Lassonde School of Engineering

Dept. of EECS
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MATH1090 A. Problem Set No1
Posted: **Sept. 19, 2020**

Due: Oct. 9, 2020; by 2:00pm, in eClass, "Assignment #1".

Q: How do I submit?

A:

(1) **Submission must be ONLY ONE file**

(2) **Accepted File Types: PDF, RTF, MS WORD, ZIP**

(3) **Deadline is strict, electronically limited.**

(4) **MAXIMUM file size = 10MB**

It is worth remembering (from the course outline):

The homework must be each individual's own work. While consultations with the instructor, tutor, and among students, are part of the learning process and are encouraged, nevertheless, *at the end of all this consultation* each student will have to produce an individual report rather than a copy (full or partial) of somebody else's report.

The concept of "late assignments" does not exist in this course.

1. (6 MARKS)

Use a Formula Construction to either *support* or *reject EACH* of the following statements:

- $(\mathbf{p})$ is a formula.
- $()$ is a formula.
- $\mathbf{p} \to \mathbf{q}$ is a formula.

2. (4 MARKS) **Prove that no wff is the empty string**.

Hint. Use induction on formulas or analyse what is happening during a formula construction.

3. **A formula schema is a tautology iff all its instances are tautologies**.

Which of the following schemata are tautologies? Show the whole process that lead to your answers.

I note that in the six sub-questions below I am *NOT always* using all the formally necessary brackets.

Be sure to insert brackets CORRECTLY before you try to answer each question.

- (1 MARK) $((A \vee B) \vee C) \equiv (A \vee (B \vee C))$
- (1 MARK) $A \to B \equiv A \vee B \equiv B$
- (1 MARK) $\top \equiv \bot \equiv \bot$
- (1 MARK) $\neg A \vee B \equiv A \vee B \equiv B$
- (1 MARK) $A \vee B \equiv A \wedge B \equiv A$
- (1 MARK) $A \to B \equiv A \wedge B \equiv A$

4. (5 MARKS) **Prove that if we have $A, B \models_{\text{taut}} C$, then we also have $\models_{\text{taut}} A \rightarrow B \rightarrow C$ and conversely.**

Or as we usually put it: "$A, B \models_{\text{taut}} C$ iff $\models_{\text{taut}} A \rightarrow B \rightarrow C$".

Here, using truth tables or truth-table shortcuts, you must prove that if you have one side of the "iff", then you must have the other. *There are two directions in your proof*!

5. (4 MARKS) **Prove that every nonempty proper *suffix* of a wff A contains an excess of *right* brackets.**

6. (3 MARKS) **Suppose we know $\models_{taut} A \wedge B$.**
Prove that we can conclude that $\models_{taut} A$ and $\models_{taut} B$.

7. (4 MARKS) **Suppose we know $\models_{taut} A \vee B$.**
Prove that the conclusion $\models_{taut} A$ *or* $\models_{taut} B$ is false.
Caution. Here you need a counter*example*! So you cannot argue with general A and B.

8. (5 MARKS) **By using truth tables, or using related shortcuts, *prove* or *disprove* the following tautological implication *claims*.**

Show the whole process that led to each of your answers and note that *disproofs* need *SPECIFIC* counter-*EXAMPLES* not a *loose general argument* via A and B.

- $A \wedge B \models_{\text{taut}} A$
- $A \models_{\text{taut}} A \wedge B$
- $A \models_{\text{taut}} A \vee B$

- $A, A \equiv B \models_{\text{taut}} B$
- $B, A \rightarrow B \models_{\text{taut}} A$

9. (6 MARKS) **Compute the result of the following substitutions, *whenever the requested substitution makes sense.***

Whenever a requested substitution does not make sense, *explain* exactly why it does not.

Remember the priorities of the various connectives as well as of the meta-expression "$[\mathbf{p} := \ldots]$"! The following formulas have not been written with all the formally required brackets.

- $\mathbf{p} \vee \mathbf{q} \rightarrow \mathbf{p}[\mathbf{p} := \mathbf{r}]$
- $(\mathbf{p} \wedge \mathbf{q})[\mathbf{p} := \mathbf{f}]$
- $(\mathbf{p} \rightarrow \mathbf{q})[\mathbf{p} := \top]$
- $\top[\top := \mathbf{p}]$
- $(q \wedge r \rightarrow p)[r' := A]$ (where A is some wff and p, q, r, r' are actual (distinct) Boolean variables; not metavariables)
- $\mathbf{p} \vee (\mathbf{q} \wedge \mathbf{r})[A := \mathbf{r}]$ (where A is some wff)