Lassonde School of Engineering

Dept. of EECS
Professor G. Tourlakis

MATH1090 A. Problem Set No. 3

Posted: **Oct. 31, 2023**

Due: Nov. 24, 2023; by 2:00pm, in eClass.

Q: How do I submit?

A:

(1) **Submission must be a SINGLE *standalone* file to eClass. Submission by email is not accepted.**

(2) **Accepted File Types: PNG, JPEG, PDF, RTF, MS WORD, OPEN OFFICE, ZIP**

(3) **Deadline is strict, electronically limited.**

(4) **MAXIMUM file size = 10MB**

Unless a required proof style (e.g., by resolution, Equational, Hilbert) is used in your answer, then your answer is graded out of 0.

(5 POINTS Max for each question) **Do all of the following:**

All resolution proofs below MUST use the graphical technique. Minimise preprocessing. You lose marks if your preprocessing is so long that it solves the problem WITHOUT doing any resolution step.

1. Use Resolution to prove $\vdash A \to \neg(\neg A \land \neg B)$.

2. Use Resolution to prove, for any A, B, C, that $\vdash (A \to B) \to (C \to A) \to (C \to B)$.

3. Use Resolution to prove, for any A, B, C, D, that

$$\vdash (A \lor B \lor C) \land (A \to D) \land (B \to D) \land (C \to D) \to D$$

4. You are in Boolean Logic.

Define: Σ is satisfiable, by definition, if some state s makes all the wff in Σ true. If no such s exists then we call Σ UNsatisfiable.

Prove that if a finite set of wff Σ is unsatisfiable, then $\Sigma \vdash \bot$.

5. Prove that for any object variables $\mathbf{x}, \mathbf{y}, \mathbf{z}$ we have the absolute theorem $\vdash \mathbf{x} = \mathbf{y} \to (\mathbf{y} = \mathbf{z}) \to (\mathbf{x} = \mathbf{z})$.

Hint. Use a **Hilbert** style proof using the axioms of equality.

Do **NOT** use the Auxiliary Hypothesis Metatheorem in THIS Problem Set!

6. Prove that $\vdash A \to B$ implies $\vdash (\exists \mathbf{x})A \to (\exists \mathbf{x})B$.

Required Methodology. Use a **Hilbert** style proof and the metatheorem from class "$\vdash A \to B$ implies $\vdash (\forall \mathbf{x})A \to (\forall \mathbf{x})B$".

7. Prove **Hilbert** style, that $\vdash (\forall \mathbf{x})(A \to B) \to (\forall \mathbf{x})A \to (\exists \mathbf{x})B$.

8. Prove **Hilbert** style, that

$$\vdash (\forall \mathbf{x})(A \lor B \to C) \to \Big((\forall \mathbf{x})(A \to C) \land (\forall \mathbf{x})(B \to C)\Big)$$