Lassonde Faculty of Engineering
EECS
MATH1090. Problem Set No. 3
Posted: November 7, 2019

Due: Nov. 21, 2019, by 2:00pm; in the course assignment box.

Administrative Stuff. It is worth remembering (from the course outline):

The homework must be each individual's own work. While consultations with the instructor, tutor, and among students, are part of the learning process and are encouraged, nevertheless, *at the end of all this consultation* each student will have to produce an individual report rather than a copy (full or partial) of somebody else's report.

The concept of "late assignments" does not exist in this course.

A brief but full justification of each proof step is required!
Do all the following problems; (5 Points Each).

Important Notes; Read First!
"Show that —or prove that— $\Gamma \vdash A$" means "write a Γ-proof that establishes A". The proof can be Equational or Hilbert-style. Equational is rather easier in Boolean Logic. But it is your choice, unless a problem explicitly asks for a particular proof style.

"**Required Method**" means that any other method will get a 0-grade.

It is all right to use the Cut Rule in each problem, unless specified otherwise!

Post's Theorem is NOT allowed in Problems 1–4.

1. Show that $A \equiv B \equiv C \vdash A \to B \to C$

Required Method: Use a **Hilbert style** proof and the Deduction Theorem.

2. Show that $\vdash A \wedge B \equiv B \wedge A$

Worth Remembering: With the above proved and with the associativity of $\wedge$ proved in the **MidTerm solutions**, we now can use without any fussing:

- In a chain of $\wedge$ connectives we can insert brackets as we please, including not at all.
- In a chain of $\wedge$ connectives we can commute the participating formulas as we please.

The proofs are exactly as those for $\vee$-chains and $\equiv$-chains.

3. For any formulas A, B and C, prove that

$$\vdash (A \to B) \to (B \to C) \to (A \to C)$$

Required Method: Use a **Hilbert style** proof and the Deduction Theorem.

4. For any formulas A, B and C, prove **again** that

$$\vdash (A \to B) \to (B \to C) \to (A \to C)$$

Required Method: Use a a **Resolution** proof.

5. For any formula A prove **two things**:

$$\vdash (\forall \mathbf{x})A \to (\forall \mathbf{x})A \vee (\forall \mathbf{z})B$$

and

$$\vdash (\forall \mathbf{x})(\forall \mathbf{y})(\forall \mathbf{z})\Big(A \to A\Big)$$

6. Use an **Equational** Proof to show

$$\vdash (\exists \mathbf{x})(\exists \mathbf{y})A \equiv (\exists \mathbf{y})(\exists \mathbf{x})A$$

7. Typo corrected. This should read:

Prove $\vdash (\forall \mathbf{x})(A \vee B \to C) \to (\forall \mathbf{x})(A \to C) \wedge (\forall \mathbf{x})(B \to C)$.