Lassonde School of Engineering

Dept. of EECS
Professor G. Tourlakis

EECS 1028 M. Problem Set No3

Posted: Feb. 19, 2022

Due: Mar. 17, 2022; by 10:00pm, in eClass.

Q: How do I submit?

A:

(1) **Submission must be a SINGLE *standalone* file to eClass. Submission by email is not accepted.**

(2) **Accepted File Types: PNG, JPEG, PDF, RTF, MS WORD, OPEN OFFICE, ZIP**

(3) **Deadline is strict, electronically limited.**

(4) **MAXIMUM file size = 10MB**

It is worth remembering (from the course outline):

The homework **must** be each individual's own work. While consultations with the instructor, tutor, and among students, are part of the learning process and are encouraged, **nevertheless**, *at the end of all this consultation* each student will have to produce an individual report rather than a *copy* (full or partial) of somebody else's report.

The concept of "late assignments" does not exist in this course, as you recall.

1. (5 MARKS) Show that it *was not necessary* to apply the *new* Principle 3 to prove that for an equivalence relation R on A, both sets, the class of equivalence classes of R —A/R— is a set.

Specifically show that this follows by Principles 0–2 implicitly —via the subclass-theorem.

Hint. You will need, of course, to find a *superset* of A/R, that is, a class X that *demonstrably* is a set, and satisfies $A/R \subseteq X$.

2. (3 MARKS) Prove that if the function f is 1-1, then f^{-1} is a function.

3. (6 MARKS) Let $f : A \to B$. Then $\mathbf{1}_B f = f$ and $f\mathbf{1}_A = f$.

Hint. You may use the fact that fg, for functions f, g, means $g \circ f$.

4. Let $f : A \to B$ be a 1-1 correspondence. Then

- (2.5 MARKS) If $gf = \mathbf{1}_A$, we have $g = f^{-1}$.
- (2.5 MARKS) If $fh = \mathbf{1}_B$, we have $h = f^{-1}$.

5. (5 MARKS) Suppose we have an enumeration of A

$$a_0, a_1, a_2, \ldots \tag{1}$$

without repetitions (i.e., all the a_i are distinct).

Show in mathematical detail how to construct a new enumeration from (1) where *each element of A is enumerated infinitely many times*.

6. (5 MARKS) We defined the relation $\sim$ between sets by

$A \sim B$ means that there there is a 1-1 correspondence $f : A \to B$

Show that $\sim$ is symmetric and transitive.