Lassonde Faculty of Engineering
EECS
MATH1090. Problem Set No. 2
Posted: October 1, 2018

Due: Oct. 23, 2018, by 3:00pm; in the course assignment box.

Administrative Stuff. It is worth remembering (from the course outline):

The homework must be each individual's own work. While consultations with the instructor, tutor, and among students, are part of the learning process and are encouraged, nevertheless, *at the end of all this consultation* each student will have to produce an individual report rather than a copy (full or partial) of somebody else's report.

The concept of "late assignments" does not exist in this course.

A brief but full justification of each proof step is required!
Do all the following problems; (5 Points Each).

Important Notes; Read First!
"Show that —or prove that— $\Gamma \vdash A$" means "write a Γ-proof that establishes A". The proof can be Equational or Hilbert-style. Equational is rather easier in Boolean Logic. But it is your choice, unless a problem explicitly asks for a particular proof style.

You may *NOT* use ***any of these tools*** in this Problem Set: **Deduction Theorem, Resolution, Post's Theorem, Cut Rule**. *Any solutions that use these tools will be discarded (**grade 0**).*

Recall that notation such as $\mathbf{p}$ is for **metavariables** or **syntactic variables** that stand for *any* Boolean variable of our alphabet $\mathcal{V}$ (that is, any of p, q, r''_{123}, etc.). In particular, just as in algebra x and y may hold the same value, similarly here $\mathbf{p}$ and $\mathbf{q}$ might stand for the same variable, say r. So, if we intend them to *stand for different variables* we will indicate this by simply saying "*where* $\mathbf{p}$ *and* $\mathbf{q}$ *are different*".

1. Let $\mathbf{p}$ and $\mathbf{q}$ be different, and let $\mathbf{q}$ be fresh for C.

Prove by *induction on formulas* C (NOT by induction on their complexity; no credit will be given for such proof) that the result of the substitution $C[\mathbf{p} := \mathbf{q}][\mathbf{q} := \mathbf{A}]$ is the same as that of the substitution $C[\mathbf{p} := A]$.

2. Show that $A \equiv C \vdash A \to (B \to C)$

3. Show that $\vdash A \equiv B \equiv A \equiv \bot \equiv B \equiv \bot$

4. Show that $\vdash A \vee B \equiv (A \to B) \to B$

5. Suppose you are given for some formulae A and B and set Γ that $\vdash A$ and $\Gamma \vdash B$. Show that $\Gamma \vdash A \vee B \to A \wedge B$.

6. For any formulas A and B show that $A \wedge \neg A \vdash B$

7. Prove that $A, B \vdash A \equiv B$.

8. For any formulas A, B and C, prove that

$$\vdash A \to B \to \Big((C \to A) \to (C \to B)\Big)$$

9. For any formula A, prove that $\bot \vdash A$.

Only an equational proof is acceptable in exercise #9 (0 points for a Hilbert-style proof).