York University

Faculties of Pure and Applied Science, Arts, Atkinson

MATH 2090. Problem Set #1. Posted September 2, 2003

Due in the Course Box by 2:00pm, September 30, 2003

Section A

In your proofs it is required that you clearly state what **tools** you use from those among the posted "Logic ToolBox"

1. (5 MARKS) In class we have proved that $\vdash (\exists x)(\forall y)A \Rightarrow (\forall y)(\exists x)A$ using the Auxiliary Variable Metatheorem.

Now prove the same thing but without the help of the Auxiliary Variable Metatheorem, instead exploiting monotonicity.

2. (5 MARKS) Prove that $\vdash ((\forall x)B \Rightarrow A) \equiv (\exists x)(B \Rightarrow A)$, *provided x is not free in A.*

Use an equational proof!

3. (5 MARKS) Let $\circ$ be a function symbol of arity 2.

Prove that $\vdash x = y \Rightarrow x \circ z = y \circ z$ no matter what variables x, y, z you use.

Careful! Do *not* just say that this "follows" from Chapter 1 stuff. It doesn't. This is a Predicate Calculus exercise—and I mean this in the strict sense! That is, you do not need any axioms about the symbol "$\circ$" in order to prove this.

4. Let P be **any predicate of arity 2**.

(a) (2 MARKS) Explain WHY $(\forall x)(\forall y)P(x, y) \Rightarrow (\forall y)P(y, y)$ is NOT an instance of **Ax2**.

(b) (5 MARKS) Nevertheless, *prove* that $(\forall x)(\forall y)P(x, y) \Rightarrow (\forall y)P(y, y)$ IS an absolute theorem.

5. (5 MARKS) Prove (absolutely) the formula $(\exists x)A \wedge (\forall x)B \Rightarrow (\exists x)(A \wedge B)$.

6. (5 MARKS) For any predicate P of arity 2 prove

$$\vdash (\forall x)(\forall y)P(x,y) \equiv (\forall y)(\forall x)P(y,x)$$

Supplementary question: Precisely how is this different from $\vdash (\forall x)(\forall y)A \equiv (\forall y)(\forall x)A$ the we proved in class?