Lecture #13, Oct. 30

Resolution

Easy and self-documenting 2-dimensional proofs.

The technique is used in the "automatic theorem proving", i.e., special computer systems (programs) that prove theorems.

It is based on proof by contradiction metatheorem:

0.0.1 Metatheorem.

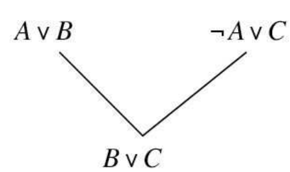
$$\Gamma, \neg A \vdash \bot \tag{1}$$

iff

$$\Gamma \vdash A \tag{2}$$

Thus, instead of proving (2) prove (1).

(1) is proved using (almost) exclusively the CUT Rule.



The technique can be easily learnt via examples:

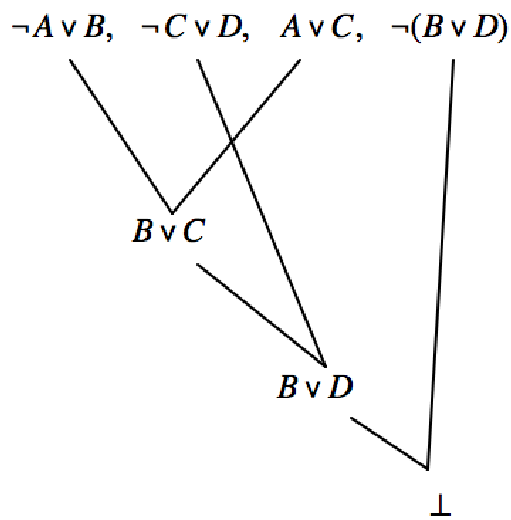
0.0.2 Example. Use Resolution to prove (1) below:

$$A \to B, C \to D \vdash A \lor C \to B \lor D \tag{1}$$

by DThm prove instead:

$$A \to B, C \to D, A \lor C \vdash B \lor D$$

By 0.0.1 prove instead that the "Γ" in the top line below proves ⊥



□

0.0.3 Example. Next prove

$$\vdash (A \to (B \to C)) \to ((A \to B) \to (A \to C))$$

By the DThm prove instead

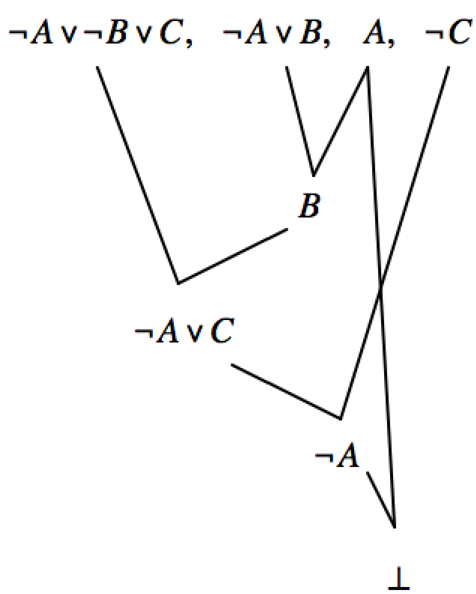
$$A \to (B \to C) \vdash (A \to B) \to (A \to C)$$

Two more applications of the DThm simplify what we will prove into the following:

$$A \to (B \to C), A \to B, A \vdash C$$

By 0.0.1, prove instead that $\Gamma \vdash \bot$ where

$$\Gamma = \{\neg A \vee \neg B \vee C, \neg A \vee B, A, \neg C\}$$



□

0.0.4 Example. Prove

$$\vdash (A \wedge \neg B) \to \neg(A \to B)$$

By DThm do insted: $A \wedge \neg B \vdash \neg(A \to B)$.

By 0.0.1 do instead

$$A \wedge \neg B, A \to B \vdash \bot$$

or

$$A \wedge \neg B, \neg A \vee B \vdash \bot$$

Use HYP Splitting, so do instead

$$A, \neg B, \neg A \vee B \vdash \bot$$

$$A, \neg B, \neg A \vee B$$

To this end, cut 1st and 3rd to get B.

Cut the latter with $\neg B$ to get $\bot$.

□

0.0.5 Example.

$\neg(A \vee B)$

$\neg A \wedge \neg B$

$\neg A$ $\neg B$

□

Bibliography

[Rob65] J.A. Robinson, *A Machine Oriented Logic Based on the Resolution Principle*, JACM **12** (1965), no. 1, 23–41.