Lassonde School of Engineering

Dept. of EECS

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EECS 1028 E. Problem Set No1

Posted: Jan. 19, 2025

Due: Feb. 7, 2025; by 6:00pm, in eClass.

Q: How do I submit?

A:

(1) **Submission must be a SINGLE *standalone* file to eClass. Submission by email is NOT accepted.**

(2) **Accepted File Types: PNG, JPEG, PDF, RTF, MS WORD, OPEN OFFICE, ZIP**

(3) **Deadline is strict, electronically limited.**

(4) **MAXIMUM file size = 10MB!!!**

It is worth remembering (from the course outline):

The homework **must** be each individual's own work. While consultations with the instructor, tutor, and among students, are part of the learning process and are encouraged, **nevertheless**, *at the end of all this consultation* each student will have to produce an individual report rather than a *copy* (full or partial) of somebody else's report.

The concept of "late assignments" does not exist in this course, as you recall.

1. True or False **and Why**. (**NOTE**: NO Why – NO Points)

(a) (3 MARKS) $\{\mathbb{U}\}$ is a set.

(b) (2 MARKS) $1 \in 1$. Do you need set-formation-by-stages to answer this?

(c) (2 MARKS) For sets or atoms c, d: $\bigcup\{\{c\}, \{d\}\} = \{c, d\}$

(d) (2 MARKS) $\emptyset \subseteq \emptyset$

(e) (2 MARKS) $\emptyset \in \{1\}$

2. (4 MARKS) Prove for any set A that $\{\{A\}\}$ is a set. No random words please! A *MATH proof* is expected that one step implies the next.

3. (3 MARKS) Let A, B, C be sets or atoms. Prove that $\{A, B, C\}$ *is a set, without* using *any* of the Principles 0, 1, 2. *Rather use results (theorems)* that we already established in class/Notes. *Which ones?*

4. Prove that Principle 2 implies that we have infinitely many stages available.

Hint. Arguing by contradiction, assume instead that we only have **finitely many** stages. So repeatedly applying Principle 2 we can form a non ending sequence of stages

$$\cdots < \Sigma' < \Sigma'' < \Sigma''' < \Sigma'''' < \cdots \tag{1}$$

If the sequence (1) contains only a *finite* number of distinct $\Sigma''^{\cdots\prime}$, then at least two of the $\Sigma''^{\cdots\prime}$ in (1) are the same stage.

(a) (1 MARK) Why is the red underlined statement above true?

(b) (4 MARKS) Use this conclusion of stage-repetition and properties of "<" to get a contradiction.

5. (3 MARKS) Find a *one element* set A for which $\mathbb{U} - A \neq \mathbb{U}$.

Prove that indeed $\mathbb{U} - A \neq \mathbb{U}$.

6. (3 MARKS) Let $\mathbb{A}$ be any class (proper or not). Prove that

$$\mathbb{U} \cup \mathbb{A} = \mathbb{U} \tag{1}$$

Caution. "Must prove" means do not wear the wig, but argue MATHEMATICALLY that (1) is true.

Statement (1) requires TWO proofs (two directions); State WHY and proceed:

1) Let $x \in lhs$. Analyse what this means and prove $x \in rhs$.

2) Let $x \in rhs$. Analyse what this means and prove $x \in lhs$.

7. (4 MARKS) Prove for any classes $\mathbb{A}, \mathbb{B}$, that

$$\mathbb{A} \cap \mathbb{B} = \mathbb{U} - \Big(\big(\mathbb{U} - \mathbb{A}\big) \cup \big(\mathbb{U} - \mathbb{B}\big)\Big)$$

Hint. This is a simple case of proving $lhs \subseteq rhs$ by doing "*Let* $x \in lhs$. BLA BLA BLA *AND* concluding $x \in rhs$", and then *ALSO* doing $rhs \subseteq lhs$ by doing "Let $x \in rhs$. BLA BLA BLA and concluding $x \in lhs$".

8. Use the notation by explicitly listing **all the members** of each rhs $\{???\}$ to complete the following incomplete equalities:

(a) (2 MARKS) $2^{\emptyset} = \{???\}$

(b) (2 MARKS) $2^{\{a,b,c\}} = \{???\}$