Lassonde School of Engineering

Dept. of EECS

Professor G. Tourlakis

EECS 1028 M. Problem Set No4

Posted: **March 19, 2022**

Due: Apr. 11, 2022; by 10:00pm, in eClass.

Q: How do I submit?

A:

(1) **Submission must be a SINGLE *standalone* file to eClass. Submission by email is not accepted.**

(2) **Accepted File Types: PNG, JPEG, PDF, RTF, MS WORD, OPEN OFFICE, ZIP**

(3) **Deadline is strict, electronically limited.**

(4) **MAXIMUM file size = 10MB**

It is worth remembering (from the course outline):

The homework **must** be each individual's own work. While consultations with the instructor, tutor, and among students, are part of the learning process and are encouraged, **nevertheless**, *at the end of all this consultation* each student will have to produce an individual report rather than a *copy* (full or partial) of somebody else's report.

The concept of "late assignments" does not exist in this course, as you recall.

1. (5 MARKS) Prove that if A is infinite and $a \in A$, then $A - \{a\}$ is also infinite.

2. (5 MARKS) We learnt in class/Notes that $A[x] \to (\forall x)A[x]$ is NOT a theorem if x indeed occurs free in A.

Yet, I think I got a proof: First, let me use DThm and prove instead $A[x] \vdash (\forall x)A[x]$:

1) $A[x]$ $\langle$hyp$\rangle$
2) $(\forall x)A[x]$ $\langle 1 + \text{Gen}\rangle$

Something must be wrong in my "PROOF"! What EXACTLY (not too many words please and above all don't say "it's not a theorem"!)

3. (5 MARKS) Prove that $\vdash (\exists x)(A \to B) \to (\forall x)A \to (\exists x)B$.

4. (5 MARKS) All the sets in this problem are subsets of $\mathbb{N}$. For any $A \subseteq \mathbb{N}$, let us use the notation

$$\overline{A} \stackrel{Def}{=} \mathbb{N} - A$$

Now prove by simple induction on n that

$$\overline{\bigcap_{1 \le i \le n} A_i} = \bigcup_{1 \le i \le n} \overline{A_i} \tag{1}$$

5. (4 MARKS) Prove by simple induction on n that

$$2^n > n$$

6. (5 MARKS) Prove by CVI that every natural number $n \ge 2$ is a product of prime numbers.

NOTE. A prime number p is defined to satisfy (a) $p > 1$ and (b) the only divisors of p are 1 and p.