Department of EECS

MATH1090 A. Problem Set No1

Posted: Sept. 14, 2024

Due: Oct. 4, 2023, by 2:00pm; in eClass.

Q: **How do I submit?**

A:

(1) **Submission must be ONLY ONE file**

(2) **Accepted File Types: PDF, RTF, MS WORD, ZIP**

(3) **Deadline is strict, electronically limited.**

(4) **MAXIMUM file size = 10MB**

It is worth remembering (from the course outline):

The homework must be each individual's own work. While consultations with the instructor, tutor, and among students, are part of the learning process and are encouraged, nevertheless, *at the end of all this consultation* each student will have to produce an individual report rather than a copy (full or partial) of somebody else's report.

The concept of "late assignments" does not exist in this course.

1. (3 MARKS) Prove that (p) is *NOT* a wff, *NOR* is the two-symbol string

$$\bot\top$$

Hint. One way to *prove* this (in the metatheory) is to analyse formula-constructions/calculations. The other is to look at the inductive definition of formulas: can it be applied to define "(p)" and $\bot\top$ as a formulas? Why?

2. (1 MARK) Let Q, P, R be wff's. Prove that so is $(((P \to Q) \to R) \wedge P)$.

In so doing you MUST use "Hilbert style layout" in *formula calculations*, that is, AT EACH STEP, *ONE string is written* at the end of a growing VERTICAL list with a number at its left (its position/row number) and required annotation to the right. Just like the example from class below.

(1)	p	$\langle$atomic$\rangle$
(2)	q	$\langle$atomic$\rangle$
(3)	$(p \vee q)$	$\langle 1 + 2 + \vee$-glue$\rangle$
(4)	r	$\langle$atomic$\rangle$
(5)	$((p \vee q) \vee r)$	$\langle 4 + 3 + \vee$-glue$\rangle$

BE MINDFUL that the CAPITAL P, Q, R are *NOT necessarily* variables or constants! *They are "general" wff we obtained earlier and we do not care of their details*!

3. (6 MARKS) Recall that a **schema** is a tautology iff *all* its *instances* are tautologies.

Which of the following six *schemata* are tautologies? Show the *whole process* that led to your answers, *including truth tables or equivalent short cuts, if you used one or the other, and words of explanation if needed*.

I note that in the six sub-questions below I am NOT *using all the formally necessary brackets. You need to reinsert missing brackets to answer correctly.*

- $A \to B \equiv \neg A \vee B$
- $\Big((A \to B) \to C\Big) \equiv \Big(A \to (B \to C)\Big)$
- $A \wedge B \equiv \neg(\neg A \vee \neg B)$
- $A \to B \to C \equiv B \to A \to C$
- $A \to B \equiv \neg B \to \neg A$
- $((A \to B) \to A) \to A$

4. (2 MARKS) Prove for all wff A, B that we have $A \equiv \neg A \models_{\text{taut}} B$.

5. (6 MARKS) By using truth tables, or using related shortcuts, examine whether or not the following tautological implications are correct.

In order to show that a tautological implication that involves *meta*-variables for formulas (capital latin letters) —i.e., it is a schema— is *incorrect* you *must* consider a *special case* that *is* incorrect (since some other special cases might *work*).

Show the whole process that led to each of your answers.

- $A \vee \neg A \models_{\text{taut}} \top$
- $A \vee \neg A \models_{\text{taut}} A \wedge B$
- $A \wedge \neg A \models_{\text{taut}} A \wedge B$
- $A, A \to B \models_{\text{taut}} B$
- $A \equiv B \models_{\text{taut}} \neg B \to \neg A$
- $A \wedge B \models_{\text{taut}} B \vee A \equiv A \equiv B$

6. (6 MARKS) Write down the most simplified result of the following substitutions, *whenever the requested substitution makes sense*. Whenever a requested substitution does not make sense, explain exactly why it does not.

Show the whole process that led to each of your answers in each case.

Remember the priorities of the various connectives as well as that of the meta-expression "[**p** := ...]"! The following formulas have not been written with all the formally required brackets.

- $(q \to p)[q := p]$
- $(q \to p)[p' := \top]$
- $p \to \top[\top := p]$
- $p \to \top[p := \mathbf{f}]$
- $(\bot \to r \to q)[q \wedge r := p]$
- $(\bot \to r \to q)[r := p][p := r]$

7. (3 MARKS) Prove by induction (on *length* of formula construction or on complexity) or directly via the recursive definition of wff that no wff is the empty string.

8. (2 MARKS) Prove by induction (on *length* of formula construction or on *complexity*) or directly via the *recursive definition* of wff that every formula has as *rightmost* symbol either a right bracket or an atomic formula.