EECS 4111/5111 —Fall 2019

Posted: Sep. 23, 2019
Due: TBA

Problem Set No. 1

NB. *All problems are equally weighted and will be assigned a letter grade; an overall letter grade for the paper will be computed using York's 0–9 gpa scale.*

The problem set list for *grad students* enrolled in EECS 5111 is the entire list here. Undergrads *should omit any problems marked "Grad"*.

This is not a course on *formal* recursion theory. Your proofs should be *informal* (but NOT sloppy), *completely argued*, correct, and informative (and if possible **short**). However, please do not trade length for correctness or readability.

All problems are from the "Theory of Computation Text", or are improvisations that I completely articulate here.

(1) For total f, prove that if its graph $y = f(\vec{x})$ is recursive, then $f \in \mathcal{R}$.

Hint. Use Unbounded Search to define the function.

(2) Show by a simple counterexample that if we omit the qualifier "total" above, then this does not work. That is, show that there is an f that has a recursive graph, but $f \notin \mathcal{R}$.

(3) Let $\|x\|$ denote the decimal length of the natural number x. Prove that $\lambda x.\|x\| \in \mathcal{PR}$.

Hint. Use Bounded Search to define the function.

(4) Prove that $\lambda x.\lfloor \log_{10} x \rfloor \in \mathcal{PR}$.

Hint. Use Bounded Search to define the function.

From Section 2.12.

(5) Do problems 6, 9, 11, 12.

(6) In class I claimed that $p_n \le 2^{2^n}$ for all n. Prove this.

Hint. Do "strong" induction (that is, I.H. will be "assume for n or less"). Work with $p_0 p_1 \cdots p_n + 1$.[†]

(7) (**Grad**). Do problems 22, 25.

Hint for 25. Prove an easy lemma (induction): $A_x(2) > x + 1$, for all $x \ge 0$.

[†] As Euclid did in order to prove that there are infinitely many primes.