

CSE 2001—Winter 2008

Problem Set No. 1
Posted: January 18, 2008
Due: TBA on the web site



All reports must be typed (except for diagrams). All assignments are due by 2:00pm on the due date in the course box.



1. (5 MARKS) Design a FA over $\{0, 1\}$ that accepts exactly all the strings of even length.

E.g., ε , 00, 0110, 0000 are all acceptable while 0, 000, 01101 are not.

Clearly justify why your automaton works as stated (**NOT** by example; give a “general argument” or a “proof” if you prefer (although a proof is not required in this problem)).

2. (5 MARKS) Design a regular expression α over $\{0, 1\}$ that defines the language over $\{0, 1\}$ of all the strings of odd length.

E.g., 0, 000, 01101 are all in $L(\alpha)$ while ε , 00, 0110, 0000 are not.

Clearly justify why your regular expression works as stated (**NOT** by example; give a “general argument” or a “proof” if you prefer (although a proof is not required in this problem)).

3. (9 MARKS) Design a FA over $\Sigma = \{0, 1, 2\}$ that adds any sequence (string!) of digits $0, 1, 2 \pmod 3$ in the following sense:

It will have three states, q_0, q_1, q_2 and the property that any string x over Σ satisfies the statement (specification):

$$q_0x \vdash^* xq_i \text{ iff } \text{mod}(\text{sum of digits of } x, 3) = i \quad (1)$$

You are **required** to prove by induction on $|x|$ that your design satisfies the equivalence (1).

Note that the problem is not concerned with acceptance of input.

4. (5 MARKS) Build an NFA that accepts precisely all the strings over $\{0, 1\}$ of length ≥ 5 that contain at least one 1 among their last 5 symbols.

You must argue that your design is correct. Again, **NOT** by example.

5. (3 MARKS) Describe in plain English the set of strings defined by the regular expression over $\{0, 1\}$: $(0 + 10)^*1^*$.
6. (6 MARKS) Consider the FA below.

	0	1
$\rightarrow q_1$	q_2	q_1
q_2	q_3	q_1
$*q_3$	q_3	q_2

Compute:

- Regular expressions for all R_{ij}^0
 - Regular expressions for all R_{ij}^1
 - Regular expressions for all R_{ij}^2
7. (6 MARKS) Convert to NFA (all over $\{0, 1\}$) without comment:
- 01^*
 - $(0+1)01$
 - $00(0+1)^*$
8. (4 MARKS) Convert the previous last two NFA to a FA.
9. (5 MARKS) We mentioned without proof in class that over any Σ , $(\alpha^*\beta^*)^* = (\alpha + \beta)^*$.
Prove this!
RECALL that $\alpha = \beta$ means really $L(\alpha) = L(\beta)$.
10. (10 MARKS) Prove that the following are not regular languages:
- (a) Over $\{0, 1, 2\}$: The set $\{0^n1^m2^n : m \geq 0 \wedge n \geq 0\}$
 - (b) Over $\{0, 1\}$: The set $\{0^n1^{2n} : n \geq 0\}$
11. (5 MARKS) Prove that $L = \{0^n : n \text{ is a perfect cube}\}$ over $\{0\}$ is not regular.