Lassonde School of Engineering

Dept. of EECS

Professor G. Tourlakis

EECS 1028 M. Problem Set No4

Posted: **March 19, 2023**

Due: Apr. 11, 2023; by 10:00pm, in eClass.

Q: How do I submit?

A:

(1) **Submission must be a SINGLE *standalone* file to eClass. Submission by email is not accepted.**

(2) **Accepted File Types: PNG, JPEG, PDF, RTF, MS WORD, OPEN OFFICE, ZIP**

(3) **Deadline is strict, electronically limited.**

(4) **MAXIMUM file size = 10MB**

It is worth remembering (from the course outline):

The homework **must** be each individual's own work. While consultations with the instructor, tutor, and among students, are part of the learning process and are encouraged, **nevertheless**, *at the end of all this consultation* each student will have to produce an individual report rather than a *copy* (full or partial) of somebody else's report.

The concept of "late assignments" does not exist in this course, as you recall.

1. (5 MARKS) Prove that if A is infinite and $A \subseteq B$, then B is also infinite.

2. (3 MARKS) Prove that an enumerable set is infinite.

3. (5 MARKS) Prove that $\vdash (\forall x)(A \to B) \to (\exists x)A \to (\exists x)B$.

4. (5 MARKS) Use simple induction to prove that $\sum_{i=1}^{n+1} i2^i = n2^{n+2} + 2$, for $n \geq 0$.

5. (3 MARKS) Consider the statement (formula)

$$(\exists x)A(x) \to A(z) \tag{1}$$

where z is a *new* variable *not free* (not an "input variable") in $A(x)$.

Find now a *specific* example of $A(x)$ over the set $\mathbb{N}$ and choose a specific value of $z \in \mathbb{N}$ so that (1) becomes **false** (meaning we cannot prove it, since proofs start from true axioms and preserve truth at every step).

6. (4 MARKS) Prove by simple induction on n that, for $n \geq 0$,

$$3^n > n$$

7. (5 MARKS) Using induction prove that $1^3 + 2^3 + \ldots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$, for $n \geq 1$.