Lassonde Faculty of Engineering
EECS
MATH1090. Problem Set No. 3
Posted: October 26, 2018

Due: Nov. 20, 2018, by 2:30pm; in the course assignment box.

Administrative Stuff. It is worth remembering (from the course outline):

The homework must be each individual's own work. While consultations with the instructor, tutor, and among students, are part of the learning process and are encouraged, nevertheless, *at the end of all this consultation* each student will have to produce an individual report rather than a copy (full or partial) of somebody else's report.

The concept of "late assignments" does not exist in this course.

A brief but full justification of each proof step is required!
Do all the following problems; (5 Points Each).

Important Notes; Read First!

"Show that —or prove that— $\Gamma \vdash A$" means "write a Γ-proof that establishes A". The proof can be Equational or Hilbert-style. Equational is rather easier in Boolean Logic. But it is your choice, unless a problem explicitly asks for a particular proof style.

"**Required Method**" means that any other method will gets a 0-grade.

It is alright to use the Cut Rule in each problem, unless specified otherwise!

Post's Theorem is NOT allowed in Problems 1–4.

1. Show that $A \equiv C \vdash A \rightarrow (B \rightarrow C)$

Required Method: Use a **Hilbert style** proof and the Deduction Theorem.

2. Show that $\vdash A \vee B \equiv (A \to B) \to B$

Required Method: Use a **Resolution** proof in each direction obtained by the ping-pong theorem.

3. For any formulas A, B and C, prove that

$$\vdash A \to B \to \Big((C \to A) \to (C \to B)\Big)$$

Required Method: Use a **Hilbert style** proof and the Deduction Theorem.

4. Now prove a somewhat different theorem than the above: For any formulas A, B and C, prove that

$$\vdash (A \to B) \to \Big((C \to A) \to (C \to B)\Big)$$

Required Method: Use a a **Resolution** proof.

5. For any formula A prove **two things**:

$$\vdash (\forall \mathbf{x})A \equiv (\forall \mathbf{x})A$$

and

$$\vdash (\forall \mathbf{x})(\forall \mathbf{x})(\forall \mathbf{z})\Big(A \equiv A\Big)$$

6. Let A, B be any formulas, and $\mathbf{x}$ a variable that is not free in B.

Prove that

$$(\forall \mathbf{x})(A \to B),\ \neg B \vdash (\forall \mathbf{x})\neg A$$

7. Use an **Equational** Proof to show

$$\vdash (\exists \mathbf{x})(\exists \mathbf{y})A \equiv (\exists \mathbf{y})(\exists \mathbf{x})A$$

8. Prove $\vdash (\forall \mathbf{x})((A \vee B) \to C) \to (\forall \mathbf{x})(A \to C) \to (\forall \mathbf{x})(B \to C)$.