Lassonde Faculty of Engineering
EECS
EECS2001Z. Problem Set No2
Posted: Feb. 24, 2019

Due: Mar. 19, 2019, by 2:30pm; in the course assignment box.

It is worth remembering (quoted from the course outline):

The answers must be typed (but you may dow symbols by hand, if it is easier for you).

The homework must be each individual's own work. While consultations with the instructor, tutor, and among students, are part of the learning process and are encouraged, nevertheless, *at the end of all this consultation* each student will have to produce an individual report rather than a copy (full or partial) of somebody else's report.

The concept of "late assignments" does not exist in this course.

1. (5 MARKS) Write a simultaneous recursion that uses absolutely no arithmetic to compute $\lambda x.rem(x, 5)$.

Hint. Use as reference the similar *completely solved* example (text/notes/class) for $rem(x, 2)$. Your recursion will be defining *exactly* five functions, one of which will be rem. Identify which one is rem and carefully explain how you obtained the five recurrence equations.

2. (5 MARKS) Imitate the diagonalisation that we used in showing the Halting Problem unsolvable, and show that $\lambda xyz.\phi_x(y) = z$ is unsolvable too.

Hint. If the problem were solvable then so would be $\lambda x.\phi_x(x) = z$ by Grz. Ops. Diagonalise to get a contradiction to the claim that $\lambda x.\phi_x(x) = z$ is solvable.

3. (5 MARKS) Prove that $A = \{x : \phi_x = \lambda x.42\}$ is not recursive; i.e., $x \in A$ is unsolvable/undecidable.

Hint. Use the technique in posted Note #7 to show (via S-m-n) that $K \leq A$.

4. (5 MARKS) Prove that the problem $x \in Q$ where $Q = \{x : 111 \in \text{ran}(\phi_x)\}$ *is* semi-computable.

Hint. Use closure properties of $\mathcal{P}_*$ and the semi-recursiveness of $\lambda xyz.\phi_x(y) = z$ from the Notes #7.

5. (5 MARKS) Prove that $B = \{x : 42 \notin \text{ran}(\phi_x)\}$ is *not* semi-computable, that is, we cannot verify that "ϕ_x will not ever print 42"

Hint. We have techniques (*and* examples!) in Notes #7 to almost directly conclude the above from them! For example, we have (essentially) proved that $\overline{B} = \{x : 42 \in \text{ran}(\phi_x)\}$ is *unsolvable* AND semi-decidable (we did the latter in problem #4 above). (Nudge-nudge: What is a set S if both it and its complement are semi-recursive?)

Alternatively, easily modify the reduction techniques of Notes #7 to show $\overline{K} \leq B$.

6. (5 MARKS) Prove that $C = \{x : \phi_x \text{ is a } \textit{total} \text{ } 0/1\text{-valued function}\}$ is NOT semi-computable.

Hint. Go by contradiction: Suppose it is. Then it is also r.e (or c.e.) Now use an easy modification of the proof that $\{x : \phi_x \in \mathcal{R}\}$ is not r.e. to prove that our C is not r.e.

7. (5 MARKS) Hm. How about removing the restriction "total" above? Prove that $D = \{x : \phi_x \text{ is a } 0/1\text{-valued function}\}$ is NOT semi-computable.

Hint. Read carefully Notes #7.

8. (5 MARKS) A "Word Problem!" Prove that the problem "Is an arbitrary URM of one input $\mathbf{x}$ eventually halting when inputed the value 42?" is undecidable.

Hint. Translate the word problem using ϕ_x notation. Then read carefully Notes #7; the answer is there somewhere.