York University

Faculties of Pure and Applied Science, Arts, Atkinson

MATH 2090. Problem Set #5 (addendum to #4).
Posted November 23, 2003

Due in the Course Box. December 4, 2003

Section A

In your proofs (all informal please) it is imperative to clearly state what **tools** you use (e.g., WLUS, sWLUS, MP, Leibniz, Monotonicity, Deduction Theorem, Generalization, Auxiliary Variable metatheorem, which axiom(s), etc.)

1. (5 Marks) Prove that $R : A \to A$ is transitive iff $R^2 \subseteq R$.

2. (5 Marks) Give an example of two equivalence relations R and S on the same set A, such that $R \cup S$ is *not* an equivalence relation.

3. (5 Marks) Let F be a partition on a set A and define R on A as in class:
$$aRb \overset{\text{Def.}}{\equiv} (\exists S \in F)(a \in S \wedge b \in S)$$
We know from class that R is an equivalence relation.

   Prove that the set of all the equivalence classes of R is exactly F. This put in symbols means that you prove two things:
$$(\forall x \in A)(\exists S \in F)[x] = S$$
and
$$(\forall S \in F)(\exists x \in A)[x] = S$$

4. (5 Marks) Prove that $f : A \to B$ has both a left and a right inverse, iff f is 1-1, onto and total, that is, iff f is a 1-1 correspondence.

   NB. Actually, the "only if" was done in class, but do write it down anyway. For the "if" part just prove that $f^{-1} : B \to A$, the inverse relation is also a function that is total, 1-1 and onto, which moreover satisfies $f \circ f^{-1} = 1_A$ and $f^{-1} \circ f = 1_B$. Of course, these last two we can also write as $f^{-1} \bullet f = 1_A$ and $f \bullet f^{-1} = 1_B$ respectively.