Lassonde School of Engineering

Dept. of EECS

Professor G. Tourlakis

EECS 1028Z. Problem Set No4

Posted: **Mar. 24, 2025**

Due: Apr. 4, 2025; by 6:00pm, in eClass.

Q: How do I submit?

A:

(1) **Submission must be a SINGLE *standalone* file to eClass. Submission by email is not accepted.**

(2) **Accepted File Types: PNG, JPEG, PDF, RTF, MS WORD, OPEN OFFICE, ZIP**

(3) **Deadline is strict, electronically limited.**

(4) **MAXIMUM file size = 10MB**

It is worth remembering (from the course outline):

The homework **must** be each individual's own work. While consultations with the instructor, tutor, and among students, are part of the learning process and are encouraged, **nevertheless**, *at the end of all this consultation* each student will have to produce an individual report rather than a *copy* (full or partial) of somebody else's report.

The concept of "late assignments" does not exist in this course, as you recall.

1. (1 MARK) Define: "A is uncountable".

2. (5 MARKS) Prove that if A is uncountable and $A \subseteq B$, then B is also uncountable.

3. (5 MARKS) Prove that $A \rightarrow B \vdash (\exists x)A \rightarrow (\exists x)B$.

4. (5 MARKS) Let

$$b_1 = 3,\ b_2 = 6$$
$$b_k = b_{k-1} + b_{k-2}, \text{ for } k \geq 3$$

Prove that 3 divides b_k, for all $k \geq 1$.

5. (3 MARKS) Consider the statement (formula)

$$(\exists x)A(x) \rightarrow A(c) \tag{1}$$

where c is a *constant*.

Find now a *specific, very simple,* example of $A(x)$ over the set $\mathbb{N}$ and choose a specific value of $c \in \mathbb{N}$ so that (1) becomes **false** (meaning we cannot prove it, since proofs start from true axioms and preserve truth at every step).

6. (5 MARKS) Using simple induction prove that

$$1^2 + 2^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

for $n \geq 1$.