Lassonde School of Engineering

Dept. of EECS

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EECS 1028E. Problem Set No4

Posted: November 19, 2024

Due: Dec. 3, 2024; by 5:00pm, in eClass.

Q: How do I submit?

A:

(1) **Submission must be a SINGLE *standalone* file to eClass. Submission by email is not accepted.**

(2) **Accepted File Types: PNG, JPEG, PDF, RTF, MS WORD, OPEN OFFICE, ZIP**

(3) **Deadline is strict, electronically limited.**

(4) **MAXIMUM file size = 10MB**

It is worth remembering (from the course outline):

The homework **must** be each individual's own work. While consultations with the instructor, tutor, and among students, are part of the learning process and are encouraged, **nevertheless**, *at the end of all this consultation* each student will have to produce an individual report rather than a *copy* (full or partial) of somebody else's report.

The concept of "late assignments" does not exist in this course, as you recall.

1. (1 MARK) Define: "A is uncountable".

2. (5 MARKS) Prove that if A is uncountable and $A \subseteq B$, then B is also uncountable.

3. (5 MARKS) Prove that $\vdash (\forall x)(A \to B) \to (\exists x)A \to (\exists x)B$.

4. (5 MARKS) Given the recurrence equations below that define b_n for all $n \geq 0$.

Use CV Induction to prove that $b_n \leq 3^n$ for $n \geq 0$.

$$\begin{aligned} b_0 &= 1,\, b_1 = 2,\, b_3 = 3 \\ \textit{and} & \\ b_k &= b_{k-1} + b_{k-2} + b_{k-3},\text{ for } k \geq 3 \end{aligned}$$

5. (3 MARKS) Consider the statement (formula)

$$(\exists x)A(x) \to A(z) \tag{1}$$

where z is a *new* variable *not free* (not an "input variable") in $A(x)$.

Find now a *specific, very simple,* example of $A(x)$ over the set $\mathbb{N}$ and choose a specific value of $z \in \mathbb{N}$ so that (1) becomes **false** (meaning we cannot prove it, since proofs start from true axioms and preserve truth at every step).

6. (5 MARKS) Using simple induction prove that

$$1^2 + 2^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

for $n \geq 1$.