Lassonde School of Engineering

Dept. of EECS

Professor G. Tourlakis

MATH1090 B. Problem Set No1

Posted: Sept. 15, 2019

Due: Oct. 7, 2019; by 2:00pm, in the course assignment box.

It is worth remembering (from the course outline):

The homework must be each individual's own work. While consultations with the instructor, tutor, and among students, are part of the learning process and are encouraged, nevertheless, *at the end of all this consultation* each student will have to produce an individual report rather than a copy (full or partial) of somebody else's report.

The concept of "late assignments" does not exist in this course.

1. (5 MARKS)

Prove that the string $\to\to$ cannot appear as a *substring* in *any* wff.

Restrictions. Your proof will be acceptable only if it is either *by induction on formulas*, or by *analysing formula-calculations*.

2. Recall that a schema is a tautology iff all its instances are tautologies.

Which of the following schemata are tautologies? Show the whole process that lead to your answers.

I note that in the six sub-questions below I am not using all the formally necessary brackets.

- (1 MARK) $((A \to B) \to A) \to A$
- (1 MARK) $A \vee B \to A \wedge B$
- (1 MARK) $A \to B \equiv \neg B \to \neg A$
- (1 MARK) $A \wedge (B \equiv C) \equiv A \wedge B \equiv A \wedge C$

- (1 MARK) $A \vee (B \equiv C) \equiv A \vee B \equiv A \vee C$
- (1 MARK) $\neg A \equiv A \equiv \bot$

3. (4 MARKS) Prove that if we have $\neg A \models_{taut} B$ and $B \models_{taut} \bot$ then we must have $\models_{taut} A$.

4. (5 MARKS) Prove that we have $A, B \models_{\text{taut}} C$, then we also have $\models_{\text{taut}} A \to B \to C$ and conversely. Or as we usually put it: "$A, B \models_{\text{taut}} C$ iff $\models_{\text{taut}} A \to B \to C$".

Here, using truth tables or truth-table shortcuts, you will show that if you have one side of the "iff", then you must have the other. *There* ***are*** *two directions in your proof*!

5. (5 MARKS) Prove that the complexity of a wff equals the number of its right brackets.

Caution. The proof **must** be by analysing formula calculations or by induction on formulas.

6. (3 MARKS) Prove that if for some formulas A and B it is the case that $A \wedge B \models_{\text{taut}} \bot$, then it is also the case that $\models_{\text{taut}} A \to \neg B$.

7. (6 MARKS) *True* or *false*? **Prove whichever answer you opt for**!

The statement

$$\models_{taut} A \text{ iff } \models_{taut} B$$

is *equivalent* (i.e., both are true, or both are false) to the statement

$$\models_{taut} A \equiv B$$

8. (5 MARKS) By using truth tables, or using related shortcuts, examine whether or not the following tautological implications are correct.

Show the whole process that led to each of your answers.

- $p \models_{\text{taut}} p \wedge q$
- $A, B \models_{\text{taut}} A \wedge B$
- $A, A \to B \models_{\text{taut}} B$
- $B, A \to B \models_{\text{taut}} A$

- $p \models_{\text{taut}} p \vee q$

9. (6 MARKS) Compute the result of the following substitutions, *whenever the requested substitution makes sense*. Whenever a requested substitution does not make sense, explain exactly why it does not.

Remember the priorities of the various connectives as well as of the meta-expression "[$\mathbf{p}$:= ...]"! The following formulas have not been written with all the formally required brackets.

- $p \vee (q \rightarrow p)[p := r]$
- $(p \vee q)[p := \mathbf{t}]$
- $(p \vee q)[p := \top]$
- $(\top \vee q)[\top := p]$
- $p \vee q \wedge r[q := A]$ (where A is some formula)
- $p \vee (q \wedge r)[q := A]$ (where A is some formula)