  public void visitAddition(Addition e) {
    Evaluator evalL = new Evaluator();
    Evaluator evalR = new Evaluator();
    e.getLeft().accept(evalL);
    e.getRight().accept(evalR);
    this.result = evalL.result() + evalR.result();
  }
}
```

Testing the Visitor Pattern

```
1  @Test
2  public void test_expression_evaluation() {
3    CompositeExpression add;
4    Expression c1, c2;
5    Visitor v;
6    c1 = new Constant(1); c2 = new Constant(2);
7    add = new Addition(c1, c2);
8    v = new Evaluator();
9    add.accept(v);
10   assertEquals(3, ((Evaluator) v).result());
11 }
```

Double Dispatch in **Line 9**:

1. ***DT*** of `add` is `Addition` ⇒ Call `accept` in `ADDITION`.

   `v.visitAddition(add)`

2. ***DT*** of `v` is `Evaluator` ⇒ Call `visitAddition` in `Evaluator`.

   visiting result of `add.left()` + visiting result of `add.right()`

To Use or Not to Use the Visitor Pattern

- In the ***visitor pattern***, what kind of ***extensions*** is easy?
  Adding a new kind of ***operation*** element is easy.
  To introduce a new operation for generating C code, we only need to introduce a new descendant class `CCodeGenerator` of `Visitor`, then implement how to handle each language element in that class.
  ⇒ *Single Choice Principle* is satisfied.
- In the ***visitor pattern***, what kind of ***extensions*** is hard?
  Adding a new kind of ***structure*** element is hard.
  After adding a descendant class `Multiplcation` of `Expression`, every concrete visitor (i.e., descendant of `Visitor`) must be amended with a new `visitMultiplication` operation.
  ⇒ *Single Choice Principle* is violated.
- The applicability of the visitor pattern depends on to what extent the ***structure*** will change.
  ⇒ Use visitor if ***operations*** (applied to structure) change often.
  ⇒ Do not use visitor if the ***structure*** changes often.

Index (1)

Index (2)

Index (3)