# Lassonde Faculty of Engineering
# EECS
## EECS2001B. Problem Set No2
**Posted: Oct. 24, 2020**

**Due: Nov. 17, 2020, by 2:00pm; in the course's eClass, "Assignment #2".**

**Q**: *How do I submit?*

**A**:

(1) **The text of all answers is expected to be typed.**

(2) **Submission must be ONLY ONE file**

(3) **Accepted File Types: PDF, RTF, MS WORD, ZIP**

(4) **Deadline is strict, electronically limited**.

(5) **MAXIMUM file size = 10MB**

It is worth remembering (quoted from the course outline):

**The answers must be typed** (but you may dow symbols by hand, if it is easier for you).

The homework must be each individual's own work. While consultations with the instructor, tutor, and among students, are part of the learning process and are encouraged, nevertheless, *at the end of all this consultation* each student will have to produce an individual report rather than a copy (full or partial) of somebody else's report.

The concept of "late assignments" does not exist in this course.

**1.** (5 MARKS) Program the function $\lambda xyz.if\ x = 0\ then\ y\ else\ z$ in the Loop-Program programming language *with the least amount of Loop-end nesting*. (That is, **no nesting** in this case.)

**Warning**. If your program is correct but has higher nesting, then it is assessed 2 MARKS.

**2.** (5 MARKS) Imitate the diagonalisation that we used in proving that the *Halting Problem* is unsolvable, and prove that $\lambda xyz.\phi_x(y) = 42$ is unsolvable too.

*Hints.*

- If the problem were solvable then so would be $\lambda x.\phi_x(x) = 42$ by Grz. Ops.

  Now modify the main diagonal of the $\phi_i(j)$-matrix we used in the *Halting Problem*, changing every entry on it from 42 to 0, and from $\neq 42$ to 42 in order to obtain a partial recursive function that *is NOT a row of the matrix*. Then say why is this a contradiction.

  Or, if you prefer, do it this way:

- Use the technique we used for $A = \{x : \phi_x \text{ is a constant}\}$ in Class/Notes/Text to show $K \leq A$. Conclude from this.

**3.** (5 MARKS) We proved using *S-m-n* and the *reduction* $K \leq A$ that $A = \{x : \phi_x \text{ is a constant}\}$ that $A$ is not recursive ($x \in A$ is "unsolvable").

**Prove that this is so also for a specific constant: That the set** $B = \{x : \phi_x = \lambda x.1000\}$ **is *not recursive*.**

*Hint.* The proof I am asking you to do is obtained by a *very easy modification* of what we did for $A$ in the Notes/Class (and Text).

**4.** (5 MARKS) Prove that the problem $x \in C$ where $C = \{x : 42 \in \text{ran}(\phi_x)\}$ *is* semi-computable.

*Hint.* Use closure properties of $\mathcal{P}_*$ and the semi-recursiveness of $\lambda xyz.\phi_x(y) = z$ from the Notes/Class/Text.

**5.** (5 MARKS) Prove that $D = \{x : 101 \notin \text{ran}(\phi_x)\}$ is *not* semi-computable, that is, we cannot verify that "$\phi_x$ will never print 101".

*Hint.* Show $\overline{K} \leq D$ by considering

$$\psi(x, y) = \begin{cases} 101 & \text{if } \phi_x(x) \downarrow \\ \uparrow & \text{othw} \end{cases}$$

You *MUST* provide ALL details *that flow from this Hint*. **Just repeating the Hint with no work of yours added is assessed as 0 MARKS.**

**6.** (5 MARKS) Prove that $E = \{x : \phi_x \text{ is a } \textit{characteristic function}\}$ is NOT semi-computable.

*Hint.* Can you modify the proof that $\{x : \phi_x \in \mathcal{R}\}$ is not r.e. to prove that our $E$ is not r.e.?

**7.** (5 MARKS) Decidable or undecidable? And WHY? $F = \{x : \phi_x(0) \downarrow\}$.

*Hint.* Can

$$\psi(x, y) = \begin{cases} 101 & \text{if } \phi_x(x) \downarrow \\ \uparrow & \text{othw} \end{cases}$$

help?