COSC 4111 3.0/5111 3.0—Winter 2006

Posted: March 19, 2006
Due: April 6, 2006

Problem Set No. 3

Remark. Due to other commitments, I will post solutions for this problem set on April 11, 2006. Each problem (e.g., 1, 2a, 2b, 5-1, 5-23, etc.) is worth 5 points.

(1) **(Grad)** "W_i" is the symbol Rogers uses for "the i-th semi-recursive set", that is,

$$x \in W_i \equiv (\exists z)T(i, x, z)$$

Question: Is there a partial recursive function $\lambda x.f(x)$ such that for all i

$$W_i \neq \emptyset \Rightarrow f(i) \downarrow \wedge f(i) = \min\{y : y \in W_i\}$$

If you think that "yes", then you **must** give a proof.

If you think that "no", then you **must** give a definitive counterexample.

(2) **Without using Rice's theorem**, prove that

(a) the set $A = \{x : \text{ran}(\phi_x) \text{ has exactly five distinct elements}\}$ is not recursive. (I.e., "$x \in A$ is unsolvable").

(b) the set $D = \{x : \phi_x \text{ is the characteristic function of some set}\}$ is not r.e.

(c) the set $E = \{x : \text{ran}(\phi_x) \text{ contains only prime numbers}\}$ is not r.e.

(3) Is the "proof" given below for the question "Prove that if $y = f(\vec{x}_n)$ is r.e., then $f \in \mathcal{P}$" correct? If not, **where exactly** does it go wrong?

Proof. Let $y = f(\vec{x}_n)$ be r.e. Then $y = f(\vec{x}_n) \equiv \psi(y, \vec{x}_n) = 0$ for some $\psi \in \mathcal{P}$. Thus $g = \lambda \vec{x}_n.(\mu y)\psi(y, \vec{x}_n)$ is in $\mathcal{P}$. But $g = f$, since the unbounded search finds the y that makes $y = f(\vec{x}_n)$ true, if $f(\vec{x}_n) \downarrow$. Thus, $f \in \mathcal{P}$. □

(4) **(Grad) Ch.7.** #3 parts (1) and (2) only.

(5) **Chapter 13** problems 1, 23, 26, 27, (**Grad, 30**), (**Grad, 45**).

(6) Prove that $T \in \mathcal{E}^3_*$.

Hint. Systematically scan the proof that $T \in \mathcal{PR}_*$ contained in the Kleene Notes and modify it to obtain this sharper result.