Exercises 2

Disussion of solutions: Wednesday, Oct 11 in class

1. **Perceptron**
   The Perceptron algorithm takes in a sequence of data points $S = ((x_1, y_1), \ldots, (x_m, y_m))$ and passes over this sequence data point by data point until none of the points in S is mislabeled.

   (a) Create a data sequence $S = ((x_1, y_1), \ldots, (x_m, y_m))$ (for example in $\mathbb{R}^2$) on which the Perceptron algorithm, after the first full pass over the data, will still make a mistake (during its second pass).

   (b) How many passes over the data will the algorithm make at most? Provide an upper bound.

2. **Uniform convergence**
   Let $\mathcal{H} \subseteq \{0,1\}^{\mathcal{X}}$ be a hypothesis class. We say that $\mathcal{H}$ *has the uniform convergence property* if the following holds:

   For all $\epsilon, \delta > 0$, there exists a $m(\epsilon, \delta) \in \mathbb{N}$ such that for all distributions P over $\mathcal{X} \times \{0,1\}$ and all $m \geq m(\epsilon, \delta)$, we have

$$\mathbb{P}_{S \sim P^m}\left[\max_{h \in \mathcal{H}}\{|L(h) - \hat{L}_m(h)|\} \leq \epsilon\right] \;\geq\; 1 - \delta$$

   (a) Give 2 examples of classes that have the uniform convergence property. (You don't need to provide a full proof, just refer to a suitable result, we 6 have seen in class.)

   (b) Show that, if classes $\mathcal{H}$ and $\mathcal{H}'$ have the uniform convergence property, then so does $\mathcal{H} \cap \mathcal{H}'$.

   (c) Show that, if classes $\mathcal{H}$ and $\mathcal{H}'$ have the uniform convergence property, then so does $\mathcal{H} \cup \mathcal{H}'$.

   (d) Show that, if a class $\mathcal{H}$ has the uniform convergence property, then so does the class $\mathcal{H}^c$ of complements of $\mathcal{H}$, defined as

$$\mathcal{H}^c := \{h \in \{0,1\}^{\mathcal{X}} \mid \text{there is a } h' \in \mathcal{H} \text{ such that } h(x) = 1 - h'(x) \text{ for all } x\}$$

   (e) Prove of refute: If a class $\mathcal{H}$ has the uniform convergence property, then so does its complement

$$\bar{\mathcal{H}} = \{0,1\}^{\mathcal{X}} \setminus \mathcal{H}$$