

Lassonde Faculty of Engineering
EECS

EECS2001B. Problem Set No1

Posted: Sept. 17, 2022

Due: Oct. 21, 2022, by 3:00pm; in the course's eClass, "Assignment #1".

Q: *How do I submit?*

A:

- (1) **The text of all answers is expected to be typed.**
- (2) **Submission must be ONLY ONE file**
- (3) **Accepted File Types: PDF, RTF, MS WORD, ZIP**
- (4) **Deadline is strict, electronically limited.**
- (5) **MAXIMUM file size = 10MB**



It is worth remembering (quoted from the course outline):

The answers must be typed (but you may draw symbols *by hand*, if it is easier for you; *MS Word has an equation editor that does almost all the symbols we use*).

The homework must be each individual's own work. While consultations with the instructor, tutor, and among students, are part of the learning process and are encouraged, *at the end of all this consultation*

each student will have to produce an individual report rather than a copy (full or partial) of somebody else's report.

The concept of “late assignments” does not exist in this course.



1. (5 MARKS) By **induction on the length of derivations** prove that $\mathcal{PR} \subseteq \mathcal{P}$.
2. (2 MARKS) Using the preceding problem conclude that $\mathcal{PR} \subseteq \mathcal{R}$.
3. (3 MARKS) Prove that $\lambda x.2^x$ is primitive recursive without using the primitive recursiveness of $\lambda xy.x^y$.
4. (5 MARKS) Prove that the function

$$\lambda x. x \text{ 2s } \left\{ \begin{array}{l} 2 \\ 2^2 \\ \dots \\ 2^{\dots 2} \end{array} \right.$$

is in \mathcal{PR} .

5. p.234 of the text, Section 2.12: Do
 - (a) (4 MARKS) Do Exercise 6.
 - (b) (5 MARKS) Do Exercise 10.
Hint for (b). Express the outputs of max and min by expressions that involve x, y and only use $+$, \div .
6. (4 MARKS) Put the recursion

$$\begin{aligned} sw(0, y) &= y \\ sw(x + 1, y) &= 0 \end{aligned}$$
 in normal primitive recursion form.
7. (3 MARKS) Show that the function in Problem 3 above can be programmed by a Loop-program that does not nest **Loop-end** instructions in depth more than two such instructions.
8. (5 MARKS) Write a very simple loop program that computes the function in Problem 4. Be sure that your program has *loop nesting equal to 3*—no more, no less!

9. (4 MARKS) Revisit problem 6 and prove that $sw \in \mathcal{PR}$ *not* by virtue of showing that the indicated recursion is indeed primitive, but rather prove it by expressing the call $sw(x, y)$ for any x, y by a composition in terms of the known to be primitive recursive $\lambda zu.z \times u$ and $\lambda zu.z \div u$.