Lassonde School of Engineering

Dept. of EECS
Professor G. Tourlakis

MATH1090 A. Problem Set No 4
Posted: Nov. 19, 2024

Due: Dec. 3, 2024; by 5:00pm, in eClass, "Assignment #4"

Q: How do I submit?

A:

(1) **Submission must be ONLY ONE file**

(2) **Accepted File Types: PDF, RTF, MS WORD, ZIP**

(3) **Deadline is strict, electronically limited**.

(4) **MAXIMUM file size = 10MB**

It is worth remembering (from the course outline):

The homework must be each individual's own work. While consultations with the instructor, tutor, and among students, are part of the learning process and are encouraged, nevertheless, *at the end of all this consultation* each student will have to produce an individual report rather than a copy (full or partial) of somebody else's report.

The concept of "late assignments" does not exist in this course.

In what follows, if I say "give a proof of $\vdash A$" or "show $\vdash A$" this means to give an Equational or Hilbert-style proof of A, unless some other proof style is required (e.g., Resolution).

Annotation is always required! Never-ever omit the "$\Leftrightarrow$" from an Equational proof!'

1. (5 MARKS) Prove using 1st-Order **Soundness** (**Required**):

$$\nvdash (\exists \mathbf{x})A \wedge (\exists \mathbf{x})B \rightarrow (\exists \mathbf{x})(A \wedge B)$$

2. (5 MARKS) *Prove* that *IF we have*

$$\vdash (\exists \mathbf{x})A \rightarrow A[\mathbf{x} := \mathbf{z}] \tag{1}$$

($\mathbf{z}$ fresh), THEN we also have

$$\vdash (\exists \mathbf{x})A \rightarrow (\forall \mathbf{x})A \tag{2}$$

Now also answer these three subsidiary questions:

(a) (2 MARKS) What does (2) say in words?

(b) (2 MARKS) Can you find a very simple example of a wff "A" over the natural numbers that makes (2) a non-theorem?

Prove that the wff you proposed *IS* a NON theorem!

(c) (2 MARKS) What can you conclude from (b) about the *validity OR not* of (1)? Why?

3. (5 MARKS) *Use the $\exists$ elimination technique —**Required**— to PROVE, for any A and B*

$$\vdash (\exists \mathbf{x})(A \wedge \neg A) \rightarrow B$$

4. (5 MARKS) Let ϕ be an arbitrary two-variable predicate.

Prove $\vdash (\forall x)(\forall y)\phi(x, y) \to (\forall y)\phi(y, y)$.

Caution: Careful with capture! You cannot allow it!