Lassonde School of Engineering

Dept. of EECS

Professor G. Tourlakis

EECS 1028 E. Problem Set No1

Posted: Sept. 14, 2024

Due: Oct. 4, 2024; by 6:00pm, in eClass.

Q: How do I submit?

A:

- (1) Submission must be a SINGLE *standalone* file to <u>eClass</u>. Submission by email is NOT accepted.
- (2) Accepted File Types: PNG, JPEG, PDF, RTF, MS WORD, OPEN OFFICE, ZIP
- (3) Deadline is strict, electronically limited.
- (4) MAXIMUM file size = 10MB



It is worth remembering (from the course outline):

The homework **must** be each individual's <u>own work</u>. While consultations with the <u>instructor</u>, <u>tutor</u>, and <u>among students</u>, are part of the <u>learning process</u> and are encouraged, **nevertheless**, at the end of all this consultation each student will have to produce an <u>individual report</u> rather than a *copy* (full or partial) of somebody else's report.

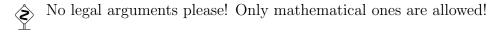
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The concept of "late assignments" does not exist in this course, as you recall.



- 1. True or False and Why. (NOTE: NO Why NO Points)
 - (a) (2 MARKS) For sets or atoms a, b: $\{\{a\}, \{b\}\} = \{a, b\}$
 - (b) (4 MARKS) $\emptyset \in \emptyset$. Do you need set-formation-by-stages to answer this?
 - (c) (2 MARKS) For sets or atoms c, d: $\bigcup \{\{c\}, \{d\}\} = \{c, d\}$
 - (d) (2 MARKS) $\emptyset \subseteq \emptyset$
 - (e) $(2 \text{ MARKS}) \emptyset \in \{1\}$
- **2.** (4 MARKS) Someone suggested in class that in order to prove that $A \cup B$ is a *set*, for any sets A and B, we can argue that $A \cup B$ can be built at the latest of Σ, Σ' when A and B were built and we do not need to wait until a stage Σ'' that is *after* both Σ and Σ'' .

If you agree with this, then prove it that way, carefully and deliberately. If you disagree, indicate definitively why we need a later (than Σ and Σ') stage Σ'' .





- **3.** (3 MARKS) Let A, B, C, D be sets or atoms. Prove that $\{A, B, C, D\}$ is a set, without using any of the Principles 0, 1, 2. Rather use results (theorems) that we already established in class/Notes.
- **4.** (3 MARKS) Now repeat 3 but this time prove it using whichever principles among 0, 1, 2 that you need.
- **5.** (5 MARKS) Prove that Principle 2 implies that we have infinitely many stages available.

Hint. Arguing by contradiction, assume instead that we only have **finitely** many stages. So repeatedly applying Principle 2 we can form a non ending sequence of stages

$$\cdots < \Sigma' < \Sigma'' < \Sigma''' < \Sigma'''' < \cdots \tag{1}$$

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If the sequence (1) contains only a *finite* number of distinct $\Sigma''^{...'}$, then at least two of the $\Sigma''^{...'}$ in (1) are the <u>same</u> stage. Use this conclusion and properties of "<" to get a contradiction

- **6.** (3 MARKS) Prove that, for any *set* A we have that $\mathbb{U} \cup A = \mathbb{U}$.
- 7. (2 MARKS) Is the above true for proper classes \mathbb{A} ? Do we have that $\mathbb{U} \cup \mathbb{A} = \mathbb{U}$?
- **8.** (4 MARKS) Prove for any classes \mathbb{A}, \mathbb{B} , that $\mathbb{A} \cup \mathbb{B} = \mathbb{U} (\mathbb{U} \mathbb{A} \cap \mathbb{U} \mathbb{B})$. Hint. This is a simple case of proving $lhs \subseteq rhs$ by doing "Let $x \in lhs$. BLA BLA BLA AND concluding $x \in rhs$ ", and then ALSO doing $rhs \subseteq lhs$ by doing "Let $x \in rhs$. BLA BLA BLA and concluding $x \in lhs$ ".
- **9.** Use the notation by explicitly listing **all the members** of each rhs {????} to complete the following incomplete equalities:
 - (a) $(2 \text{ MARKS}) 2^{\{1\}} = \{???\}$
 - (b) (2 MARKS) $2^{\{1,2,3,4\}} = \{???\}$

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