This page must be submitted as the first page of your exam-paper answer pages.

York University
Department of Electrical Engineering and Computer Science
Lassonde School of Engineering

EECS 1028 M. FINAL EXAM, April 9, 2020; 9:00-11:00am
Professor George Tourlakis

By putting my name and student ID on this Exam page, I attest to the fact that my answers included here and submitted by Moodle are my own work, and that I have acted with integrity, abiding by the *Senate Policy on Academic Honesty* that the instructor discussed at the beginning of the course and *linked the full Policy to the Course Outline*.

Student NAME (**Clearly**):________________________________

Student NUMBER (**Clearly**):________________________________

DATE (**Cearly**):________________________________

README FIRST! **INSTRUCTIONS:**

1. Please read ALL these instructions carefully before you start writing.

2. **TIME-LIMITED ON LINE EXAM**. You have **TWO hours to answer the Exam questions, plus another 45 minutes to allow a proper uploading job**. Overall Exam End and last opportunity to upload is thus at 11:45 am. **Only ONE file can be uploaded per student**.

3. If you submit photographed copy **it still must be ONE file that you submit**. Either ZIP the PNG images **OR** import them in MS Word and submit *ONE* Word *file* with the PNGs attached.

4. Using the time allotted for the uploading mechanisms (45 min) for the exam-answering part is at your own *discretion*. But also at your own **risk**. **Exam not uploaded = Exam not written**.

5. Please write your answers by hand **as you normally do for assignments** or use a word processor that can convert to PDF. Microsoft Word is acceptable to upload as is (without conversion to PDF).

6. Whichever theorems were *proved* in class or appeared in the assignments you may use without proof, **unless I am asking you to prove them in this Exam**. If you are not sure whether some statement has **indeed** been proved *in class*, I recommend that you prove it in order to be "safe".

Question	MAX POINTS	MARK
1	4	
2	5	
3	6	
4	4	
5	5	
6	4	
7	4	
8	4	
9	4	
TOTAL	40	

Question 1. (a) (1 MARK) Does **Principle 2** refer to "stages"? (Simply: **Yes/No**)

(b) (3 MARKS) Using said principle prove that if R is an equivalence relation on a *set* A then A/R that denotes **the set of all the equivalence classes of** R is also a *set*.

Question 2. Consider the congruence modulo 5, "$\equiv_5$".

We know that it is an equivalence relation on $\mathbb{Z}$.

(a) (1 MARK) How many equivalence classes does $\equiv_5$ have?

(b) (4 MARKS) Display **each of the equivalence classes** of $\equiv_5$ as specific sets of integers.
Caution. A "display" that is just the *definition* of an equivalence class as in

$$[y] \stackrel{Def}{=} \{x \in \mathbb{Z} : x \equiv_5 y\}$$

will **NOT** do (0 marks). Your display must NOT refer to the symbol "$\equiv_5$".

Question 3. Given the relation R on $A = \{a, b, c, d, e\}$ by the pairs

$$R = \{(a, b), (c, b), (b, d), (e, d)\}$$

(a) (2 MARKS) Display the transitive closure R^+ of R as a set of pairs.

(b) (1 MARK) **Explain why** R^+ is an order.
Caution: An order has **two** *defining* properties.

(c) (2 MARKS) Display the **Hasse diagram** of the order R^+

(d) (1 MARK) Display the set of minimal members of R^+.

Question 4. (4 MARKS) We know that for relations R, Q and P we have

$$(R \circ Q) \circ P = R \circ (Q \circ P)$$

Prove that for **functions** f, g and h we have

$$(fg)h = f(gh)$$

Hint. Careful with the notation "fg"!

Question 5. (5 MARKS) Solve the recurrence below in closed form for T_n:

$$\begin{cases} T_0 & = 0 \\ \text{and, for } n > 0, & \\ T_n & = nT_{n-1} + n! \end{cases}$$

The notation "$n!$" above is the "n factorial", that is, "$1 \cdot 2 \cdot 3 \cdot 4 \cdot \ldots \cdot n$".

Hint. Do NOT use generating functions. Use the "*telescoping series*" trick for *additive recurrences* from our lecture notes. But *before* you do that it is **strongly recommended** to divide by "$n!$" both sides of "=" (in the second equation) and work with

$$t_n \stackrel{Def}{=} \frac{T_n}{n!}$$

instead of T_n.

Question 6. (4 MARKS) Prove that $\sum_{i=1}^{n} i^5 = O(n^6)$.

Question 7. (4 MARKS) Prove using the short north-east diagonals

$$\nearrow$$

or any other mathematical method of your preference, that if A is *enumerable*, then it is also countable with an enumeration that lists each of its members *exactly three* (3) times.

Hint. Your proof will consist of constructing an enumeration with the stated requirement.

Question 8. (4 MARKS) **Prove** $A \to (\forall x)B \vdash (\forall x)(A \to B)$.

Hint. It is recommended that you use Axiom 2 (in its "simple form") early on in your proof.

Question 9. (4 MARKS) Use (simple) induction on n to prove that for $n \geq 0$, $7^n - 2^n$ is divisible by 5.