Lassonde School of Engineering

Dept. of EECS

Professor G. Tourlakis

EECS 1028 M. Problem Set No2

Posted: Jan. 26, 2020

Due: Feb. 12, 2020; by 3:00pm, in the course assignment box.

It is worth remembering (from the course outline):

The homework **must** be each individual's own work. While consultations with the instructor, tutor, and among students, are part of the learning process and are encouraged, **nevertheless**, *at the end of all this consultation* each student will have to produce an individual report rather than a *copy* (full or partial) of somebody else's report.

The concept of "late assignments" does not exist in this course, as you recall.

1. (3 MARKS) Use a theorem that we proved in class, but do *NOT* use an argument involving stages, to prove that if A is a set, then so is $\{A\}$.

2. (3 MARKS) By picking two particular very small sets A and B show that $A - B = B - A$ is not true for all sets A and B.

Is it true of all classes? **WHY**?

3. (3 MARKS) What is $\bigcup F$ if $F = \emptyset$? Set or proper class? **WHY**? "Compute" which class exactly it is.

4. (6 MARKS) Prove that if a *reflexive* R (on some set A) satisfies

$$xRy \wedge xRz \rightarrow yRz \qquad (1)$$

for all x, y, z, then it is an *equivalence relation*.

Caution. Note the order of the x, y, z in (1)!

5. (6 MARKS) Show that if for a relation R we know that $R^2 \subseteq R$, then R is transitive, *and conversely*.

Hint. There are two directions in the sought proof.

6. (6 MARKS) Show that for any relation R we have

$$R \text{ is transitive } \textit{iff } R = R^+$$

Hint. There are two directions in the sought proof.

7. (8 MARKS) In class we proved that if R is on $A = \{a_1, a_2, \ldots, a_n\}$, where the a_i are distinct, then

$$R^+ = \bigcup_{i=1}^{n} R^i$$

Prove that if R is moreover reflexive, then

$$R^+ = R^{n-1}$$

That is, taking a *union* of R^i is not needed; just one term, R^{n-1} will do!

8. (5 MARKS) Give an example of two equivalence relations R and S on the set $A = \{1, 2, 3\}$ such that $R \cup S$ is *not* an equivalence relation.