COSC 4111/5111 3.0—Fall 2004

Date: Nov 6, 2004
Due: TBA in class and Web page. (Approx. shelf life=3 weeks)

Problem Set No. 2

Most problems are from "Computability", **Chapters 3, 7, 8**. All relate to material in said chapters.

(1) **Ch.3.** Nos. 6, 26, 29, and 30.

(2) Prove that if f is total and $\lambda\vec{x}y.y = f(\vec{x})$ is in $\mathfrak{R}_*$, then $f \in \mathcal{R}$.

(3) Prove that there exists a partial recursive h that satisfies

$$h(y,x) = \begin{cases} y & \text{if } x = y+1 \\ h(y+1,x) & \text{otherwise} \end{cases}$$

Which function is $\lambda x.h(0,x)$?

(4) Given $\lambda y\vec{x}.f(y,\vec{x}) \in \mathfrak{P}$. Prove that there exists a partial recursive g that satisfies

$$g(y,\vec{x}) = \begin{cases} y & \text{if } f(y,\vec{x}) = 0 \\ g(y+1,x) & \text{otherwise} \end{cases}$$

How can you express $\lambda x.g(0,x)$ in terms of f?

(5) (**Grad**) Express the projections K and L of $J(x,y) = (x+y)^2 + x$ in *closed form*—that is, without using $(\mu y)_{<z}$ or bounded quantification.

(*Hint.* Solve for x and y the *Diophantine equation* $z = (x+y)^2 + x$. The term $\lfloor\sqrt{z}\rfloor$ is involved in the solution.)

(6) **Ch.7.** Nos. 5, 6 (**Do not use Rice theorems**).

(7) (**Grad**) Prove that a recursively enumerable set of sentences $\mathcal{T}$ over a finitely generated language (e.g., like that of arithmetic) admits a recursive set of axioms, i.e., for some recursive Γ, $\mathcal{T} = \mathbf{Thm}_{\Gamma}$.

(*Hint.* Note that for any $\mathcal{A} \in \mathcal{T}$, any two sentences in the sequence

$$\mathcal{A}, \mathcal{A} \wedge \mathcal{A}, \mathcal{A} \wedge \mathcal{A} \wedge \mathcal{A}, \ldots$$

are logically equivalent. Now see if Theorem 1 and its corollaries (section 8.2 in "Computability") can be of any help.)