A SIMPLE INDUCTION PROOF

Hi all,
Here is the SOLUTION.

BTW, I had a Prof. at U of T who used to say, "Do all the exercises; actually, do the ones you *can't* do; do not do the ones you *can* do."

If he *was* right (I hope he was), then the fact that the last two practices you did *NOT do* ought to mean that you *know* how to do them. *This is good!*

OK

Find a simple Big-O upper bound in terms of a simple function of n (and *prove* why it is an upper bound) for

$$1 + 2 + 3 + \ldots + n \tag{1}$$

or more precisely

$$\sum_{i=1}^{n} i$$

Best solution:

$$1 + 2 + 3 + \ldots + n \le \overbrace{n + n + n + \ldots + n}^{n\ terms} = n^2$$

So,

$$1 + 2 + 3 + \ldots + n = O(n^2) \qquad \square$$

So-so solution:

$$1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2} = n^2/2 + n/2 = O(n^2), \text{ by 6.1.2 or 6.1.3 from notes \#11}$$

Why "so-so"? I explained that already below. $\square$

While you can easily find such a bound if you *know* the closed form formula for (1), this is a bad way of going about it (do only if you are desperate :)

The thing is, we do not always *know* a closed form for a sum like

$$\sum_{i=1}^{n} f(i)$$

where f is some function. E.g., do you know a closed form for $\sum_{i=1}^{n} i^5$? I don't either, but I can sure give you a "tight"[2] big-oh bound!

[2] Almost any sum you throw at me is $O(2^{2^{2^{2^{2^{2^n}}}}})$, but this is TOO pessimistic. A tight bound is as close as it gets to what you are bounding!

So, read the 6.1 in the last chapter that I uploaded today (March 22, 2020; Notes #11), and send me your solutions *tomorrow* between 2:00pm – 3:00pm via Moodle upload (Moodle area "practice #3").