Lassonde School of Engineering

Dept. of EECS
Professor G. Tourlakis

MATH1090 A. Problem Set No 3
Posted: Nov. 6, 2020

Due: Nov. 23, 2020; by 2:00pm, in eClass, "Assignment #3"

Q: How do I submit?

A:

(1) **Submission must be ONLY ONE file**

(2) **Accepted File Types: PDF, RTF, MS WORD, ZIP**

(3) **Deadline is strict, electronically limited.**

(4) **MAXIMUM file size = 10MB**

A proof that I ask you to write can be either Hilbert or Equational, UNLESS I ask for one of those styles specifically.

If so, the other proof style is worth 0 (F).

A brief but full justification of each proof step is required!
Do all the following problems; (5 Points Each).

Important Notes; Read First!

"**Required Method**" means that any other method will get a 0-grade.

It is all right to use the Cut Rule in each problem, unless specified otherwise!

Post's Theorem is NOT allowed in Problems 1–5.

1. Show that $\vdash (A \equiv B \equiv C) \to A \to B \to C$

Required Method: Use a **Hilbert style** proof and the Deduction Theorem.

2. Show that if $\vdash A$ and $\vdash B$, then also $\vdash B \equiv A$.

3. Prove that for any formula A, we have $\bot \vdash A$.

You may NOT use Post's Theorem.

4. For any formulas A, B and C, prove that

$$\vdash (A \equiv B) \to (B \equiv C) \to (A \to C)$$

Required Method: Use a **Hilbert style** proof and the Deduction Theorem.

5. For any formulas A, B and C, prove that

$$\vdash (A \to B) \wedge (B \to C) \to (A \to C)$$

Required Method: Use a a **Resolution** proof.

6. For any formulas A and B prove **two things** WITH THE LEAST AMOUNT OF TOOLS:

$$\vdash (\forall \mathbf{x})A \wedge (\forall \mathbf{z})B \to (\forall \mathbf{z})B$$

and

$$\vdash (\forall \mathbf{x})(\forall \mathbf{y})(\forall \mathbf{z})\Big((\forall \mathbf{w})A \to A\Big)$$

7. Let A, B be any formulas, and $\mathbf{x}$ a variable that is not free in B.

Prove that

$$(\forall \mathbf{x})(A \to B), \neg B \vdash (\forall \mathbf{x})\neg A$$

8. Prove that $\vdash (\forall \mathbf{x})A \vee (\forall \mathbf{x})B \to (\forall \mathbf{x})(A \vee B)$.