Lassonde School of Engineering

Dept. of EECS

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EECS 1028 M. Problem Set No3

Posted: Feb. 17, 2023

Due: Mar. 17, 2023; by 6:00pm, in eClass.

Q: How do I submit?

A:

(1) **Submission must be a SINGLE *standalone* file to eClass. Submission by email is not accepted.**

(2) **Accepted File Types: PNG, JPEG, PDF, RTF, MS WORD, OPEN OFFICE, ZIP**

(3) **Deadline is strict, electronically limited.**

(4) **MAXIMUM file size = 10MB**

It is worth remembering (from the course outline):

The homework **must** be each individual's own work. While consultations with the instructor, tutor, and among students, are part of the learning process and are encouraged, **nevertheless**, *at the end of all this consultation* each student will have to produce an individual report rather than a *copy* (full or partial) of somebody else's report.

The concept of "late assignments" does not exist in this course, as you recall.

1. (5 MARKS) Show that if $\mathbb{F}$ is a function and $\mathrm{dom}(\mathbb{F})$ is a set then $\mathbb{F}$ is a set.

2. (4 MARKS) True or False and WHY? (without the correct "WHY" this maxes out to 0 (zero)). If $\mathbb{P}$ is a relation and $\mathrm{dom}(\mathbb{P})$ is a set then $\mathbb{P}$ is a set.

3. (a) (1 MARK) Define "Left Field" for a Relation $\mathbb{R}$ and a Function $\mathbb{F}$.

(b) (4 MARKS) True or False and WHY? (No WHY nets a zero for guessing.) A function f is 1-1 by definition precisely if $f(x) = f(y)$ *implies* $x = y$ for all x, y in the function's left field.

4. (3 MARKS) Prove that if the function f is 1-1, then f^{-1} is a function.

Caution! The ONLY assumptions here are 1) f is a function and 2) it is 1-1. It MAY be nontotal, non onto and a lot of other "non" that you may NOT assume, NOR negate!

5. (5 MARKS) Let $f : A \to B$. Then $\mathbf{1}_B f = f$ and $f \mathbf{1}_A = f$.

Hint. You may use the fact that fg, for functions f, g, means $g \circ f$.

6. (4 MARKS) Let $<$ be an abstract (strict) order and $\mathbb{B}$ class.

Prove that $< \cap (\mathbb{B} \times \mathbb{B})$ is an order ON $\mathbb{B}$.

7. Suppose we know that each of A_n, $n \geq 0$, is countable.

Show that

(a) (3 MARKS) $\{A_0, A_1, \ldots, A_n, \ldots\}$ is a set.
If you used some of the Principles 0–3 in this subquestion, be explicit!

(b) (4 MARKS) Prove that $\bigcup_{i \geq 0} A_i$ is countable.

(c) (2 MARKS) Did you need the Axiom of Choice in any of the sub-questions here? Explain clearly in a FEW words.

8. (4 MARKS) Prove that the relation $\sim$ between sets is *symmetric* and *transitive*.