Lassonde School of Engineering

Dept. of EECS

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EECS 1028 E. Problem Set No1

Posted: Sept. 14, 2024

Due: Oct. 4, 2024; by 6:00pm, in eClass.

Q: How do I submit?

A:

(1) **Submission must be a SINGLE *standalone* file to eClass. Submission by email is NOT accepted.**

(2) **Accepted File Types: PNG, JPEG, PDF, RTF, MS WORD, OPEN OFFICE, ZIP**

(3) **Deadline is strict, electronically limited.**

(4) **MAXIMUM file size = 10MB**

It is worth remembering (from the course outline):

The homework **must** be each individual's own work. While consultations with the instructor, tutor, and among students, are part of the learning process and are encouraged, **nevertheless**, *at the end of all this consultation* each student will have to produce an individual report rather than a *copy* (full or partial) of somebody else's report.

The concept of "late assignments" does not exist in this course, as you recall.

1. True or False **and Why**. (**NOTE**: NO Why – NO Points)

(a) (2 MARKS) For sets or atoms a, b: $\{\{a\}, \{b\}\} = \{a, b\}$

(b) (4 MARKS) $\emptyset \in \emptyset$. Do you need set-formation-by-stages to answer this?

(c) (2 MARKS) For sets or atoms c, d: $\bigcup\{\{c\}, \{d\}\} = \{c, d\}$

(d) (2 MARKS) $\emptyset \subseteq \emptyset$

(e) (2 MARKS) $\emptyset \in \{1\}$

2. (4 MARKS) Someone suggested in class that in order to prove that $A \cup B$ is a *set*, for any sets A and B, we can argue that $A \cup B$ can be built at the latest of Σ, Σ' when A and B were built and we do not need to wait until a stage Σ'' that is *after* both Σ and Σ''.

If you agree with this, then prove it that way, carefully and deliberately.

If you disagree, indicate definitively why we need a later (than Σ and Σ') stage Σ''.

No legal arguments please! Only mathematical ones are allowed!

3. (3 MARKS) Let A, B, C, D be sets or atoms. Prove that $\{A, B, C, D\}$ *is a set, without* using *any* of the Principles 0, 1, 2. *Rather use results (theorems)* that we already established in class/Notes.

4. (3 MARKS) Now repeat 3 but this time prove it *using whichever principles among* $0, 1, 2$ *that you need*.

5. (5 MARKS) Prove that Principle 2 implies that we have infinitely many stages available.

Hint. Arguing by contradiction, assume instead that we only have **finitely many** stages. So repeatedly applying Principle 2 we can form a non ending sequence of stages

$$\cdots < \Sigma' < \Sigma'' < \Sigma''' < \Sigma'''' < \cdots \tag{1}$$

If the sequence (1) contains only a *finite* number of distinct $\Sigma''^{\cdots'}$, then at least two of the $\Sigma''^{\cdots'}$ in (1) are the same stage. Use this conclusion and properties of "<" to get a contradiction

6. (3 MARKS) Prove that, for any *set* A we have that $\mathbb{U} \cup A = \mathbb{U}$.

7. (2 MARKS) Is the above true for proper classes $\mathbb{A}$? Do we have that $\mathbb{U} \cup \mathbb{A} = \mathbb{U}$?

8. (4 MARKS) Prove for any classes $\mathbb{A}, \mathbb{B}$, that $\mathbb{A} \cup \mathbb{B} = \mathbb{U} - \Big(\mathbb{U} - \mathbb{A} \cap \mathbb{U} - \mathbb{B}\Big)$.

Hint. This is a simple case of proving $lhs \subseteq rhs$ by doing "Let $x \in lhs$. BLA BLA BLA *AND* concluding $x \in rhs$", and then *ALSO* doing $rhs \subseteq lhs$ by doing "Let $x \in rhs$. BLA BLA BLA and concluding $x \in lhs$".

9. Use the notation by explicitly listing **all the members** of each rhs $\{???\}$ to complete the following incomplete equalities:

(a) (2 MARKS) $2^{\{1\}} = \{???\}$

(b) (2 MARKS) $2^{\{1,2,3,4\}} = \{???\}$