Lassonde School of Engineering
EECS
MATH1090. Problem Set No. 4
Posted: Nov. 21, 2019

Due: Dec. 4, 2019, by 2:00pm; in the course assignment box.

It is worth remembering (from the course outline):

The homework must be each individual's own work. While consultations with the instructor, tutor, and among students, are part of the learning process and are encouraged, nevertheless, *at the end of all this consultation* each student will have to produce an individual report rather than a copy (full or partial) of somebody else's report.

The concept of "late assignments" does not exist in this course.

In what follows, "give a proof of $\vdash A$" means to give an equational or Hilbert-style proof of A, unless some other proof style is required (e.g., Resolution).

Annotation is always required!

*Do the following problems (**5 MARKS/Each**).*

1. Show that

$$\vdash (\forall \mathbf{x})(A \to B \to C) \to (\forall \mathbf{x})(A \to B) \to (\forall \mathbf{x})(A \to C)$$

2. Prove using **soundness**: $\nvdash (\forall \mathbf{x})(A \vee B) \to (\forall \mathbf{x})A \vee (\forall \mathbf{x})B$.

3. Prove $\vdash (\forall \mathbf{x})A \vee (\forall \mathbf{x})B \to (\forall \mathbf{x})(A \vee B)$.

4. Use the $\exists$ elimination technique —and ping-pong if/where needed— to show

$\vdash (\exists \mathbf{x})A \to (\exists \mathbf{x})(B \to A)$.

5. Use the $\exists$ elimination technique —and ping-pong if/where needed— to show $\vdash (\exists \mathbf{x})(A \to B) \equiv (\exists \mathbf{x})(\neg A \vee B)$.

*Do **NOT** use WL for the above (0 marks for such solutions).*

6. Let ψ be a binary predicate. Prove $\vdash (\forall x)(\forall y)\psi(x, y) \to (\forall y)\psi(y, y)$.

7. Prove "$\exists$-Monotonicity": If $\vdash A \to B$ then also $\vdash (\exists \mathbf{x})A \to (\exists \mathbf{x})B$.