Efficient Mining of Active Components in a Network of Time Series

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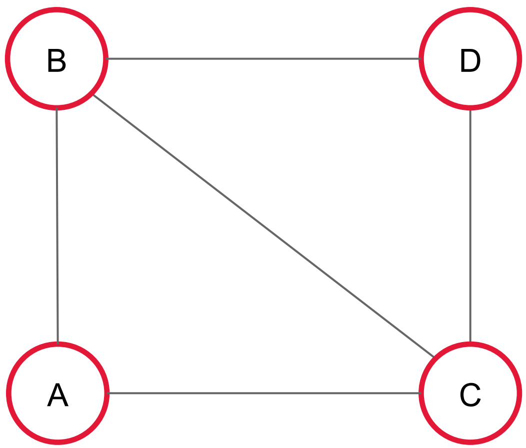
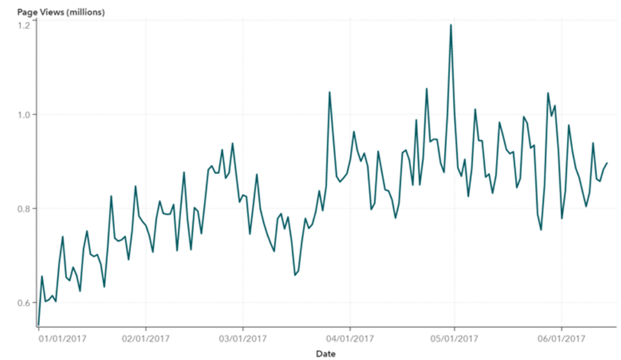
Introduction

Networks and Time Series are Ubiquitous

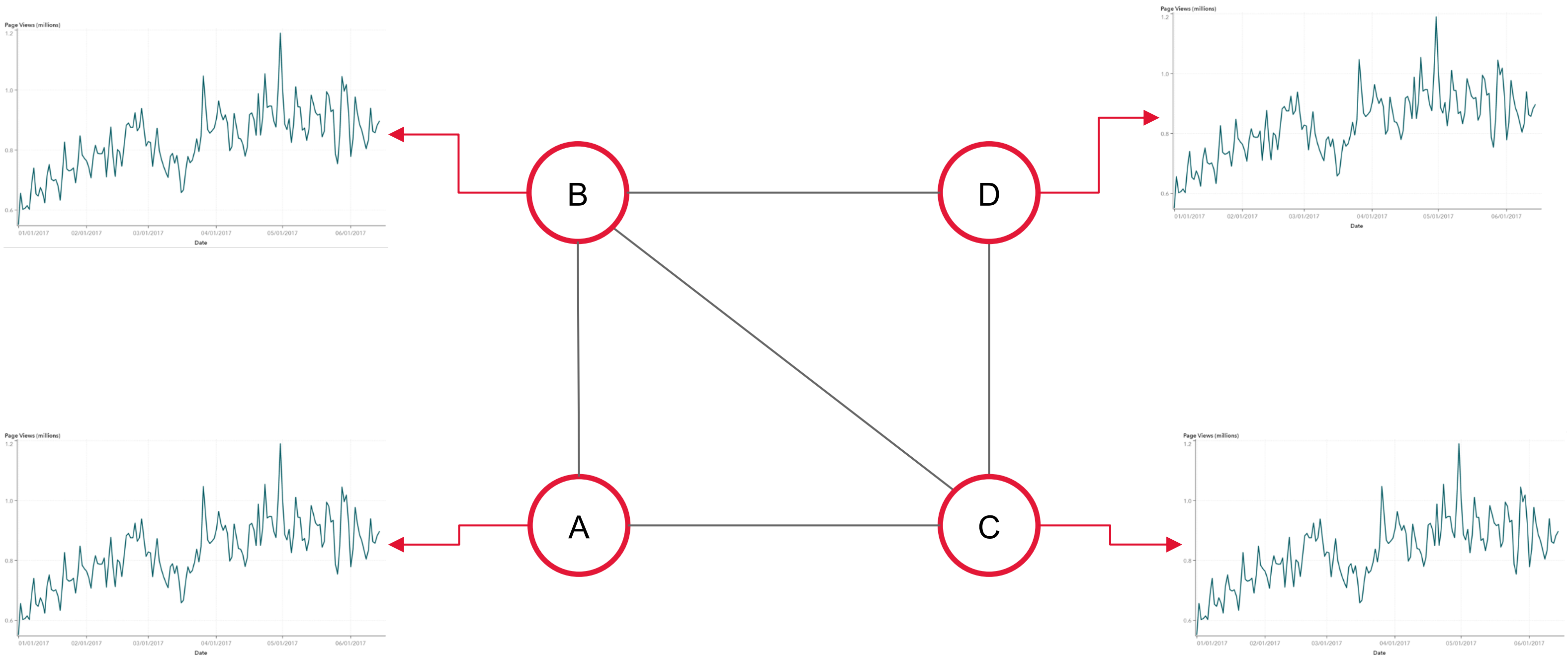




networks

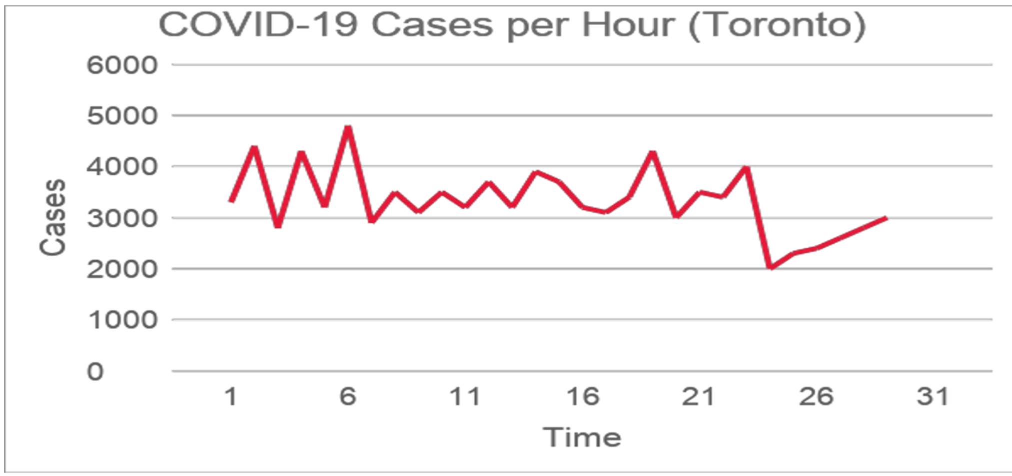
time series

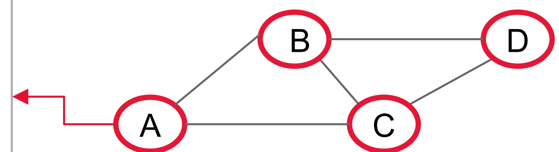
Network of Time Series

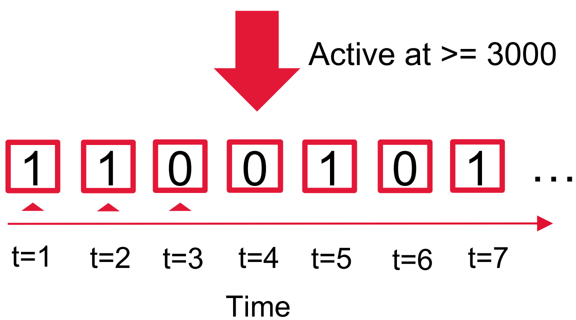


static topology

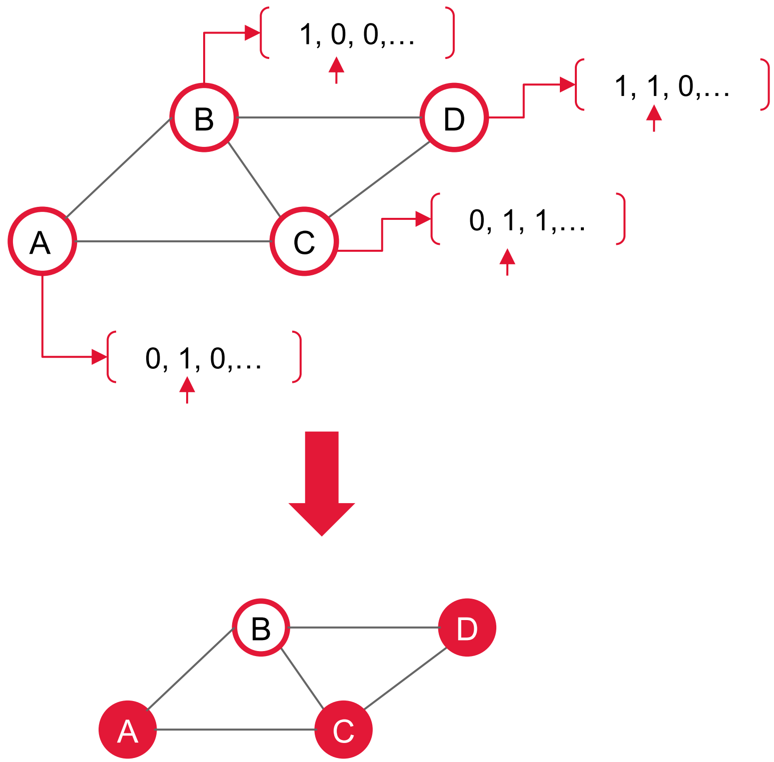
Binary Time Series





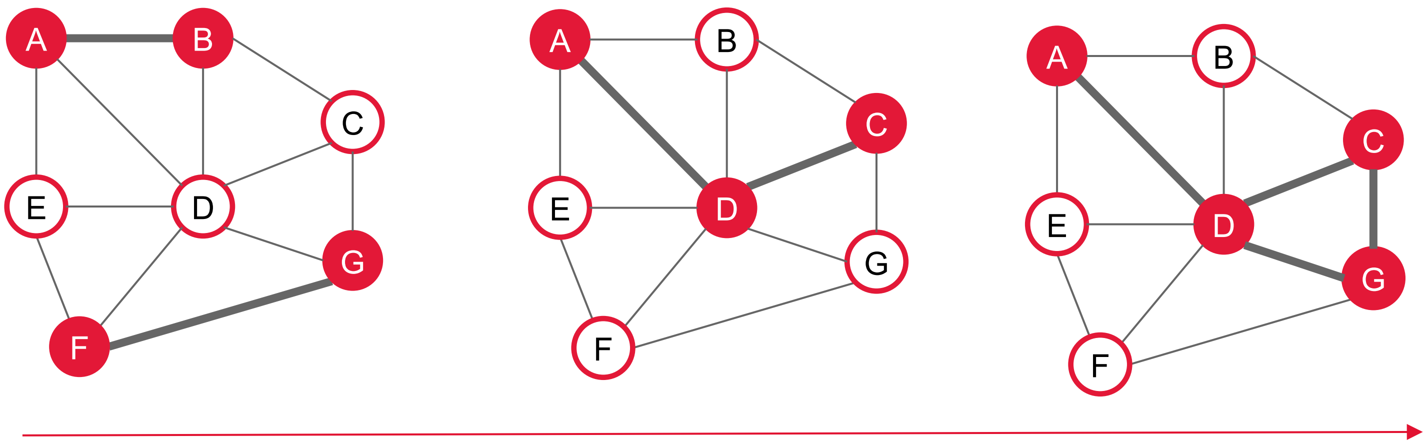


Active Nodes



Nodes at t = 2

Active Components



t = 1 t = 2 t = 3

{ {AB}, {FG} } {{ADC}} {{ ADCG}}

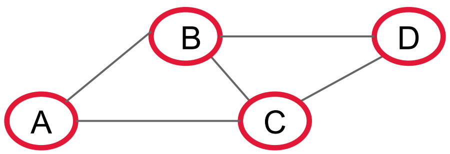
Problem Statement

Problem Statement

Given:

- A graph, G
- An $n \times h$ matrix X representing the time series, where X_{it} is the status of node i at time t

We want to compute C_t, the set of active components at time step t , for $t = 1, 2, \ldots, h$



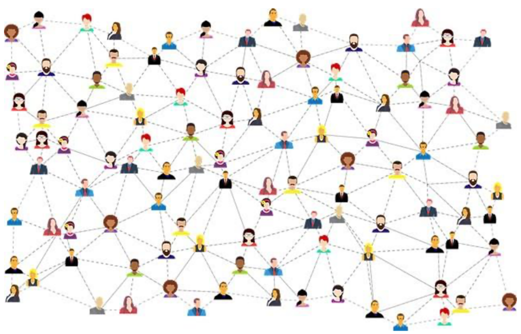
Time Steps

Vertices

	1	2	3	4	5
A	1	1	1	1	0
B	1	1	0	0	0
C	0	1	1	1	0
D	0	1	1	1	0
E	0	1	1	1	1
F	0	0	0	1	1

Why Do We Care?

Applications



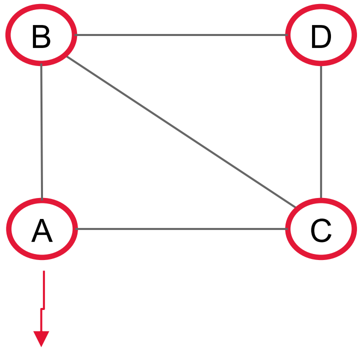
Social networks

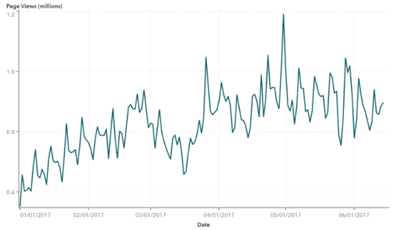
Technological networks

Biological networks

Road Networks

Road Network



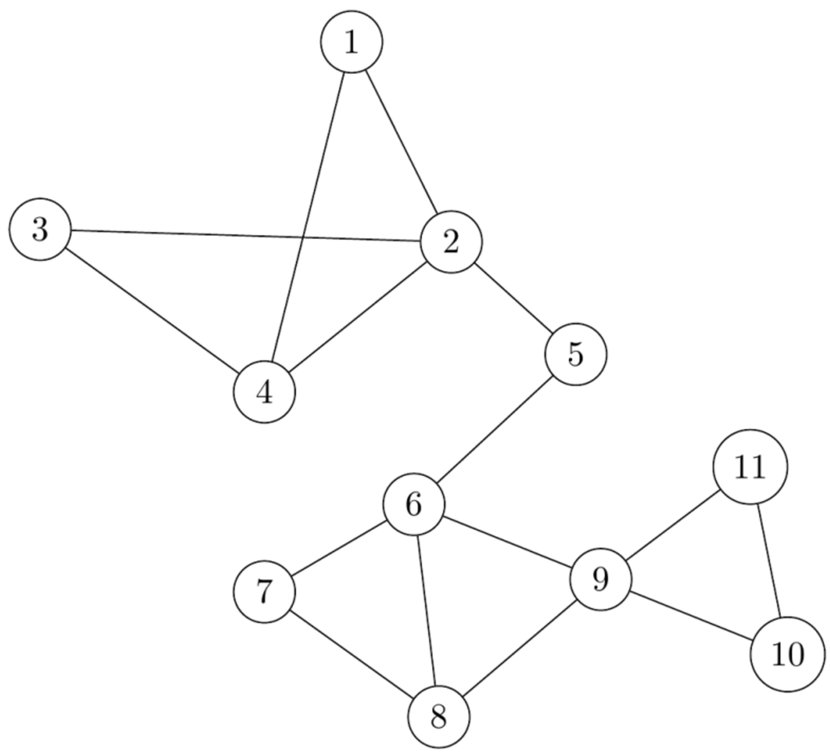


Background

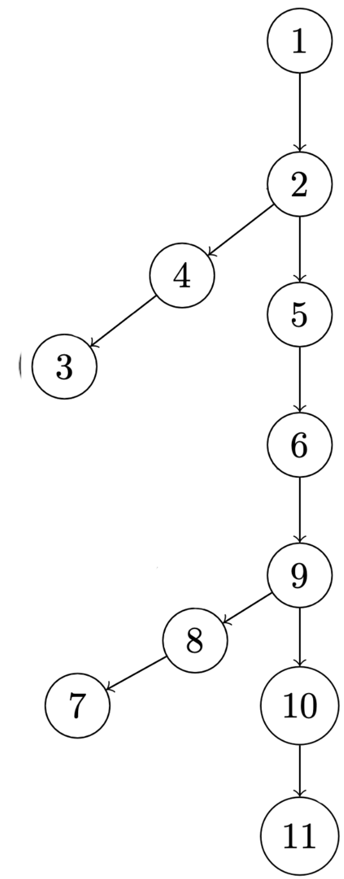
Graph Theory Background

- DFS-tree
- Heavy-Light Decomposition & Shallow Tree
- DFS-tree Enumeration

DFS-tree



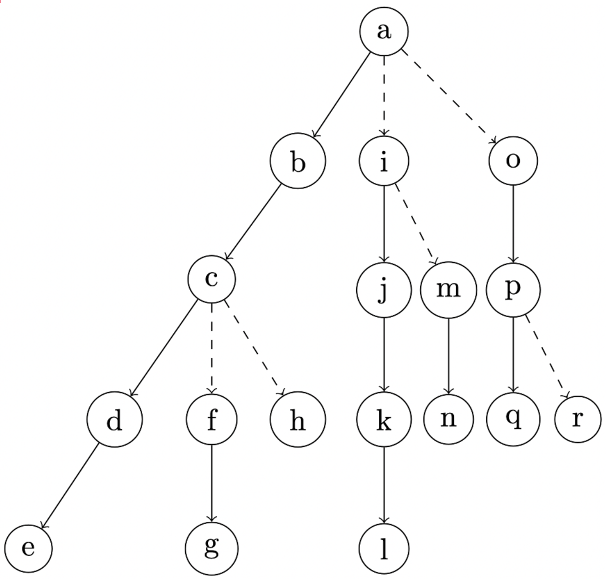
graph G



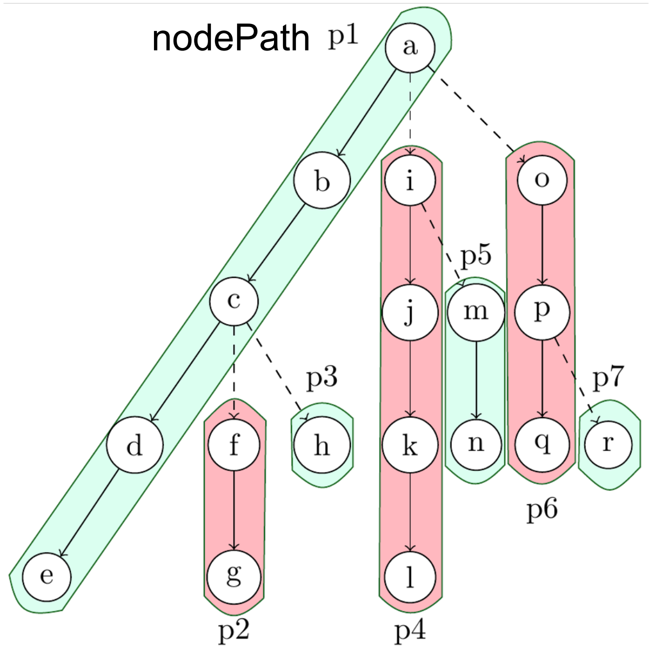
none-tree edges

DFS-tree of G

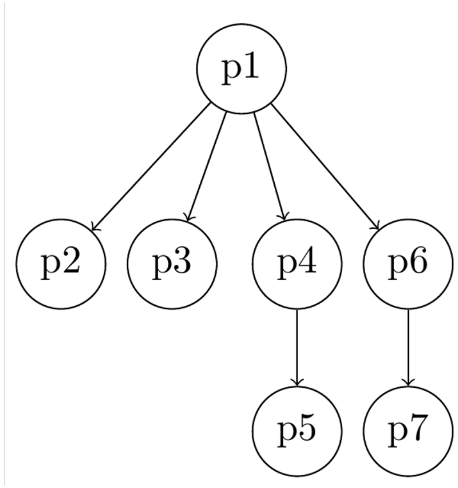
Heavy Light Decomposition & Shallow Tree



DFS-tree

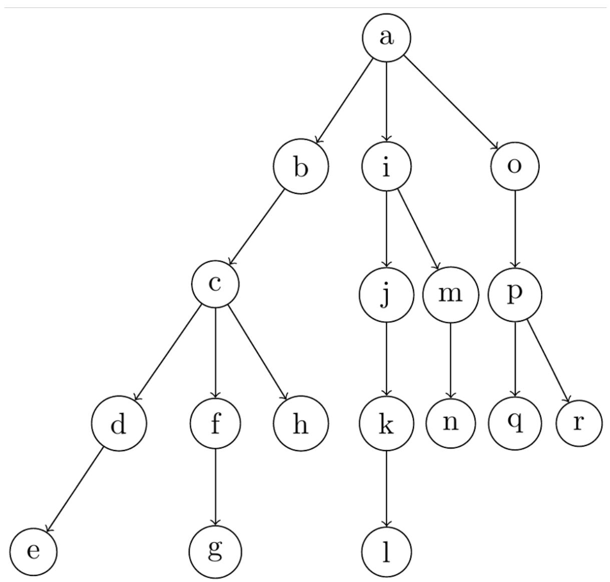


heavy light decomposition



shallow tree

DFS-tree Enumeration (dfs-id)

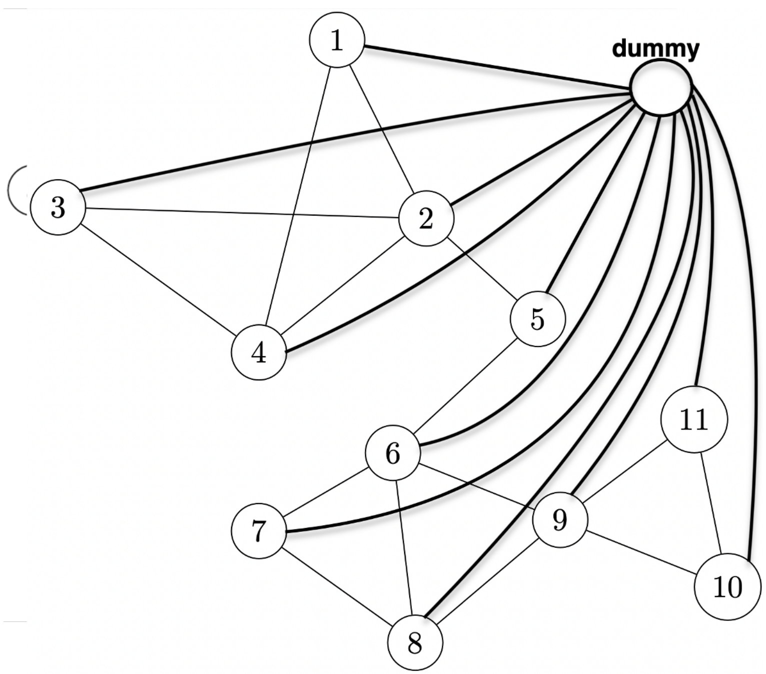


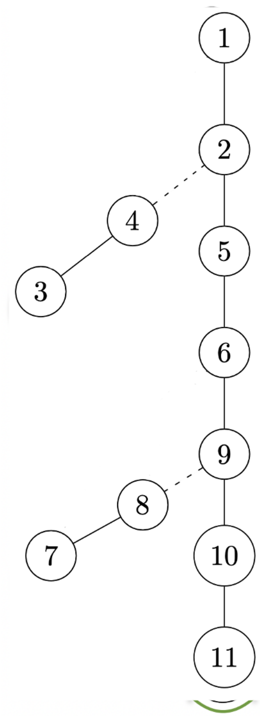
DFS-tree ***T***

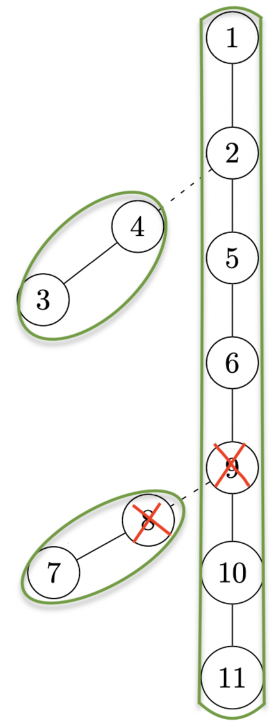
(a,1)
(b,2)
(c,3)
(d,4)
(e,5)
(f,6)
(g,7)
(h,8)
(i,9)
(j,10)
(k,11)
(l,12)
(m,13)
(n,14)
(o,15)
(p,16)
(q,17)
(r,18)

Methodology

Overview

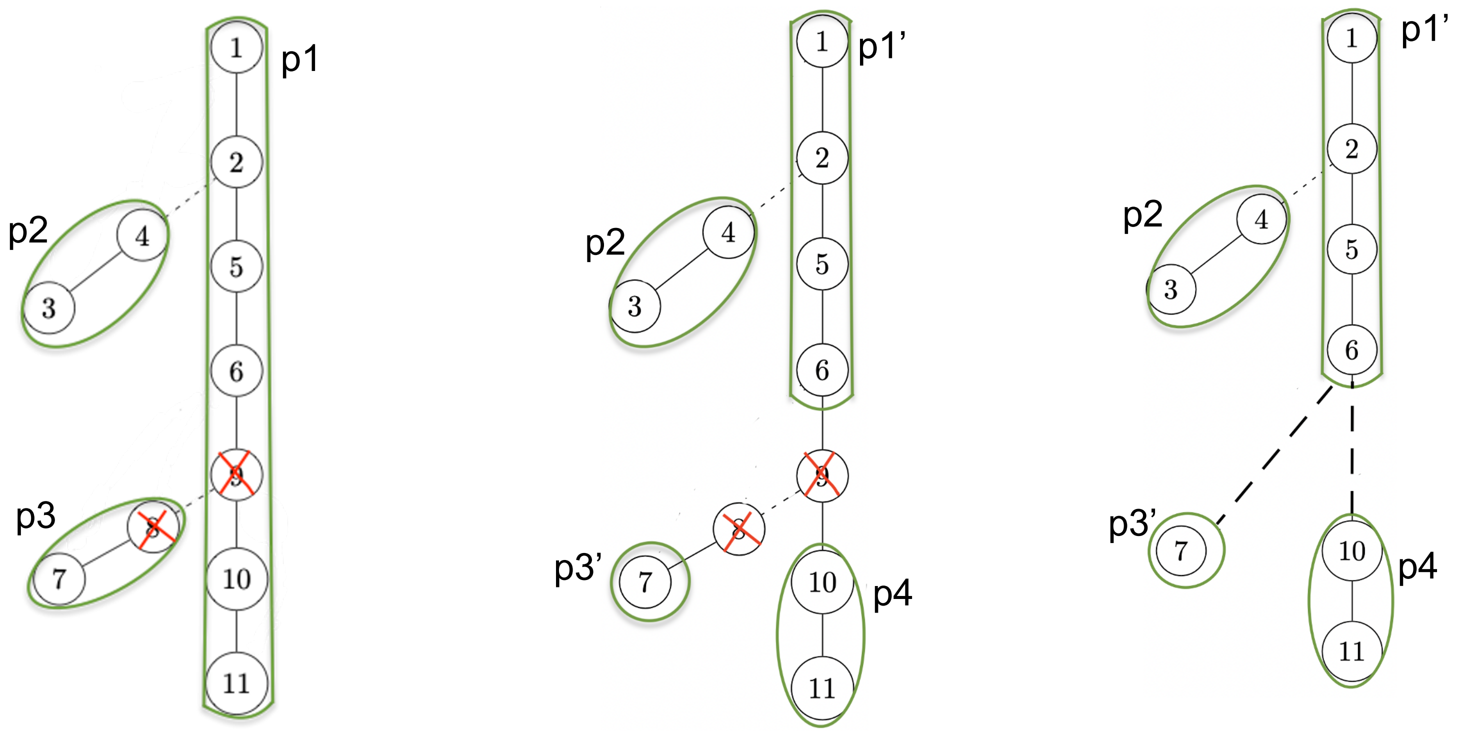






Updating the Shallow Tree

1. Delete inactive nodes from ***ST***
2. Set the ***nodePath*** for all active nodes
3. Set the parent of each ***nodePath***



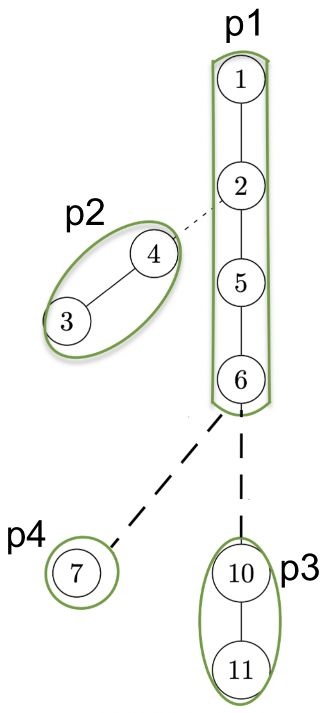
ActiveComp

ActiveComp(***G***, ***T***, ***T****, ***u***)

u.nodePath = ***p***

DFS on the shallow tree:

- Add the *nodePath* ***p*** to the partially grown spanning-tree ***T****
- Fill *Efficient_AL* for all the nodes on ***p***
- Continue with calling ActiveComp recursively on nodes in *Efficient_AL* for all the nodes on ***p***

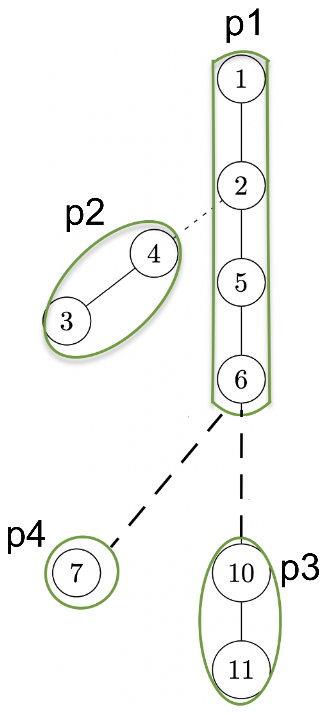


Query to the Data Structure

query(node ***u***, nodePath ***p***)

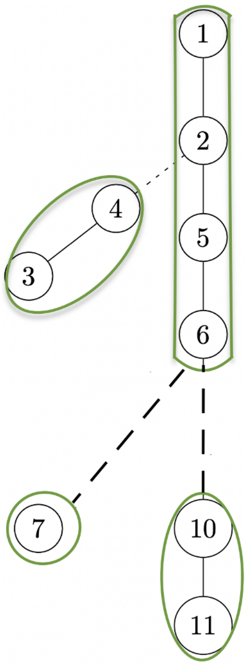
query(7,p1)

- returns a vertex on the ***p*** that is connected to ***u*** if there is any otherwise returns *nullptr*
- ***u*** has to be on one of the ***p***'s descendant



Computing Efficient Adjacency List

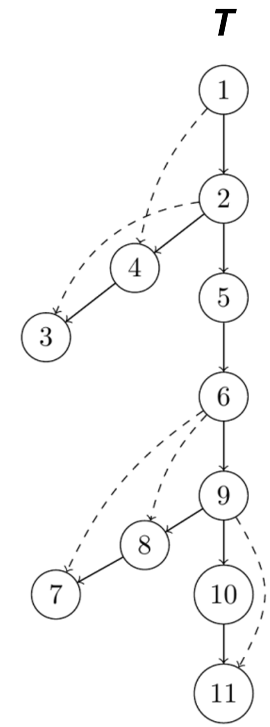
Vertices on a *nodePath* ***p*** can only have edges to the ***p***'s ancestors or descendants in the shallow tree



For each node ***u*** on the *nodePath* ***p*** that is being attached to ***T**** we need to make two categories of queries to fill its *Efficient_AL*:

1. queries from ***u*** to an ancestor of ***p*** which is not attached yet
2. queries from nodes on each descendant of ***p*** to ***p***

Data Structure



For each node ***u*** of the original graph and its corresponding DFS-tree ***T*** we save ***u***'s ancestors in ***T*** that ***u*** has an edge to.

For example:

dfs id:

$$(1,1),(2,2),(5,3),(6,4),(9,5),(8,6),(7,7),(10,8),(11,9),(4,10),(3,11)$$

Data structure *Anc_Nbr*:

node id	1	2	3	4	5	6	7	8	9	10	11
ancestors	{}	{1}	{2,4}	{1,2}	{2}	{5}	{6,8}	{6,9}	{6}	{9}	{9,10}

Time Complexity

Time complexity of *ActiveComp* is bounded by *Compute_Efficient_AL*

Compute_Efficient_AL time complexity is bounded by the number of calls to function *query*

For each node ***u*** we make a query to a path in the two following scenarios:

- ***u*** is not visited, one of ***u***.nodePath's ancestors is being attached to ***T****
- ***u***.nodePath is being attached to ***T**** and the query is from ***u*** to one of ***u***.nodePath's ancestors who are not attached

The height of shallow tree ***ST*** is d

Maximum number of calls to query from all ***n*** nodes is ***nd***

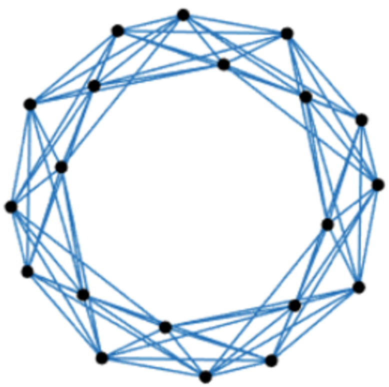
Each query takes *log*(n)

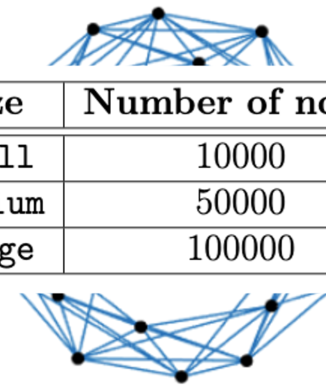
In total we the time complexity is $O(nd\log(n))$

Experimental Evaluation

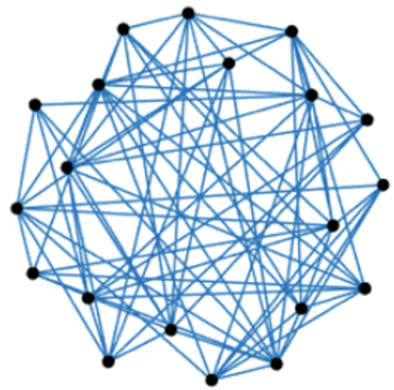
Synthetic Graph generation

Using *small-world* model





Size	Number of nodes
`small`	10000
`medium`	50000
`large`	100000



regular network

high average shortest path
high clustering coefficient

small-world network

low average shortest path
high clustering coefficient

random network

low average shortest path
low clustering coefficient

Synthetic Time Series Data Generation

$n \times h$ matrix X representing time series, nodes' status through time

Two scenarios:

1. Random
2. Forest Fire

Time Steps

Vertices

	1	2	3	4	5
A	1	1	1	1	0
B	1	1	0	0	0
C	0	1	1	1	0
D	0	1	1	1	0
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F	0	0	0	1	1

Other Baselines

- Simple DFS
- Dynamic DFS [Baswana et al.,2019]
- Edge Deletion [Albert et al., 1997]

Percentage Active Nodes

We show the result in two different categories:

1 . High percentage active nodes

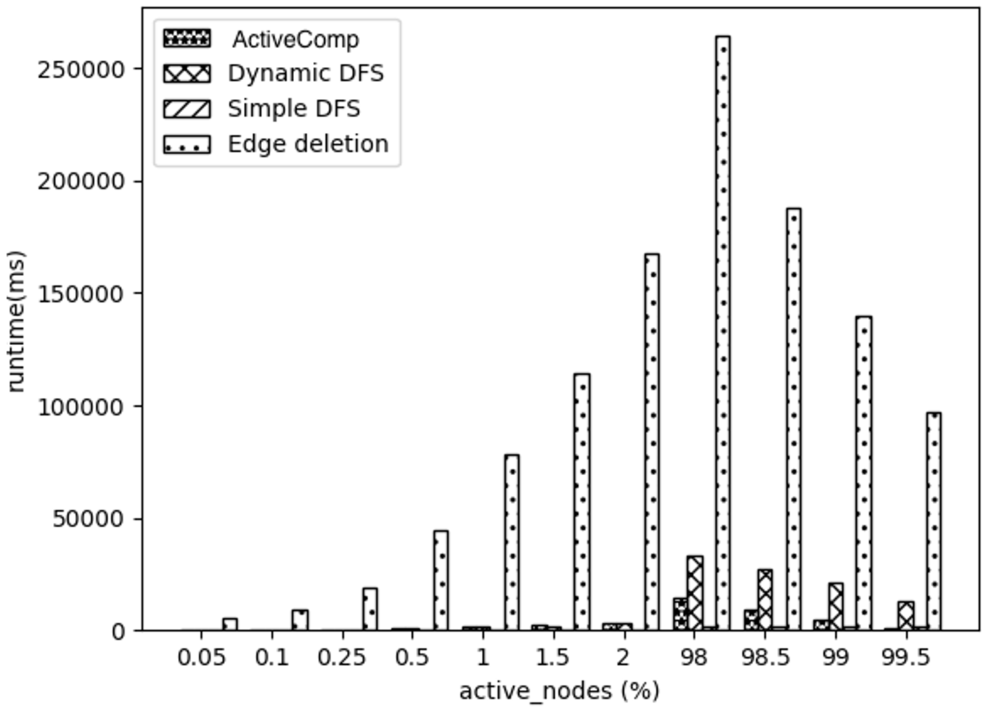
more than 99 percentage of nodes are on

2 . Low percentage active nodes

less than 1.5 percentage of nodes are on

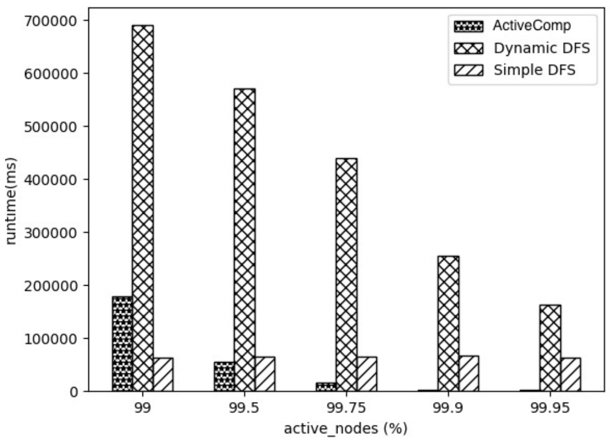
Performance of all Four Algorithms

- small-world network
- 10,000 nodes
- random scenario
- 1000 time steps

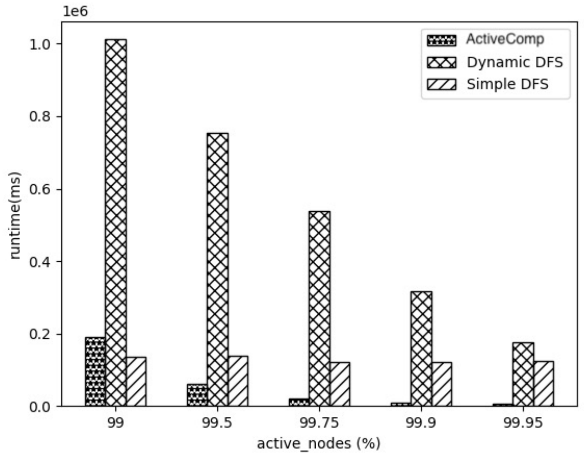


High Percentage Active Nodes

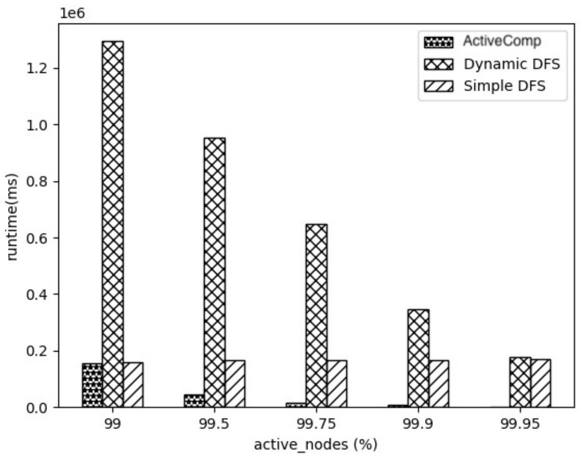
- small-world network with 50,000 nodes
- random scenario with 1000 time steps



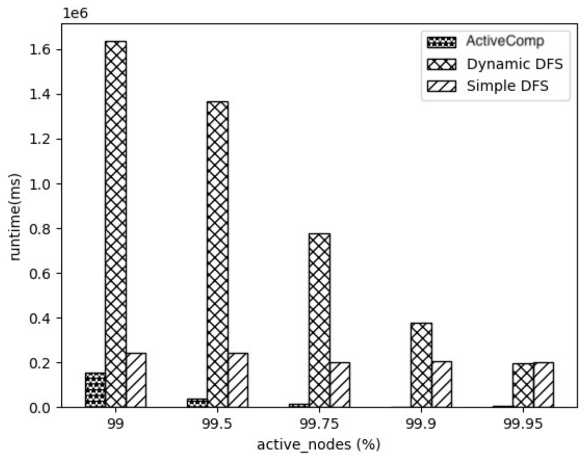
n=50k, k=250



n=50k, k=500

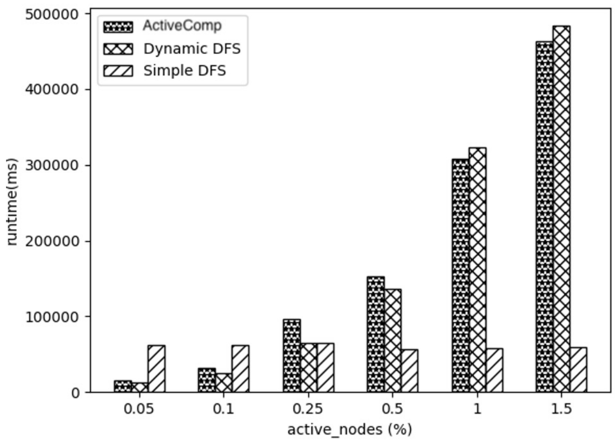


n=50k, k= 750

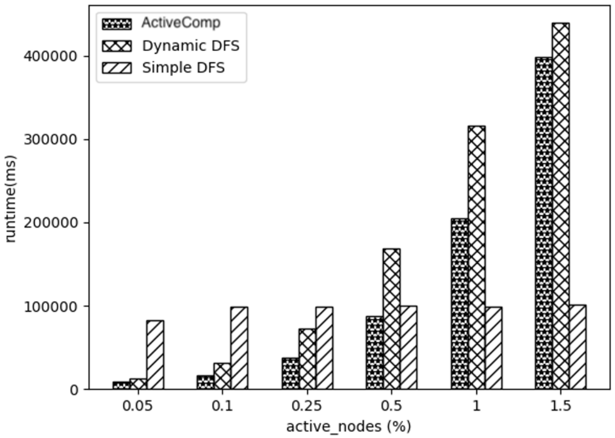


n=50k, k =1000

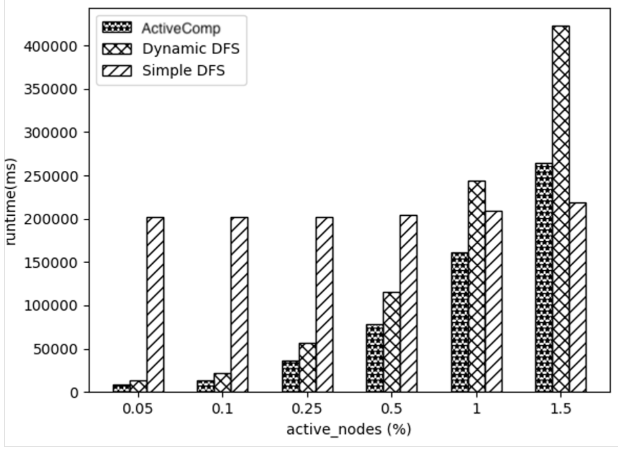
Low Percentage Active Nodes



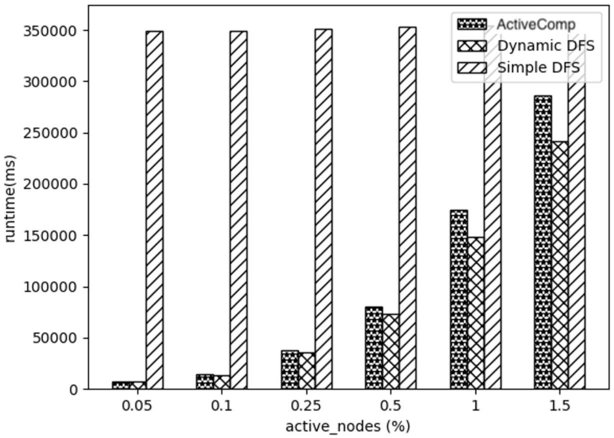
n=50k, k=250



n=50k, k =500



n=50k, k =750

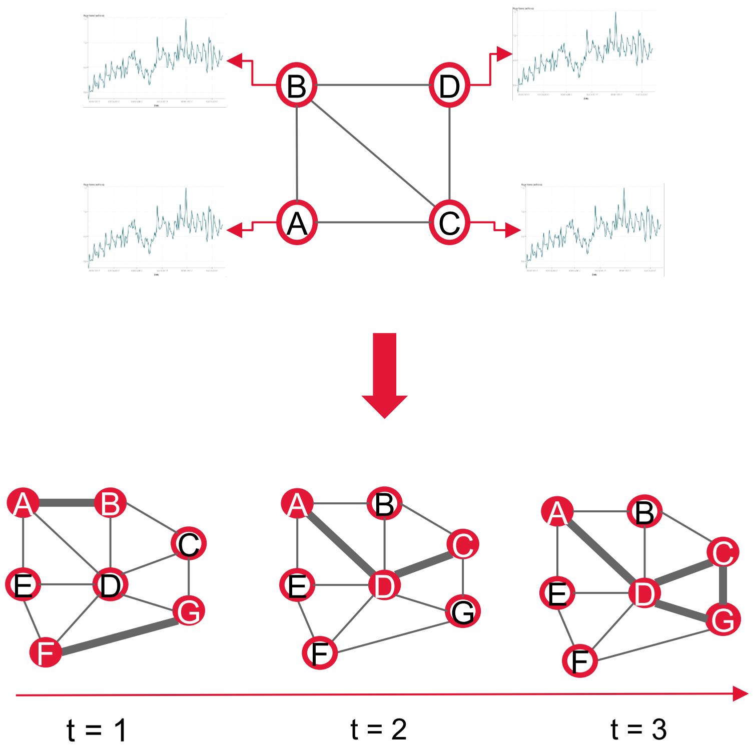


n=50k, k=1000

Conclusion, Limitations & Future Work

Conclusion

- Introduce a new problem of finding active components in a network of time series.
- Introduce ActiveComp
- Show the time complexity
- Empirically compare it to other baselines



Limitations

1. Scalability to Very Large Graphs
2. Underperforming in Specific Instances of the Problem

Future Work

1. Employing Parallel Computations
2. Fine-tuning to Accommodate Different Network Topologies
3. Extending the Comparative Empirical Analysis

Thank You