Lassonde School of Engineering

Dept. of EECS

Professor G. Tourlakis

EECS 1028E. Problem Set No2

Posted: Oct. 5, 2024

Due: Oct. 27, 2024; by 6:00pm, in eClass.

Q: How do I submit?

A:

1. **Submission must be a SINGLE *standalone* file to eClass. Submission by email is not accepted.**

2. **Accepted File Types: PNG, JPEG, PDF, RTF, MS WORD, OPEN OFFICE, ZIP**

3. **Deadline is strict, electronically limited.**

4. **MAXIMUM file size = 10MB**

It is worth remembering (from the course outline):

The homework **must** be each individual's own work. While consultations with the instructor, tutor, and among students, are part of the learning process and are encouraged, **nevertheless**, *at the end of all this consultation* each student will have to produce an individual report rather than a *copy* (full or partial) of somebody else's report.

The concept of "late assignments" does not exist in this course, as you recall.

1. (3 MARKS) Prove that $\bigcup \mathbb{V}$ is a proper class.

2. (4 MARKS) "**Grade**" the following proof of the **Claim** below, that is, show me **where exactly** it went wrong.

Claim. *Let P be a relation on A that satisfies $aPb \wedge aPc \to bPc$. Prove that P is an equivalence relation on A.*

Faulty Proof: I will show that (1) it is reflexive, (2) it is symmetric and (3) its transitive.

(1) *Reflexivity*: By logic, I have $aPb \wedge aPb$. Thus bPb, so I have reflexivity.

(2) *Symmetry*: Let aPb. Using reflexivity that we proved above, I have aPa. Thus using the given red property $aPb \wedge aPa$ gives me bPa and I got symmetry.

Finally

(3) *Transitivity*: Let $aPb \wedge bPc$. By Symmetry, I have bPa. Now $bPa \wedge bPc$ gives me aPc via the red property I was given. Done! $\square$

3. (3 MARKS) Prove that $\mathbb{U} \times \mathbb{U}$ is not a set.

4. (3 MARKS) Provide a relation $\mathbb{R}$ such that for some object a has the property that $(a)\mathbb{R}$ is a proper class.

The underlined words are significant.

Relation $\mathbb{R}$: Must be defined clearly which relation $\mathbb{R}$ we are talking about.

Object a must be specified exactly.

The "Property" of $(a)\mathbb{R}$ according to which it is a proper class MUST be **proved**.

5. (3 MARKS) In class I noted but did not prove, that $\mathbb{N}^2$ is an equivalence relation on $\mathbb{N}$.

Prove this claim.

6. (6 MARKS) Let R on set A be **symmetric** and **reflexive**. Show that the same is true of R^n for the arbitrary $n > 0$: It is on A and is **reflexive** and **symmetric**.

Hint. No need for any induction!! Work by noting (known from class that) $R^n = \overbrace{R \circ \cdots \circ R}^{n\ R}$.

7. (5 MARKS) Let R on A be reflexive and symmetric. **Prove** that R^+ is an equivalence relation.

8. (4 MARKS) If $<$ is a *linear* order on $\mathbb{A}$, then **prove that** every minimal element is also minimum.