Fortran@York/SLATEC

# Integration Example

The table of contents leads us to the integration routine GAUS8, which Integrate a real function of one variable over a finite interval using an adaptive 8-point Legendre-Gauss algorithm. It is intended primarily for high accuracy integration or integration of smooth functions. Note that there are other routines for improper integrals.

There are two such routines, S and D, and we will use the D one. The API can be found in the file: **dgaus8.f.html**. Here is the content of that file:

## dgaus8.f

```
      SUBROUTINE DGAUS8 (FUN, A, B, ERR, ANS, IERR)
C***BEGIN PROLOGUE  DGAUS8
C***PURPOSE  Integrate a real function of one variable over a finite
C            interval using an adaptive 8-point Legendre-Gauss
C            algorithm.  Intended primarily for high accuracy
C            integration or integration of smooth functions.
C***LIBRARY   SLATEC
C***CATEGORY  H2A1A1
C***TYPE      DOUBLE PRECISION (GAUS8-S, DGAUS8-D)
C***KEYWORDS  ADAPTIVE QUADRATURE, AUTOMATIC INTEGRATOR,
C             GAUSS QUADRATURE, NUMERICAL INTEGRATION
C***AUTHOR  Jones, R. E., (SNLA)
C***DESCRIPTION
C
C     Abstract  *** a DOUBLE PRECISION routine ***
C        DGAUS8 integrates real functions of one variable over finite
C        intervals using an adaptive 8-point Legendre-Gauss algorithm.
C        DGAUS8 is intended primarily for high accuracy integration
C        or integration of smooth functions.
C
C        The maximum number of significant digits obtainable in ANS
C        is the smaller of 18 and the number of digits carried in
C        double precision arithmetic.
C
C     Description of Arguments
C
C        Input--* FUN, A, B, ERR are DOUBLE PRECISION *
C        FUN - name of external function to be integrated.  This name
C              must be in an EXTERNAL statement in the calling program.
C              FUN must be a DOUBLE PRECISION function of one DOUBLE
C              PRECISION argument.  The value of the argument to FUN
C              is the variable of integration which ranges from A to B.
C        A   - lower limit of integration
C        B   - upper limit of integration (may be less than A)
C        ERR - is a requested pseudorelative error tolerance.  Normally
C              pick a value of ABS(ERR) so that DTOL .LT. ABS(ERR) .LE.
C              1.0D-3 where DTOL is the larger of 1.0D-18 and the
```

```
C             double precision unit roundoff D1MACH(4).  ANS will
C             normally have no more error than ABS(ERR) times the
C             integral of the absolute value of FUN(X).  Usually,
C             smaller values of ERR yield more accuracy and require
C             more function evaluations.
C
C             A negative value for ERR causes an estimate of the
C             absolute error in ANS to be returned in ERR.  Note that
C             ERR must be a variable (not a constant) in this case.
C             Note also that the user must reset the value of ERR
C             before making any more calls that use the variable ERR.
C
C        Output--* ERR,ANS are double precision *
C        ERR - will be an estimate of the absolute error in ANS if the
C              input value of ERR was negative.  (ERR is unchanged if
C              the input value of ERR was non-negative.)  The estimated
C              error is solely for information to the user and should
C              not be used as a correction to the computed integral.
C        ANS - computed value of integral
C        IERR- a status code
C            --Normal codes
C               1 ANS most likely meets requested error tolerance,
C                 or A=B.
C              -1 A and B are too nearly equal to allow normal
C                 integration.  ANS is set to zero.
C            --Abnormal code
C               2 ANS probably does not meet requested error tolerance.
C
C***REFERENCES  (NONE)
C***ROUTINES CALLED  D1MACH, I1MACH, XERMSG
C***REVISION HISTORY  (YYMMDD)
C   810223  DATE WRITTEN
C   890531  Changed all specific intrinsics to generic.  (WRB)
C   890911  Removed unnecessary intrinsics.  (WRB)
C   890911  REVISION DATE from Version 3.2
C   891214  Prologue converted to Version 4.0 format.  (BAB)
C   900315  CALLs to XERROR changed to CALLs to XERMSG.  (THJ)
C   900326  Removed duplicate information from DESCRIPTION section.
C           (WRB)
C***END PROLOGUE  DGAUS8
```

As an example, let us write a program to compute the first-quadrant area of an ellipse with semi-major axis = 10 and semi-minor one = 5. Its equation is:

```
y = 0.5 * sqrt(100 - x * x)
```

The program and the results are shown on the next page.

```
program quad
implicit none
real*8 a, b, answer, EPS
integer*2 status
parameter (EPS = 1.E-4)
external ellipse

print*, "Enter integration limits for Ellipse:"
read*, a, b
call dGaus8(ellipse, a, b, EPS, answer, status)

print*, answer, status
end
```

---

```
real*8 function ellipse(x)
implicit none
real*8 x
ellipse = 0.5 * sqrt(100 - x*x)
end
```

---

Running the above program yields:
39.2714...

The exact answer (based on $\pi ab/4$) leads to:
39.2699081699...

And if we changed e to 1.e-8, the answer would become:
39.2699082