Lassonde School of Engineering

Dept. of EECS

Professor G. Tourlakis

EECS 1028 Z. Problem Set No3

Posted: Mar. 10, 2025

Due: Mar. 24, 2025; by 6:00pm, in eClass.

Q: How do I submit?

A:

(1) **Submission must be a SINGLE *standalone* file to eClass. Submission by email is not accepted.**

(2) **Accepted File Types: PNG, JPEG, PDF, RTF, MS WORD, OPEN OFFICE, ZIP**

(3) **Deadline is strict, electronically limited.**

(4) **MAXIMUM file size = 10MB**

It is worth remembering (from the course outline):

The homework **must** be each individual's own work. While consultations with the instructor, tutor, and among students, are part of the learning process and are encouraged, **nevertheless**, *at the end of all this consultation* each student will have to produce an individual report rather than a *copy* (full or partial) of somebody else's report.

The concept of "late assignments" does not exist in this course, as you recall.

1. (4 MARKS) Show that if $\mathbb{F}$ is a function and $\text{dom}(\mathbb{F})$ is a set then $\mathbb{F}$ is a set.

2. (3 MARKS) Show by an easy counterexample that "if $\mathbb{F}$ is a function and $\text{ran}(\mathbb{F})$ is a set then $\mathbb{F}$ is a set" is false.

3. (4 MARKS) Define a **DIFFERENT implementation**

$$\langle x, y\rangle \stackrel{Def}{=} \Big\{\{x\}, \{x, y\}\Big\}$$

for **ordered pair** where this time we denote the latter as "$\langle x, y\rangle$" (angular brackets).

Prove that

$$\langle x, y\rangle = \langle x', y'\rangle \to x = x' \wedge y = y' \tag{1}$$

Caution. This does NOT require arguments via "set formation by stages".

4. (a) (2 MARKS) State the definition given in Class/NOTEs/Text for

$$\boxed{A \text{ is countable}} \tag{2}$$

(b) (4 MARKS) Prove that the Definition from Class/NOTEs/Text is equivalent to

$$\boxed{A \text{ is countable } \textbf{iff}\text{, there is a } \underline{\text{total}} \text{ and } \underline{\text{1-1}}\ f : A \to \mathbb{N}} \tag{3}$$

5. (4 MARKS) Prove transitivity of $\sim$, that is, if $A \sim B \sim C$, then $A \sim C$.