Lassonde School of Engineering

Dept. of EECS

Professor G. Tourlakis

EECS 1028 M. Problem Set No4

Posted: March 14, 2020

Due: From Apr. 4, 2:00pm until Apr. 6, 2020; by 2:00pm,

Submit by Moodle to area named "ASSIG #4".

It is worth remembering (from the course outline):

The homework **must** be each individual's own work. While consultations with the instructor, tutor, and among students, are part of the learning process and are encouraged, **nevertheless**, *at the end of all this consultation* each student will have to produce an individual report rather than a *copy* (full or partial) of somebody else's report.

The concept of "late assignments" does not exist in this course, as you recall.

1. (6 MARKS) We are given an enumeration of A by a 1-1 correspondence $f : \mathbb{N} \to A$. *This enumeration has no repetitions.*

Hint. Prove that you can use the enumeration f to obtain a different enumeration where each member of A is enumerated infinitely many times. You will find the traversal with north-east arrows, $\nearrow$, profitable. Traversal of what? (This I leave to you :)

2. (5 MARKS) Prove that if A is infinite and $a \in A$, then $A - \{a\}$ is also infinite.

3. (5 MARKS) Prove that $\vdash (\forall x)(A \to B) \to A \to (\forall x)B$, **provided** x is not free in A.

4. (5 MARKS) Prove that $\vdash (\forall x)(A \to B) \to (\exists x)A \to (\exists x)B$.

5. (5 MARKS) All the sets in this problem are subsets of $\mathbb{N}$. For any $A \subseteq \mathbb{N}$, let us use the notation

$$\overline{A} \stackrel{Def}{=} \mathbb{N} - A$$

Now prove by simple induction on n that

$$\overline{\bigcup_{1 \le i \le n} A_i} = \bigcap_{1 \le i \le n} \overline{A_i} \tag{1}$$

6. (5 MARKS) Prove by simple induction on n that

$$\sum_{k=1}^{n} k^2 = n(n+1)(2n+1)/6 \tag{2}$$

7. (5 MARKS) Prove by induction that

$$\sum_{0 \le k \le n} (-2)^k = (1/3)(1 - 2^{n+1})$$

for all *odd positive* n.

Hint. An odd positive n has the form $2m + 1$ for $m \ge 0$.

8. (6 MARKS) Let A denote a finite alphabet. Let A^* denote the set of all strings over A, including the empty string λ. Let us define the set P as the *closure* of the *initial set* $\mathcal{I} = \{\lambda\} \cup A$, under the *operations* on strings, O_a, given by

$$x \mapsto \boxed{O_a} \mapsto axa$$

for every $a \in A$.

So the set of operations $\mathcal{O}$ is $\{\boxed{O_a} : a \in A\}$. Thus, $P = \mathrm{Cl}(\mathcal{I}, \mathcal{O})$.

We are also told that $\widehat{P}$, the set of *palindromes* is defined over A^* as

$$\{x : x \text{ is identical to its reversal}\}$$

Prove that $P = \widehat{P}$.

Hint. For $\subseteq$ do induction on P. For $\supseteq$ do induction on the length of strings in $\widehat{P}$ to show that each such string has an $(\mathcal{I}, \mathcal{O})$-derivation.