Lassonde School of Engineering
EECS
MATH1090. Problem Set No. 3
Posted: Oct. 21, 2016

Due: Nov. 16, 2016, by 3:00pm; in the course assignment box.

It is worth remembering (from the course outline):

The homework must be each individual's own work. While consultations with the instructor, tutor, and among students, are part of the learning process and are encouraged, nevertheless, *at the end of all this consultation* each student will have to produce an individual report rather than a copy (full or partial) of somebody else's report.

The concept of "late assignments" does not exist in this course.

In what follows, "give a proof of $\vdash A$" means to give an equational or Hilbert-style proof of A, unless some other proof style is required (e.g., Resolution).

Annotation is always required!

*Do the following problems (**5 MARKS/Each**).*

1. In class we proved $\vdash A \equiv A$ using the "trick" of applying Leibniz with "mouth" a variable $\mathbf{p}$ that does *not* occur in A.

Re-prove this theorem, but this time NOT using this trick. *Be sure your proof is NOT "circular" —i.e., must not use any theorem from class that relies already on* $\vdash A \equiv A$.

2. Use resolution —but *not* Post's theorem— to prove $\vdash (A \to B) \to (B \to C) \to (A \to C)$.

3. Prove

$$\vdash (\neg B \to \neg A) \to (\neg B \to A) \to B$$

A proof by Resolution is the only *acceptable method.*

You are NOT allowed to use Post's theorem in this question!

4. Use Ping-Pong and Resolution (but *not* Post's theorem!) to prove

$$\vdash (A \to B \wedge C) \equiv (A \to B) \wedge (A \to C)$$

A proof by Resolution is the only *acceptable method.*

5. Given two formulae A and B. Suppose that the statement "$\vdash A$ iff $\vdash B$" is true.

Can we conclude $\vdash A \equiv B$?

Exactly WHY yes, or **Exactly WHY no**?

6. Let ϕ be a ternary (three-place) predicate symbol.

Prove

$$\vdash (\forall x)\phi(x, y, z) \equiv (\forall x)\phi(x, y, z) \tag{1}$$

and

$$\vdash (\forall x)\Big(\phi(x, y, z) \to \phi(x, y, z)\Big) \tag{2}$$

7. Let ψ be a binary predicate. Is $(\forall x)(\forall y)\psi(x, y) \to (\forall y)\psi(y, y)$ an instance of **Ax2**? **Why**?

8. Establish that $\vdash (\forall x)(\forall y)\psi(x, y) \to (\forall y)\psi(y, y)$.

9. By induction on terms, (meta)Prove that if x *does not occur* in term s, and t is another term, then the result of $s[x := t]$ is just s.

10. Let ϕ', ψ be any *unary* predicates, x, z distinct variables, and c a constant.

Prove that

$$(\forall x)(\phi'(x) \to \psi(x)),\ (\forall z)\phi'(z) \vdash \psi(c)$$

11. Prove $\vdash (\forall x)(\forall y)(A \lor B \lor C) \equiv (\forall x)(A \lor (\forall y)(B \lor C))$, *on the condition* that y is not free in A.