

## Faculty of Science and Engineering

### MATH1090. Problem Set No1

**Posted:** Sept. 22, 2010

**Due:** Oct. 6, 2010, by 2:00pm; **in the course assignment box.**



It is worth remembering (from the course outline):

The homework must be each individual's own work. While consultations with the instructor, tutor, and among students, are part of the learning process and are encouraged, *at the end of all this consultation* each student will have to produce an individual report rather than a copy (full or partial) of somebody else's report.

The concept of "late assignments" does not exist in this course.



1. (3 MARKS) Prove that the *last* symbol of a *well-formed-formula* cannot be  $\vee$ .

*Hint.* Analyse formula-calculations, or use induction on (the complexity of) formulae.

2. (3 MARKS) Prove that  $\perp \top$  is *not* a wff.

*Hint.* Analyse formula-calculations.

3. (1 MARK) Prove that  $((p \rightarrow \perp) \equiv \top)$  is a wff.

4. (6 MARKS) Recall that a schema is a tautology iff all its *instances* are tautologies.

Which of the following six schemata are tautologies? Show the whole process that led to your answers, *including truth tables or equivalent short cuts, and words of explanation.*

I note that in the six sub-questions below I am *not* using all the formally necessary brackets.

- $((A \rightarrow B) \rightarrow A) \rightarrow A$

- $A \equiv B \rightarrow A \vee B$
- $(A \equiv B) \rightarrow A \rightarrow B$
- $A \rightarrow B \rightarrow \neg B \rightarrow \neg A$
- $(A \rightarrow B) \rightarrow \neg B \rightarrow \neg A$
- $A \wedge B \rightarrow A \rightarrow B$

5. (5 MARKS) Prove that if we have  $A, C \models_{\text{taut}} B$ , then we also have  $\models_{\text{taut}} A \rightarrow C \rightarrow B$  and conversely. Or as we usually put it: “ $A, C \models_{\text{taut}} B$  iff  $\models_{\text{taut}} A \rightarrow C \rightarrow B$ ”.

Here, using truth tables or truth-table tricks, you will show that if you have one side, then you must have the other. There are two directions in your proof!

6. (6 MARKS) By using truth tables, or using related shortcuts, examine whether or not the following tautological implications are correct.

 In order to show that a tautological implication that involves *meta*-variables for formulas —i.e., it is a schema— is *incorrect* you *must* consider a special case that *is* incorrect (since some other special cases might work). 

Show the whole process that led to each of your answers.

- $p \vee q \models_{\text{taut}} \perp$
- $p \vee q \models_{\text{taut}} \top$
- $A \models_{\text{taut}} A \wedge B$
- $A, B \rightarrow A \models_{\text{taut}} B$
- $q, p \rightarrow q \models_{\text{taut}} q$
- $p \wedge q \models_{\text{taut}} p \rightarrow q$

7. (6 MARKS) Write down the most simplified result of the following substitutions, *whenever the requested substitution makes sense*. Whenever a requested substitution does not make sense, explain exactly why it does not.

Show the whole process that led to each of your answers in each case.



Remember the priorities of the various connectives as well as that of the meta-expression “[**p** := ...]”! The following formulae have not been written with all the formally required brackets.



- $p \vee q \rightarrow p[q := r]$
- $(p \rightarrow q)[p := \mathbf{t}]$
- $(p \vee q)[p := \top]$
- $(\perp \vee r \rightarrow q)[\perp := p]$
- $p \vee q \wedge r[q := A]$  (where  $A$  is some formula)
- $p \vee (q \wedge r)[q := A]$  (where  $A$  is some formula)