Faculty of Science and Engineering

MATH1090. Problem Set No1

Posted: Sept. 20, 2009

Due: Oct. 2, 2009; in the course assignment box.

It is worth remembering (from the course outline):

The homework must be each individual's own work. While consultations with the instructor, tutor, and among students, are part of the learning process and are encouraged, nevertheless, *at the end of all this consultation* each student will have to produce an individual report rather than a copy (full or partial) of somebody else's report.

The concept of "late assignments" does not exist in this course.

1. (3 MARKS) Prove that the *first* symbol of a *fully written formula* formula cannot be $\neg$.

Hint. Analyse formula-calculations, or use induction on (the complexity of) formulae.

2. (3 MARKS) Prove that () is *not* a formula.

Hint. Analyse formula-calculations.

3. (6 MARKS) Recall that a schema is a tautology iff all its *instances* are tautologies.

Which of the following six schemata are tautologies? Show the whole process that led to your answers, *including truth tables or equivalent short cuts, and words of explanation*.

I note that in the six sub-questions below I am not using all the formally necessary brackets.

- $((A \to B) \to A) \to A$
- $A \wedge B \to A \equiv B$
- $(A \equiv B) \to A \wedge B$

- $A \to B \equiv \neg B \to \neg A$
- $\neg(A \equiv B) \equiv (\neg A \equiv \neg B)$
- $A \wedge B \to (A \equiv B)$

4. (3 MARKS) Prove that if for some formulae A and B it is the case that $A, B \models_{\text{taut}} \bot$, then it is also the case that $\models_{\text{taut}} B \to \neg A$.

Here, using truth tables or truth-table tricks, you will show that if you have the left side of the "then" in the statement, then you must have the right side as well.

5. (5 MARKS) By using truth tables, or using related shortcuts, examine whether or not the following tautological implications are correct.

Show the whole process that led to each of your answers.

- $p \vee q \models_{\text{taut}} p$
- $A \models_{\text{taut}} A \wedge B$
- $A, A \to B \models_{\text{taut}} B$
- $q, p \to q \models_{\text{taut}} p$
- $p \wedge q \models_{\text{taut}} p$

6. (5 MARKS) Evaluate the statement below for validity:

if $\models_{\text{taut}} A \vee B$, then it must be the case that $\models_{\text{taut}} A$ or $\models_{\text{taut}} B$

Since the statement is a schema you have to act as follows:

1) If you think it is valid, then justify your belief by using a technique similar to the one you used in problem #4 above.

2) If you think it is *not* valid, then show this by providing a "counterexample", that is, an *instance* of the statement which is demonstrably invalid (through a clear *demonstration* via truth tables, for example, that you will provide).

7. (6 MARKS) Compute the most simplified result of the following substitutions, *whenever the requested substitution makes sense*. Whenever a requested substitution does not make sense, explain exactly why it does not.

Show the whole process that led to each of your answers in each case.

Remember the priorities of the various connectives as well as of the meta-expression "[$\mathbf{p}$:= ...]"! The following formulae have not been written with all the formally required brackets.

- $p \lor q \rightarrow p[p := r]$
- $(p \lor q)[p := \mathbf{f}]$
- $(p \lor q)[p := \bot]$
- $(\bot \lor q)[\bot := p]$
- $p \lor q \land r[p := A]$ (where A is some formula)
- $p \lor (q \land r)[r := A]$ (where A is some formula)