Network Effects and Cascading Behavior

Thanks to Jure Leskovec, Stanford and Panayiotis Tsaparas, Univ. of Ioannina for slides

Agenda

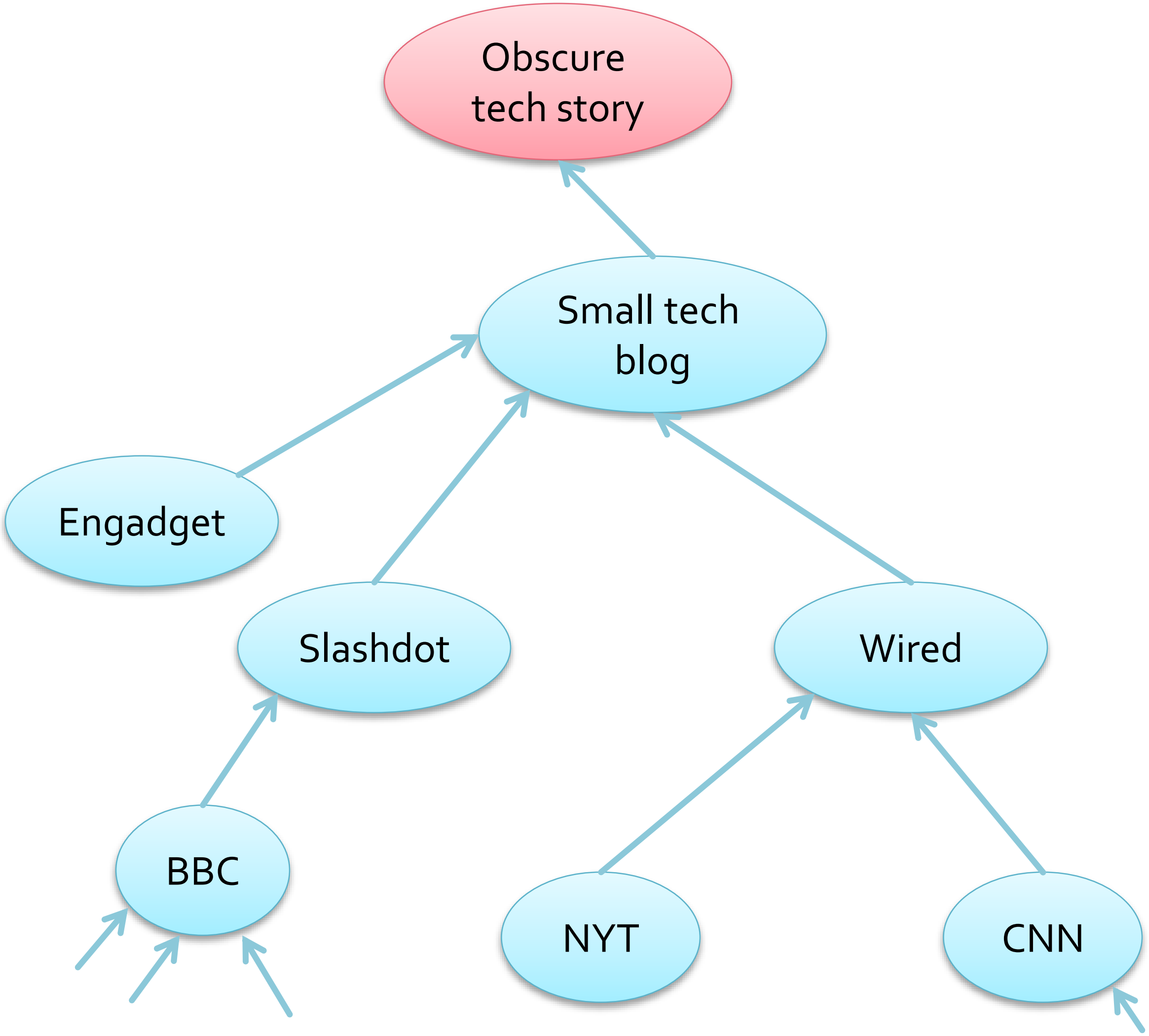
- Spreading Through Networks
- Granovetter’s Model of Collective Action
- Decision Based Model of Diffusion
  - Game Theoretic Model of Cascades

Spreading Through Networks

- **Spreading through networks:**
  - Cascading behavior
  - Diffusion of innovations
  - Network effects
  - Epidemics
- **Behaviors that cascade from node to node like an epidemic**

- **Examples:**
  - **Biological:**
    - Diseases via contagion
  - **Technological:**
    - Cascading failures
    - Spread of information
  - **Social:**
    - Rumors, news, new technology
    - Viral marketing

Information Diffusion: Media



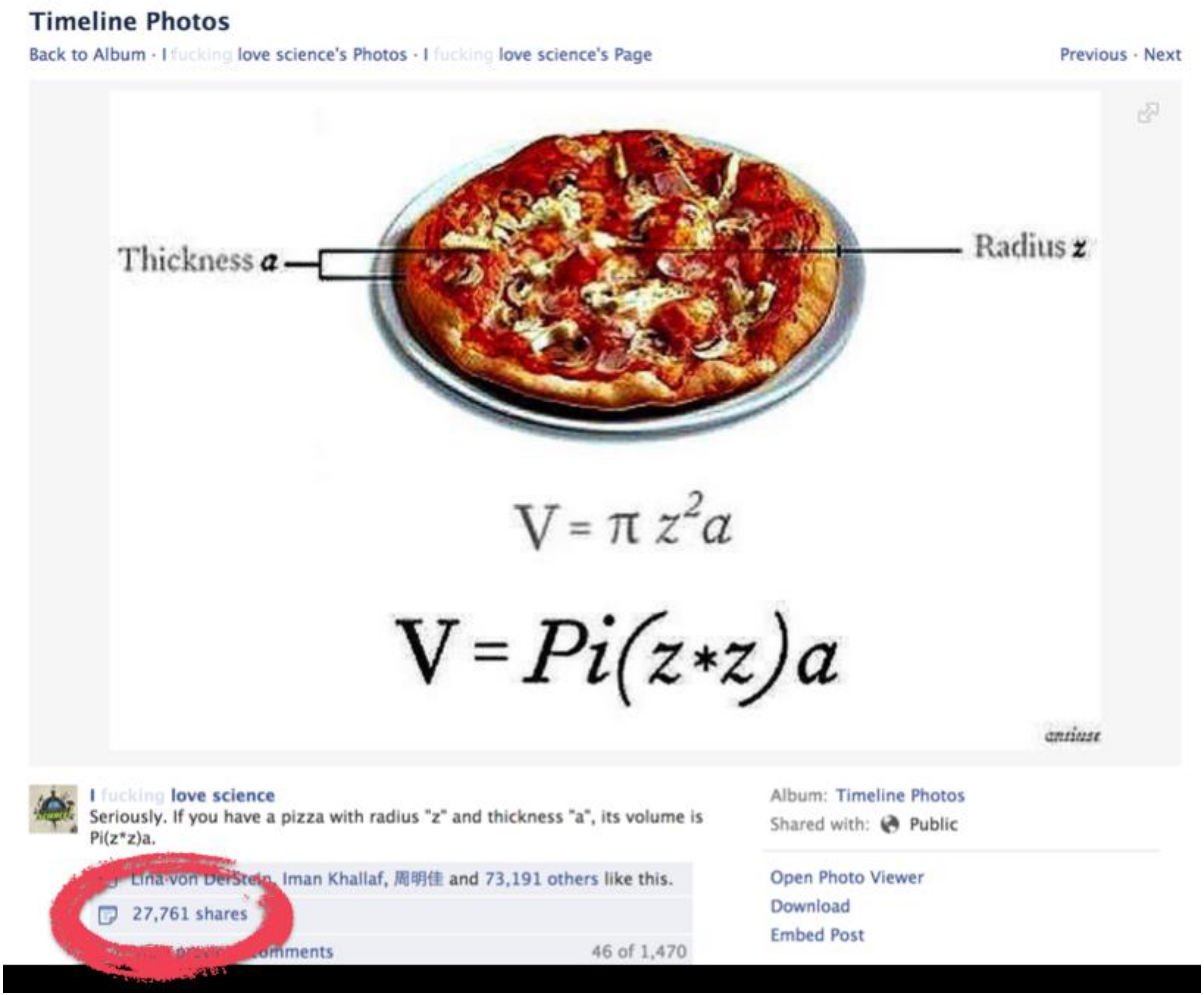
Twitter & Facebook post sharing

Lada Adamic shared a link via Erik Johnston.
January 16, 2013

When life gives you an almost empty jar of nutella, add some ice cream... (and other useful tips)

50 Life Hacks to Simplify your World
twistedsifter.com

Life hacks are little ways to make our lives easier. These low-budget tips and trick can help you organize and de-clutter space; prolong and preserve your products; or teach you...



$$V = \pi z^2 a$$

$$V = Pi(z*z)a$$

I fucking love science
Seriously. If you have a pizza with radius "z" and thickness "a", its volume is Pi(z*z)a.

Lina von DerStein, Iman Khallaf, 周明佳 and 73,191 others like this.

27,761 shares

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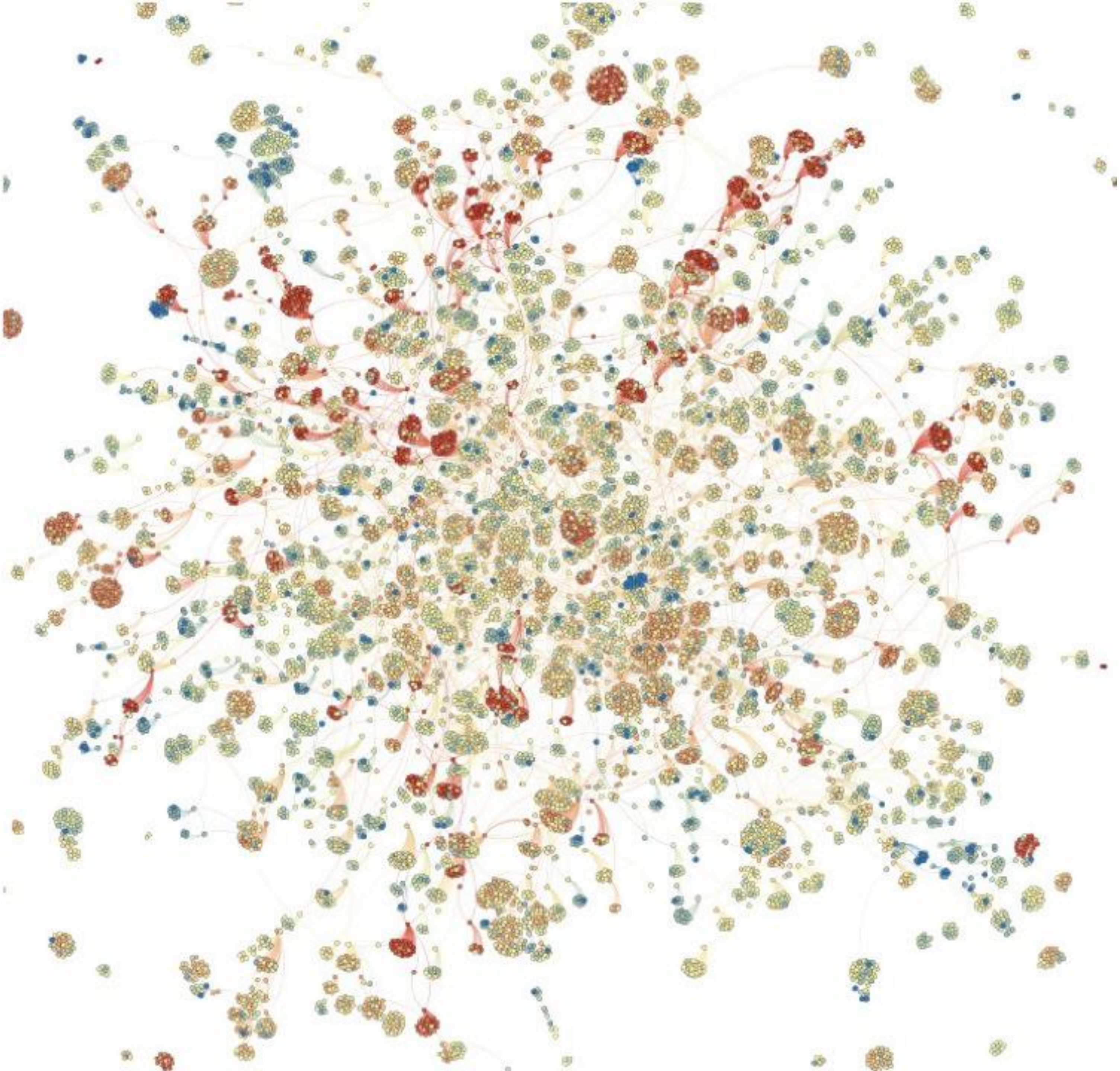
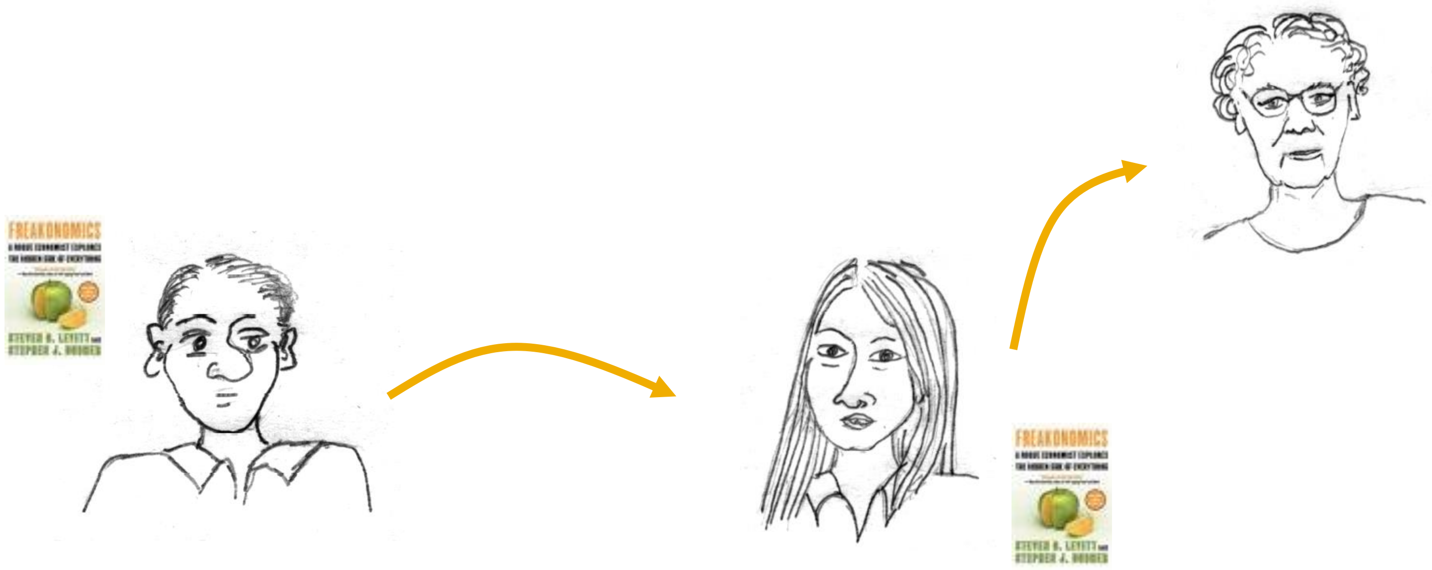
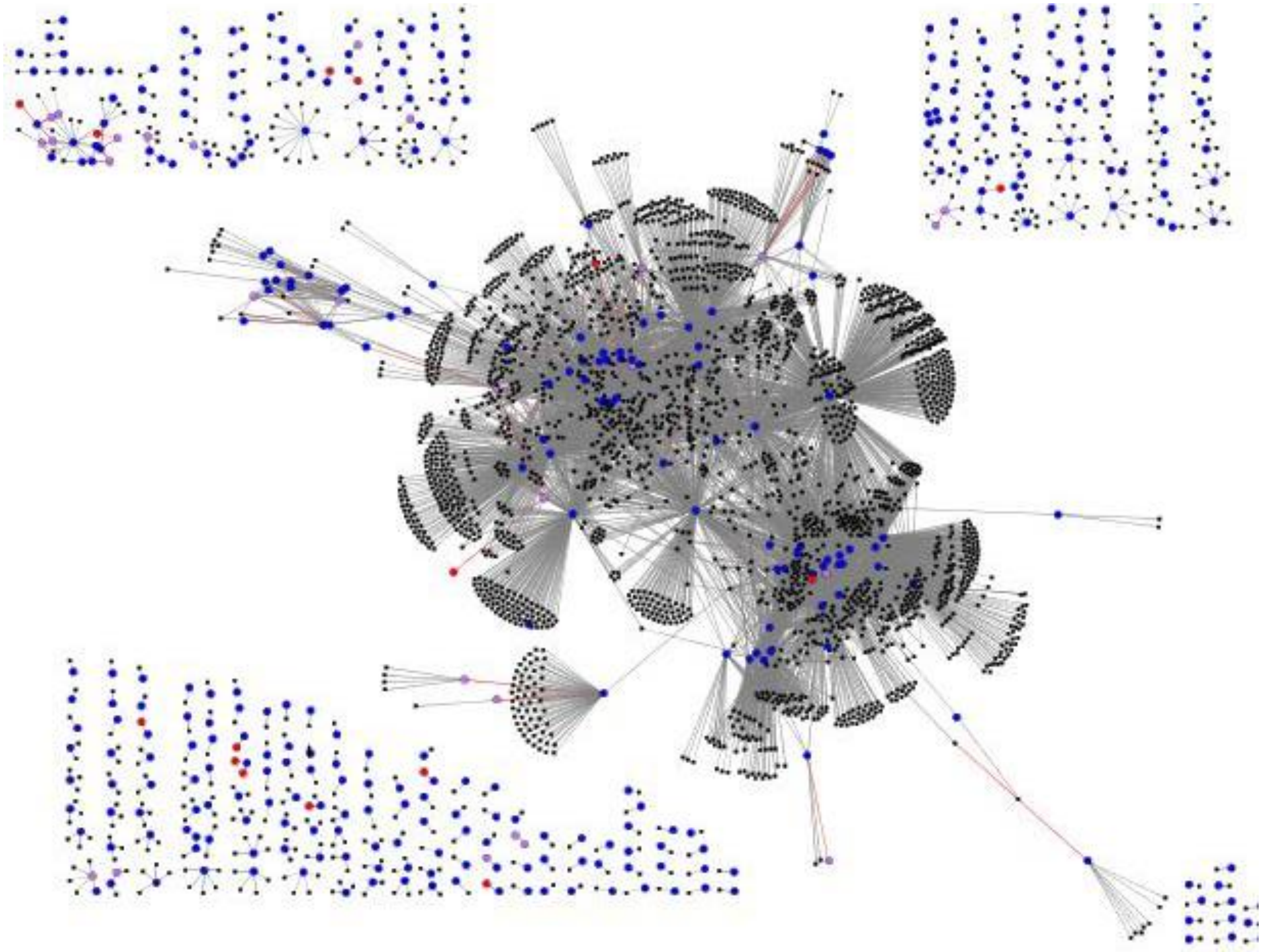


Diffusion in Viral Marketing

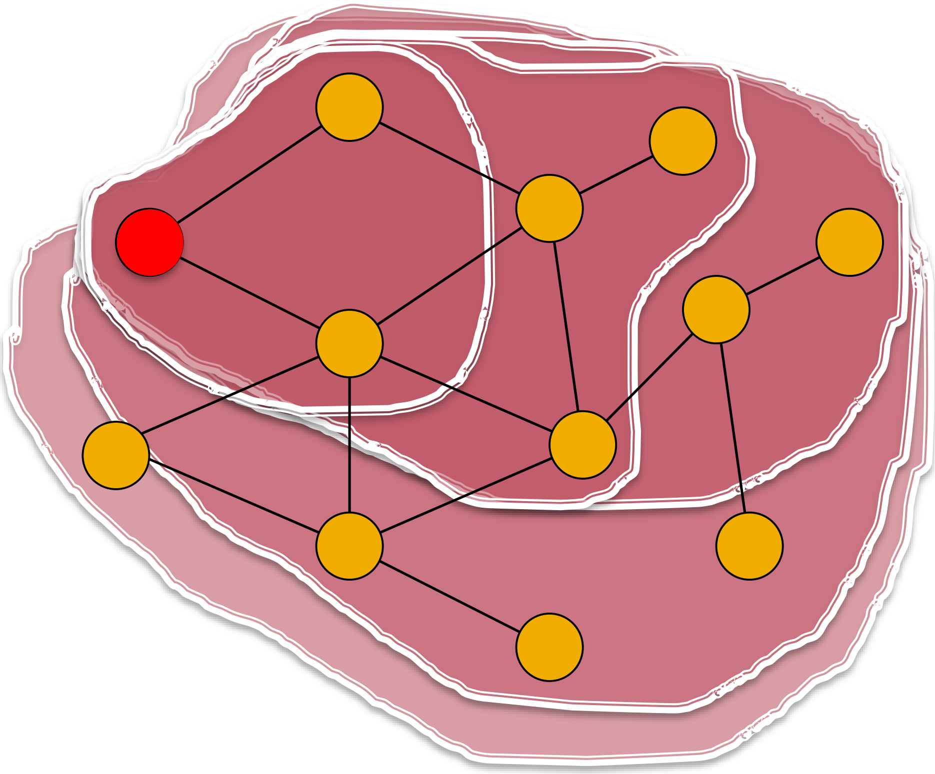
- **Product adoption:**
  - **Senders and followers of recommendations**



Diffusion in Viral Marketing

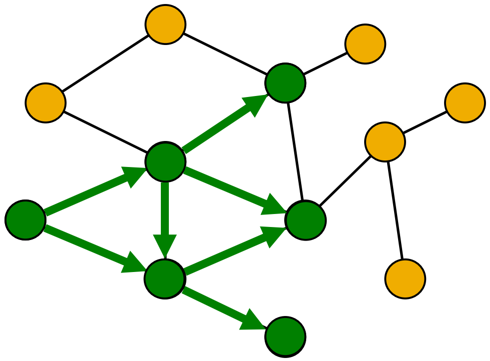


Spread of Diseases (e.g., Ebola)

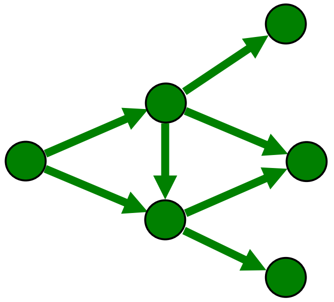


Network Cascades

- **Contagion that spreads over the edges of the network**
- It creates a propagation tree, i.e., **cascade**



Network



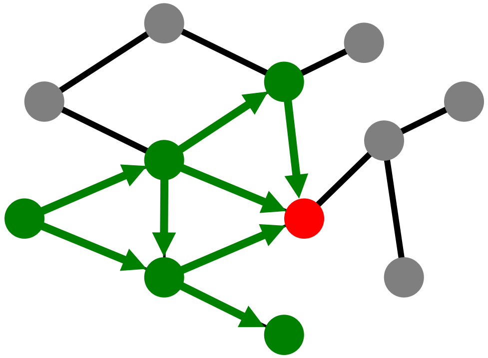
Cascade
(propagation graph)

Terminology:

- Stuff that spreads: Contagion
- “Infection” event: Adoption, infection, activation
- We have: Infected/active nodes, adopters

How Do We Model Diffusion?

- **Decision based models (today!):**
  - Models of product adoption, decision making
    - A node observes decisions of its neighbors and makes its own decision
  - **Example:**
    - You join demonstrations if *k* of your friends do so too
- **Probabilistic models (later):**
  - **Models of influence or disease spreading**
    - An infected node tries to "push" the contagion to an uninfected node
  - **Example:**
    - You "catch" a disease with some prob. from each active neighbor in the network



Granovetter’s Model of Collective Action

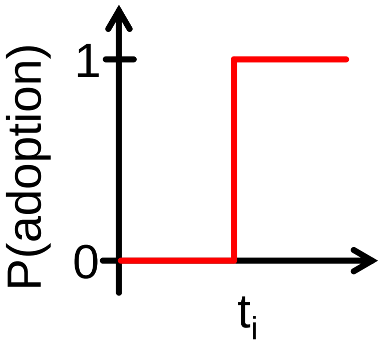
[Granovetter ‘78]

Decision Based Models

- **Collective Action** [Granovetter, ‘78]
  - **Model where everyone sees everyone else’s behavior (that is, we assume a complete graph)**
  - **Examples:**
    - Clapping or getting up and leaving in a theater
    - Keeping your money or not in a stock market
    - Neighborhoods in cities changing ethnic composition
    - Riots, protests, strikes
- **How does the number of people participating in a given activity grow or shrink over time?**

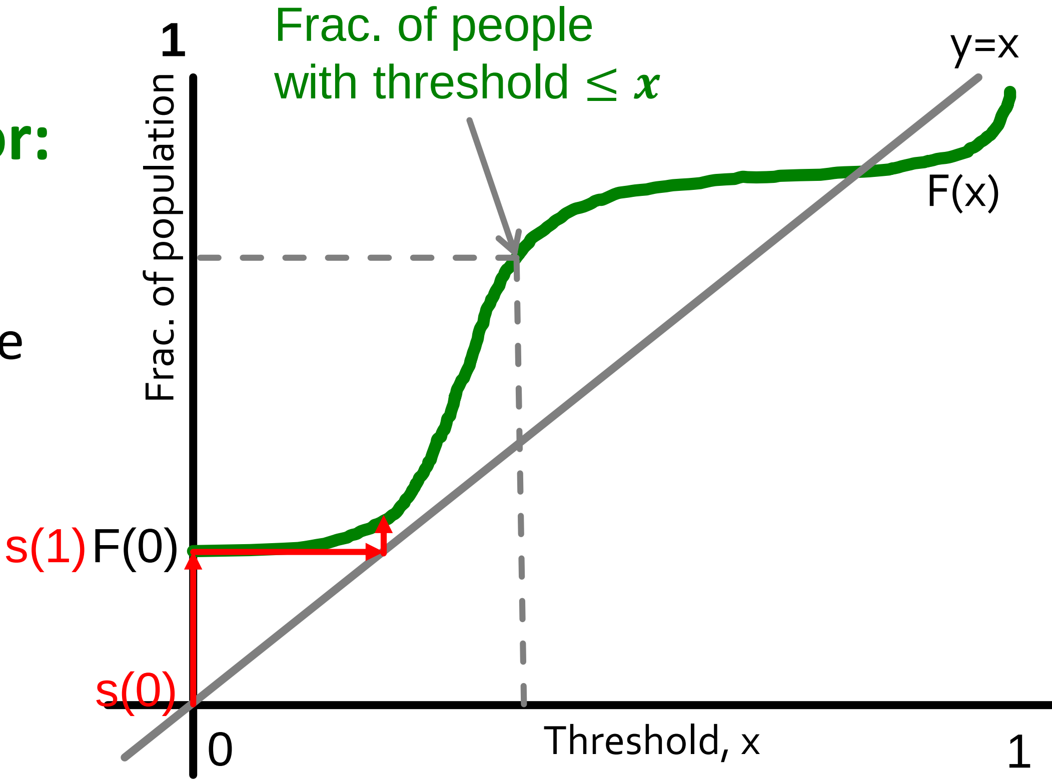
Collective Action: The Model

- ***n* people – everyone observes all actions**
- Each person ***i*** has a threshold t_i $(0 \leq t_i \leq 1)$
  - Node ***i*** will adopt the behavior iff at least t_i fraction of people have already adopted:
    - **Small t_i:** early adopter
    - **Large t_i:** late adopter
  - Time moves in discrete steps
- **The population is described by $\{t_1,\ldots,t_n\}$**
  - **F(x)** … fraction of people with threshold $t_i \leq x$
    - F(x) is a property of the contagion given to us. F(x) is the **c.d.f.** of *x*



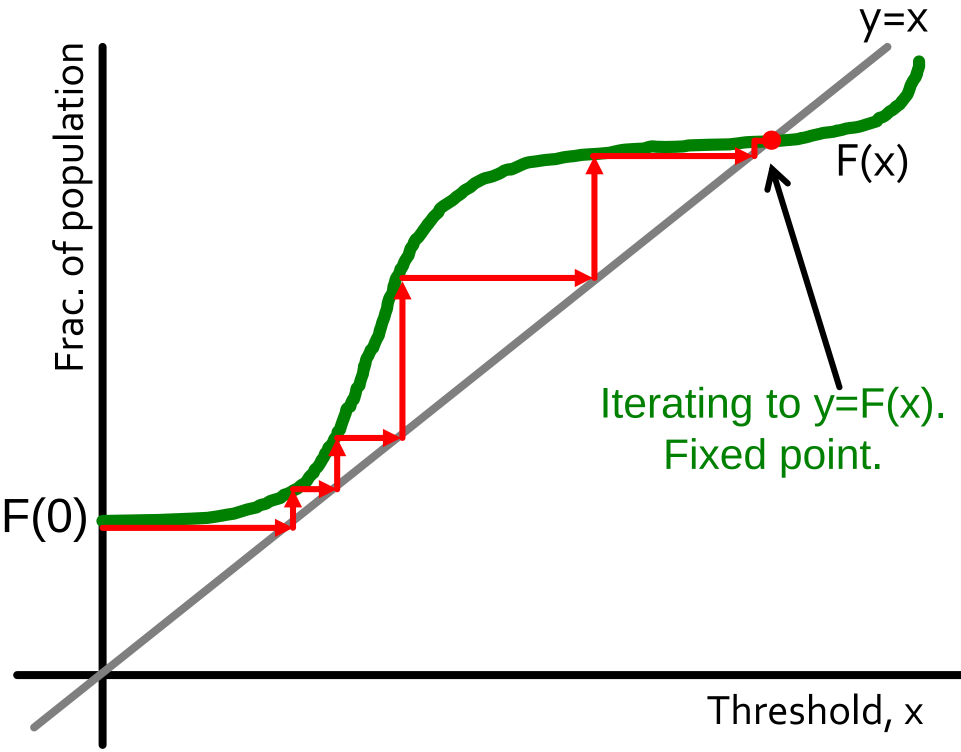
Collective Action: Dynamics

- **F(x)** … fraction of people with threshold $t_i \leq x$
  - **F(x) is non-decreasing:** $F(x+\varepsilon) \geq F(x)$
- **The model is dynamic:**
  - **Step-by-step change in number of people adopting the behavior:**
    - **F(x)** … frac. of people with threshold ≤ **x**
    - **s(t)** … number of people participating at time **t**
  - **Simulate:**
    - s(0) = 0
    - s(1) = F(0)
    - s(2) = F(s(1)) = F(F(0))



Collective Action: Dynamics

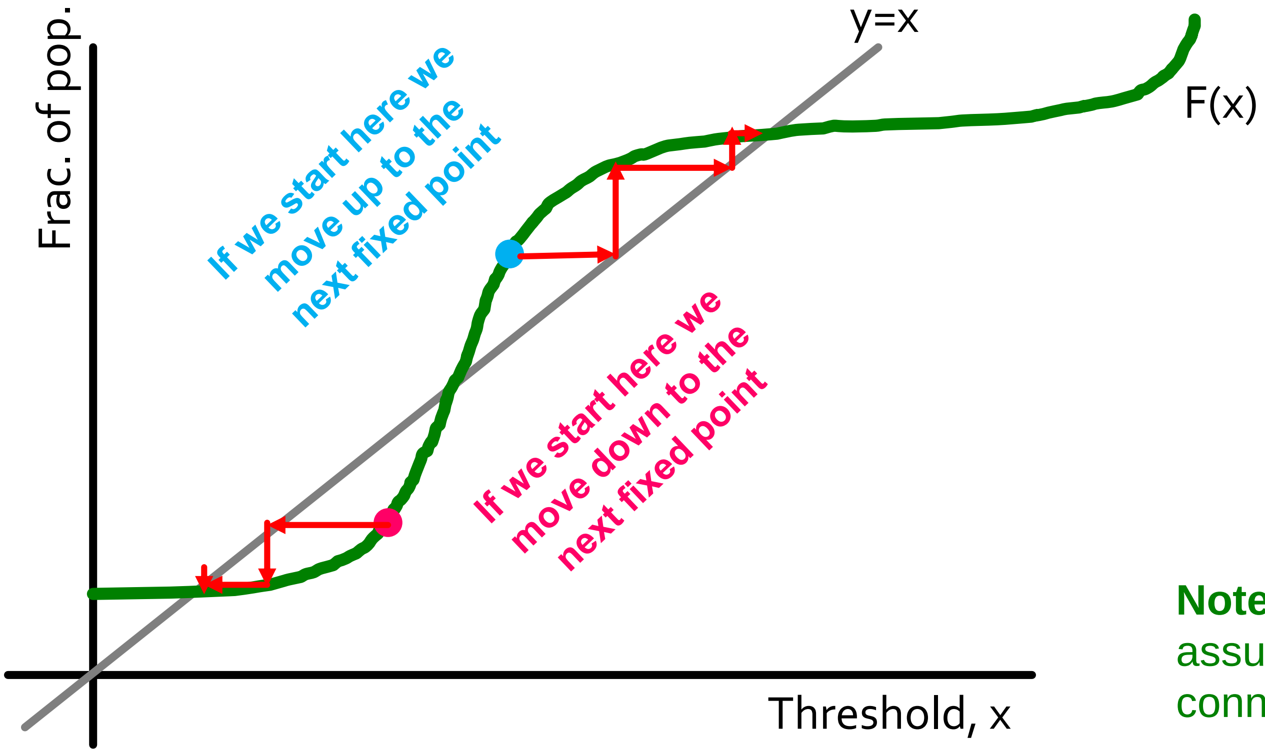
- **Step-by-step change in number of people :**
  - **F(x)** … fraction of people with threshold $\leq$ **x**
  - **s(t)** … number of participants at time **t**
- **Easy to simulate:**
  - $s(0) = 0$
  - $s(1) = F(0)$
  - $s(2) = F(s(1)) = F(F(0))$
  - $s(t+1) = F(s(t)) = F^{t+1}(0)$
- **Fixed point: F(x)=x**
  - Updates to **s(t)** to converge to a stable fixed point
  - There could be other fixed points but starting from **0** we only reach the first one



Iterating to y=F(x).
Fixed point.

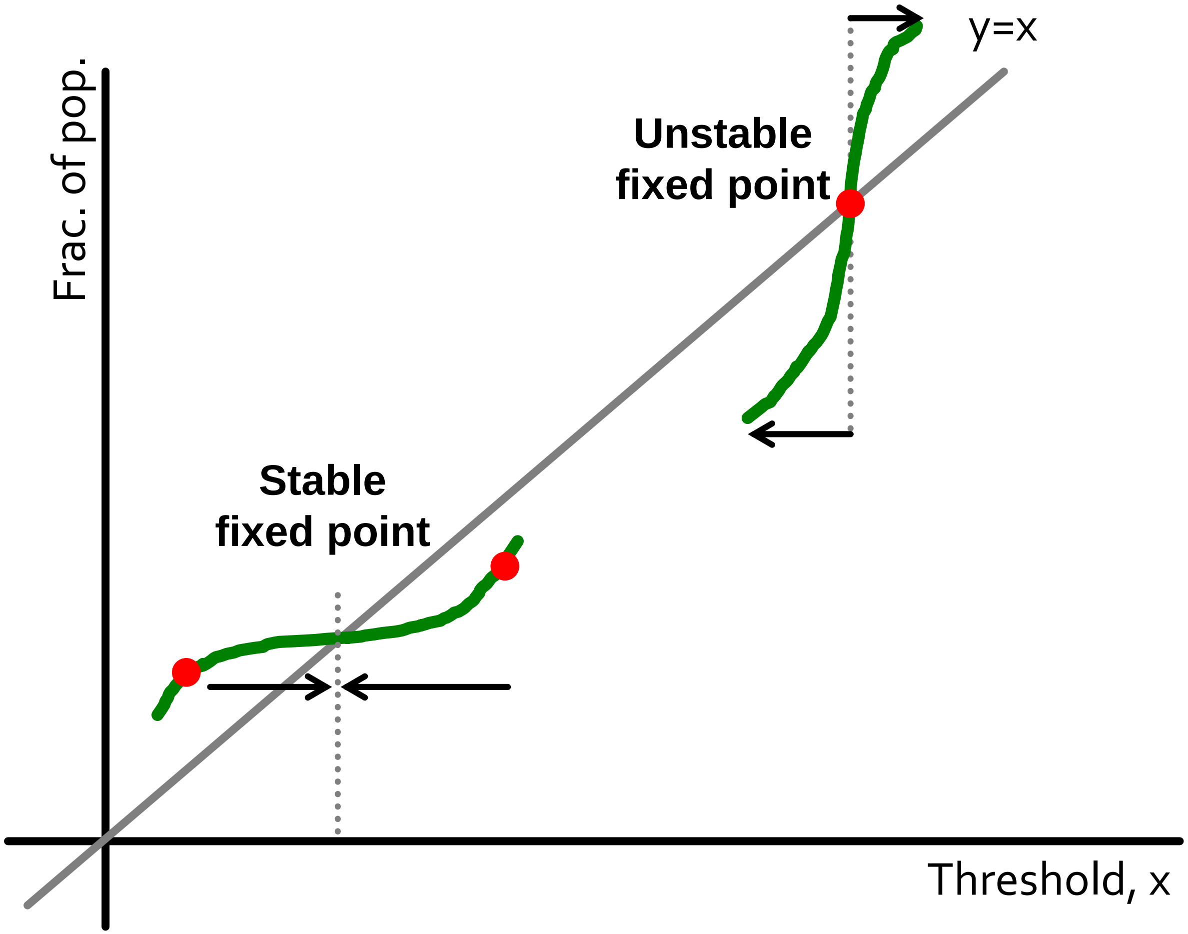
Starting Elsewhere

- **What if we start the process somewhere else?**
  - We move up/down to the next fixed point
  - **How is market going to change?**



Note: we are assuming a fully connected graph

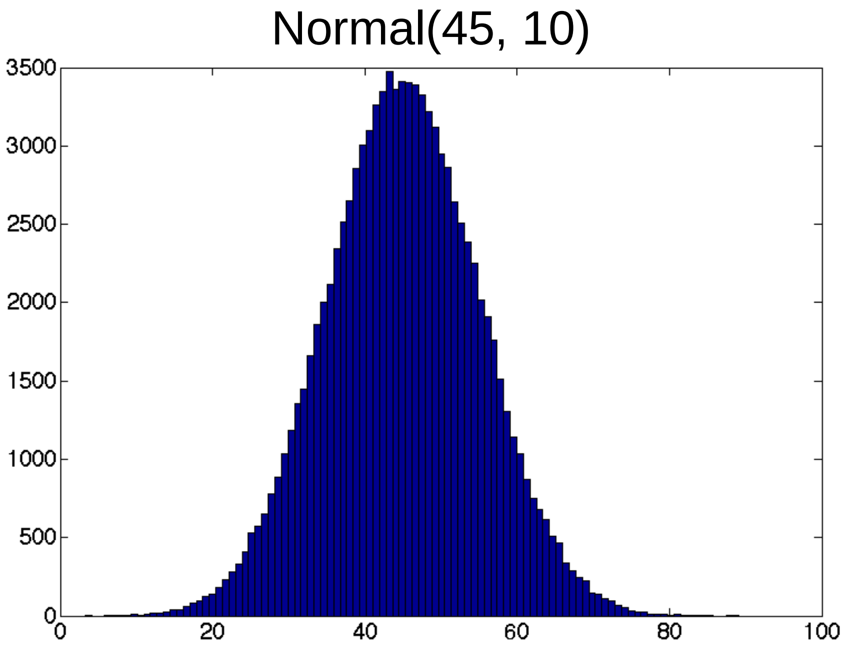
Stable vs. Unstable Fixed Point



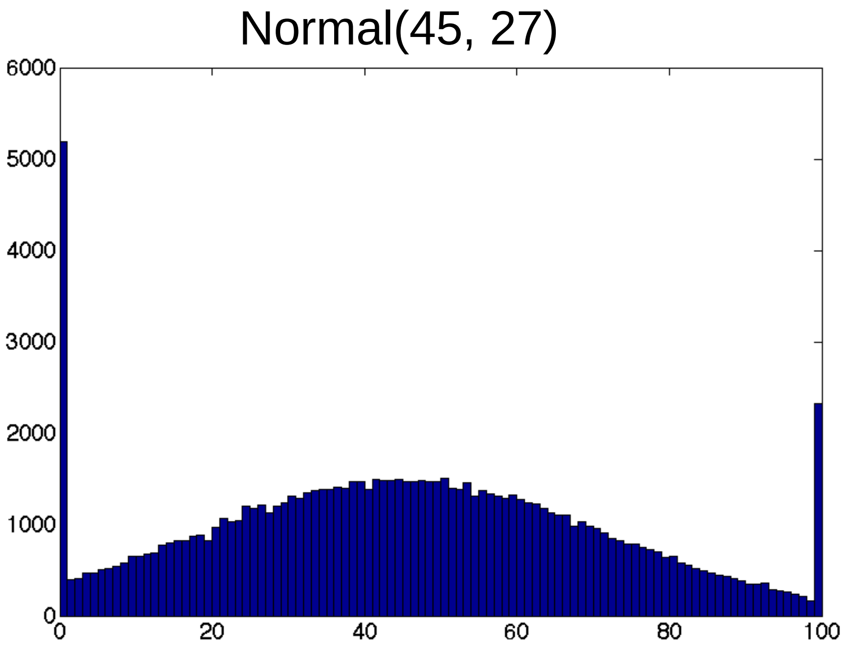
Discontinuous Transition

- Each threshold t_i is drawn independently from some distribution ***F(x) = Pr[thresh*** $\le$ ***x]***
  - **Suppose:** Normal with **μ=n/2**, variance **σ**

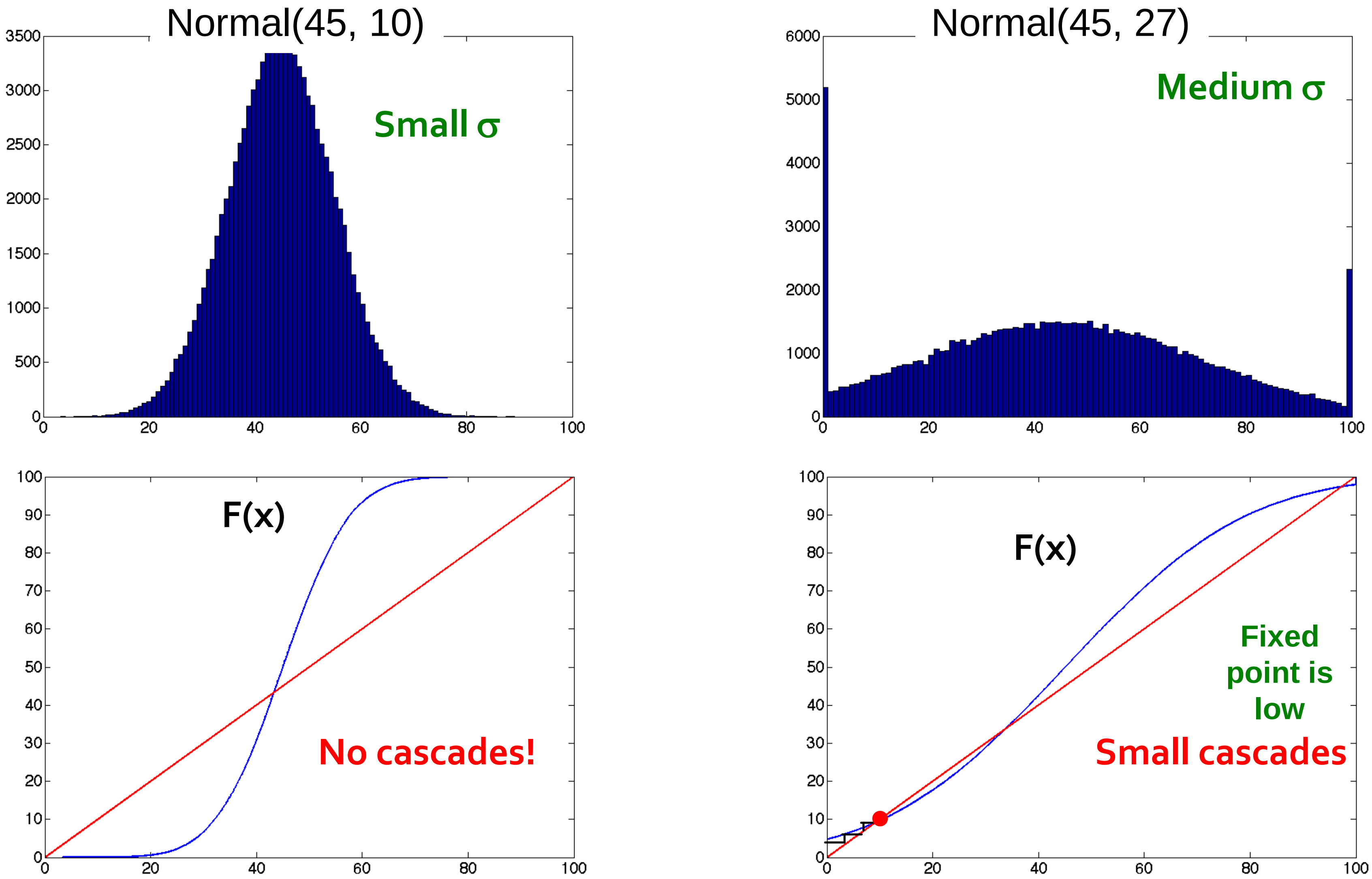
Small σ:



Medium σ:



Discontinuous Transition



Normal(45, 10)

Small σ

Normal(45, 27)

Medium σ

F(x)

No cascades!

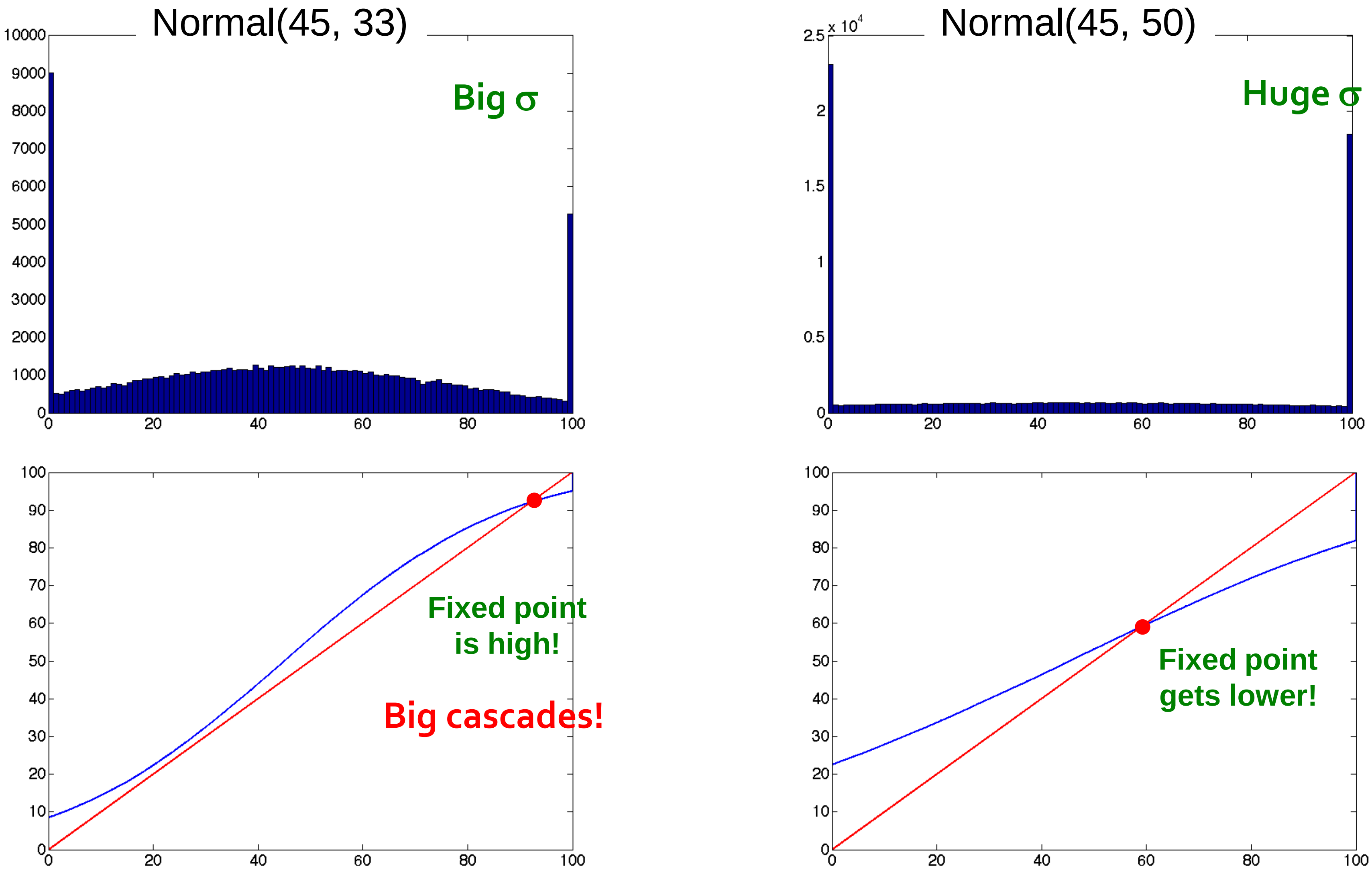
F(x)

Fixed point is low

Small cascades

Bigger variance let you build a bridge from early adopters to mainstream

Discontinuous Transition


Normal(45, 33)

Big σ

Fixed point is high!

Big cascades!

Normal(45, 50)

Huge σ

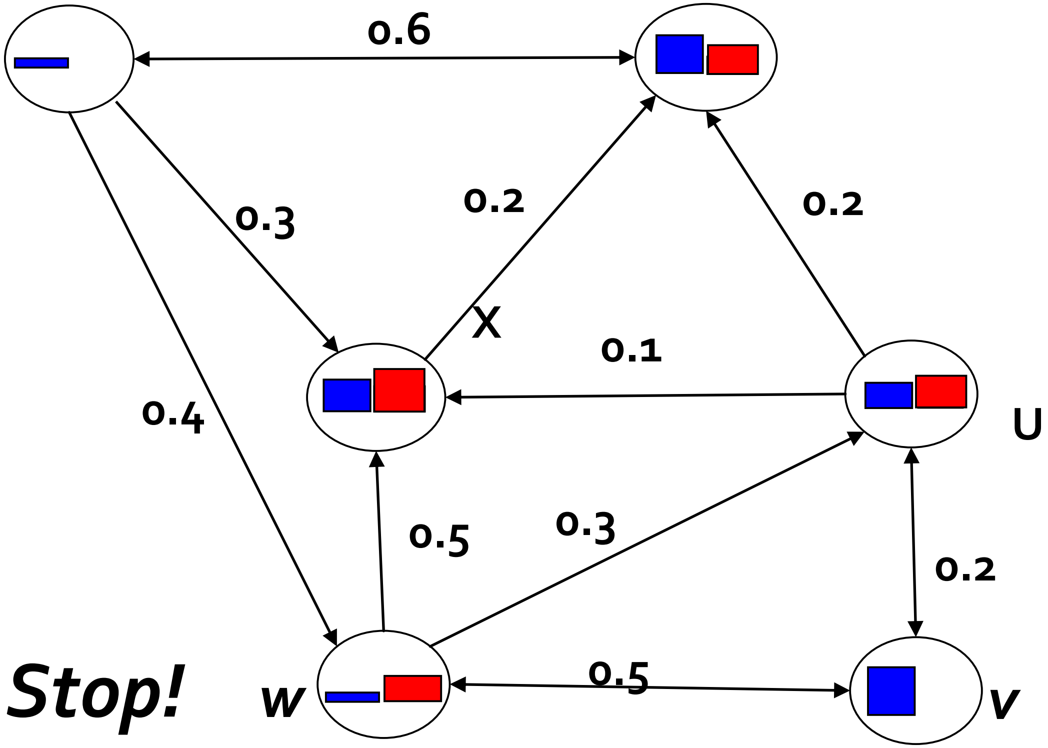
Fixed point gets lower!

But if we increase the variance the fixed point starts going down

Weaknesses of the Model

- **No notion of social network:**
  - Some people are more influential
  - It matters who the early adopters are, not just how many
- **Models people's awareness** of size of participation **not just actual number of people participating**
  - Modeling perceptions of who is adopting the behavior vs. who you believe is adopting
  - Non-monotone behavior – dropping out if too many people adopt
  - People get "locked in" to certain choice over a period of time
- **Modeling thresholds**
  - Richer distributions
  - Deriving thresholds from more basic assumptions
    - game theoretic models

Linear Threshold Model



Example

- ○ Inactive Node
- 🟢 Active Node
- 🟦 Threshold
- 🟥 Active neighbors

Thresholds:

$$\vartheta_v \sim U[0,1]$$

Influenced when:

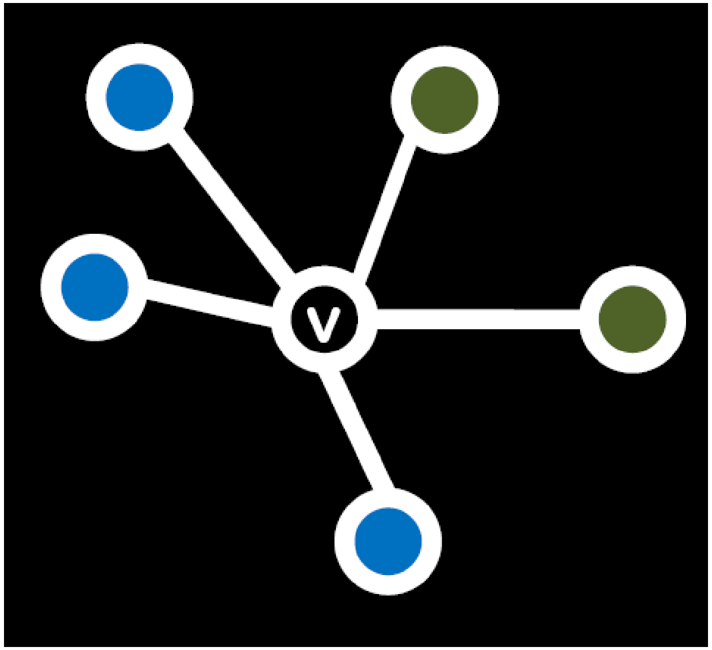
$$\sum_{w \text{ active neighbor of } v} b_{v,w} \geq \theta_v$$

Decision Based Model of Diffusion

[Morris 2000]

Game Theoretic Model of Cascades

- **Based on 2 player coordination game**
  - 2 players – each chooses technology A or B
  - Each person can only adopt **one** “behavior”, **A** or **B**
  - You gain more payoff if your friend has adopted the **same** behavior as you



Local view of the network of node **v**

Example: VHS vs. BetaMax

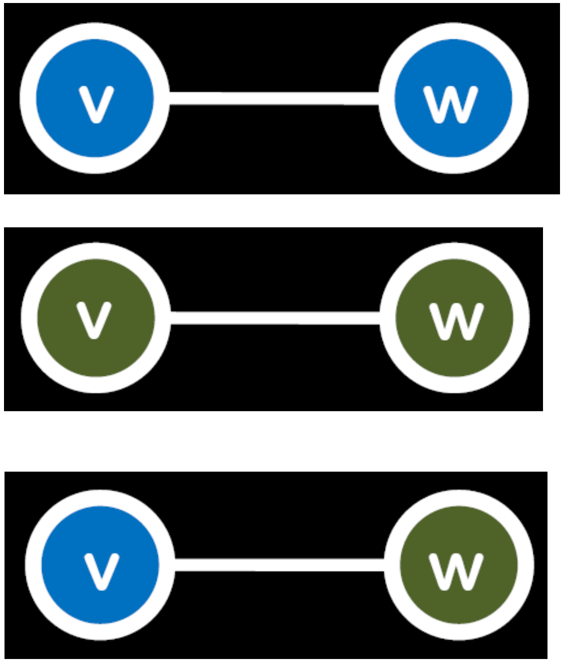
Example: BlueRay vs. HD DVD

Blu-ray Disc

HD DVD

The Model for Two Nodes

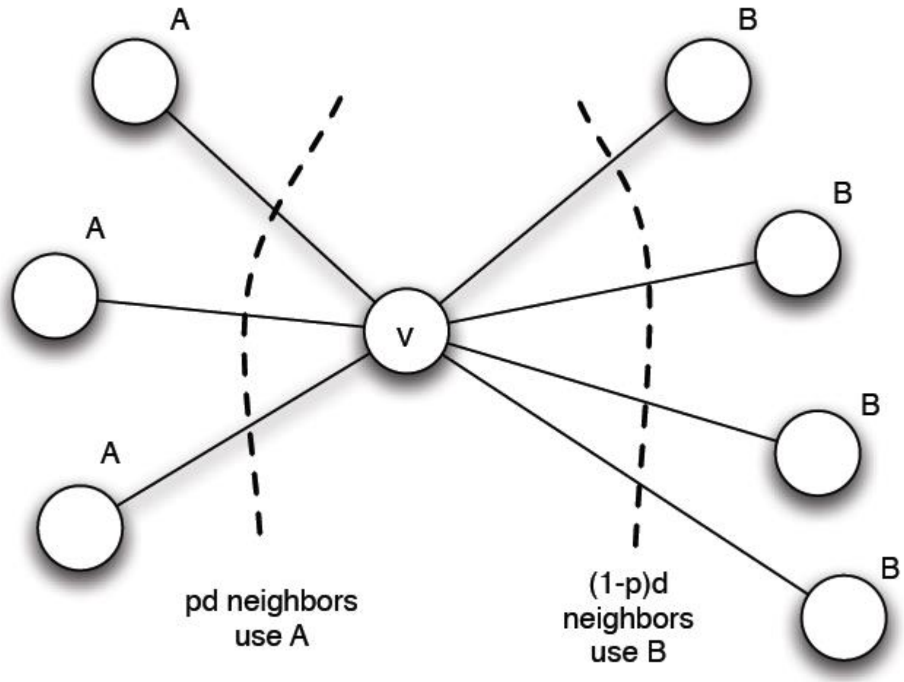
- ***Payoff matrix:***
  - If both **v** and **w** adopt behavior **A**, they each get payoff ***a* > *0***
  - If ***v*** and ***w*** adopt behavior ***B***, they each get payoff ***b* > *0***
  - If ***v*** and ***w*** adopt the opposite behaviors, they each get ***0***



- **In some large network:**
  - Each node ***v*** is playing a copy of the game with each of its neighbors
  - **Payoff**: sum of node payoffs per game

		w	
		A	B
v	A	a, a	o,o
	B	o,o	b,b

Calculation of Node v



Threshold:

v chooses ***A*** if

$$p > \frac{b}{a+b} = q$$

p… frac. v's nbrs. with A

q… payoff threshold

- Let **v** have ***d*** neighbors
- Assume fraction ***p*** of ***v***'s neighbors adopt ***A***
  - $Payoff_v = a \cdot p \cdot d$, if v chooses A
    $= b \cdot (1-p) \cdot d$, if v chooses B
- **Thus: *v* chooses A if: $a \cdot p \cdot d > b \cdot (1-p) \cdot d$**

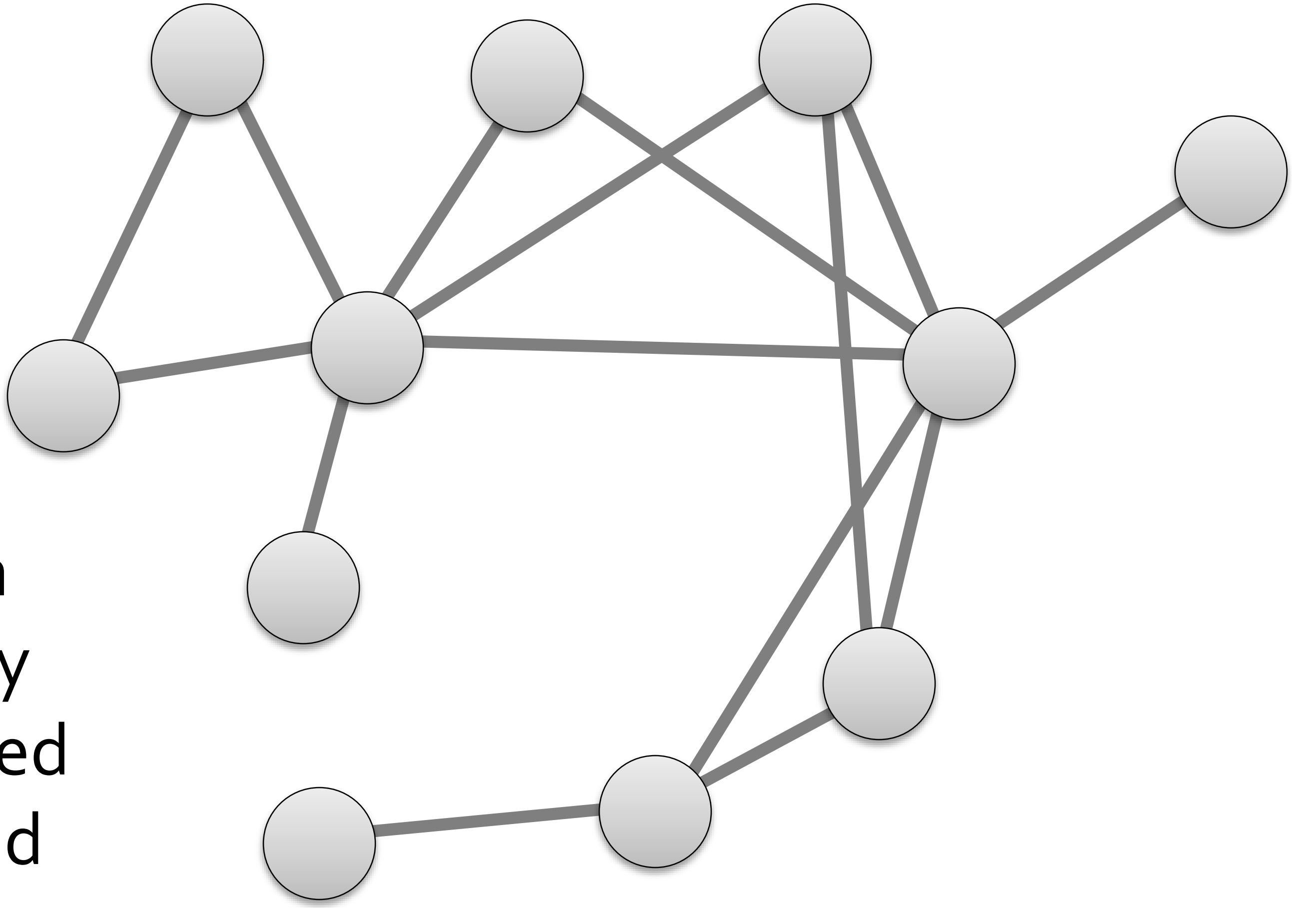
Example Scenario

- **Scenario:**
  Graph where everyone starts with ***B***
  Small set ***S*** of early adopters of ***A***
  - Hard-wire ***S*** – they keep using ***A*** no matter what payoffs tell them to do
- **Assume payoffs are set in such a way that nodes say:**
  If more than 50% of my friends take A
  I'll also take A

  (this means: $\mathbf{a = b\text{-}\varepsilon}$ and $\mathbf{q>1/2}$)

Example Scenario

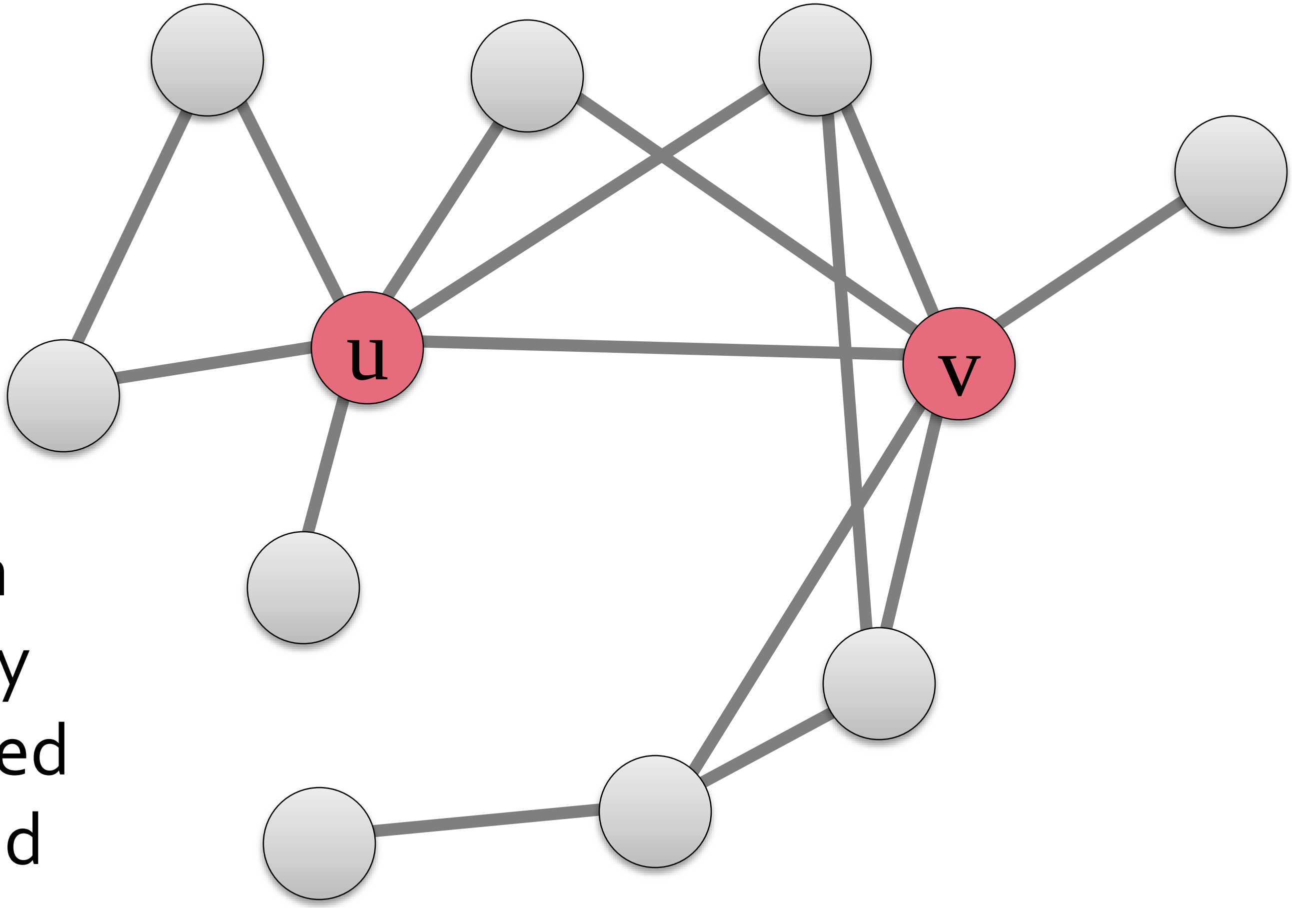
$S = \{u, v\}$



If **more** than **q=**50% of my friends are red I'll also be red

Example Scenario

$S = \{u, v\}$



If **more** than
q=50% of my
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Example Scenario

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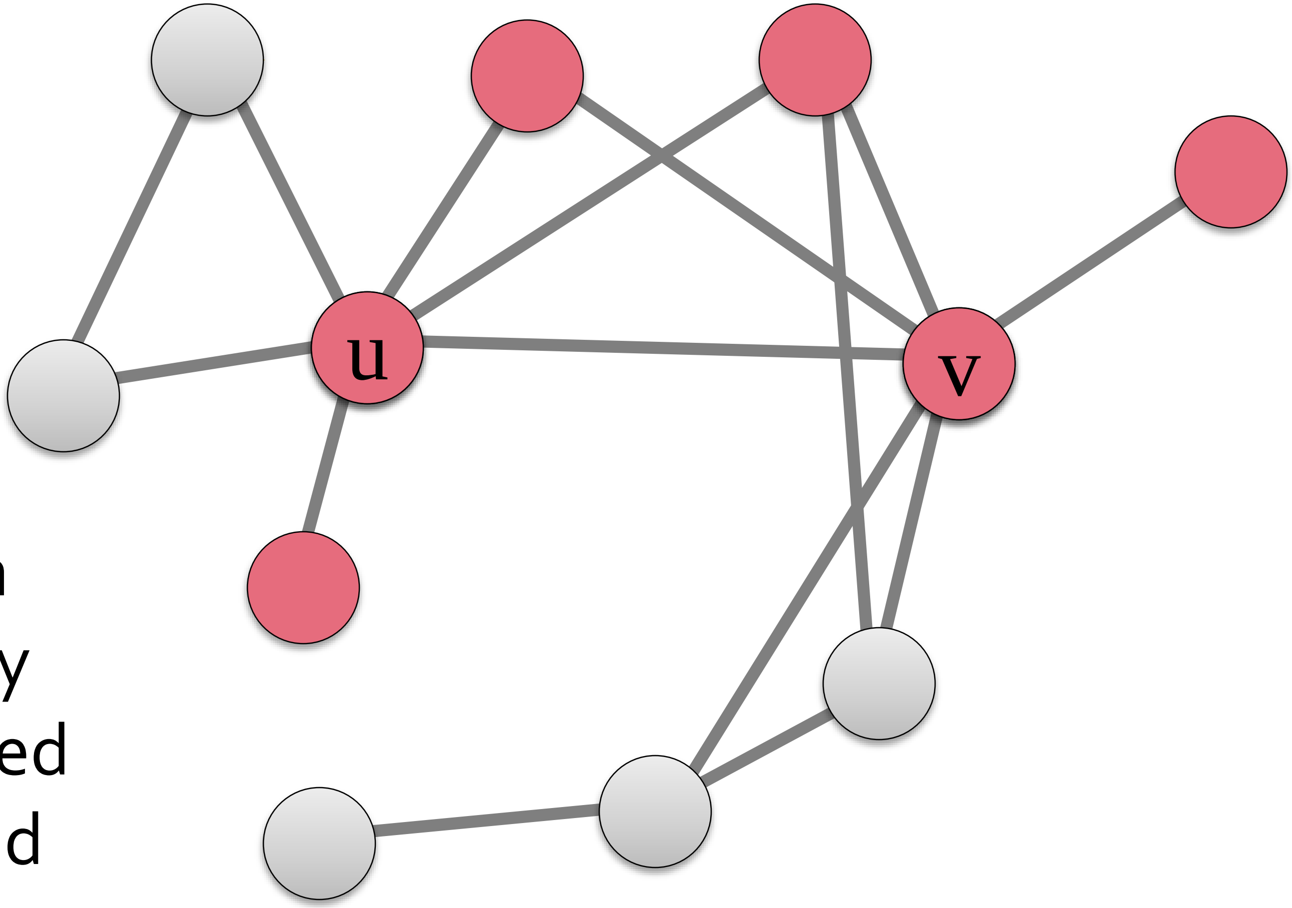

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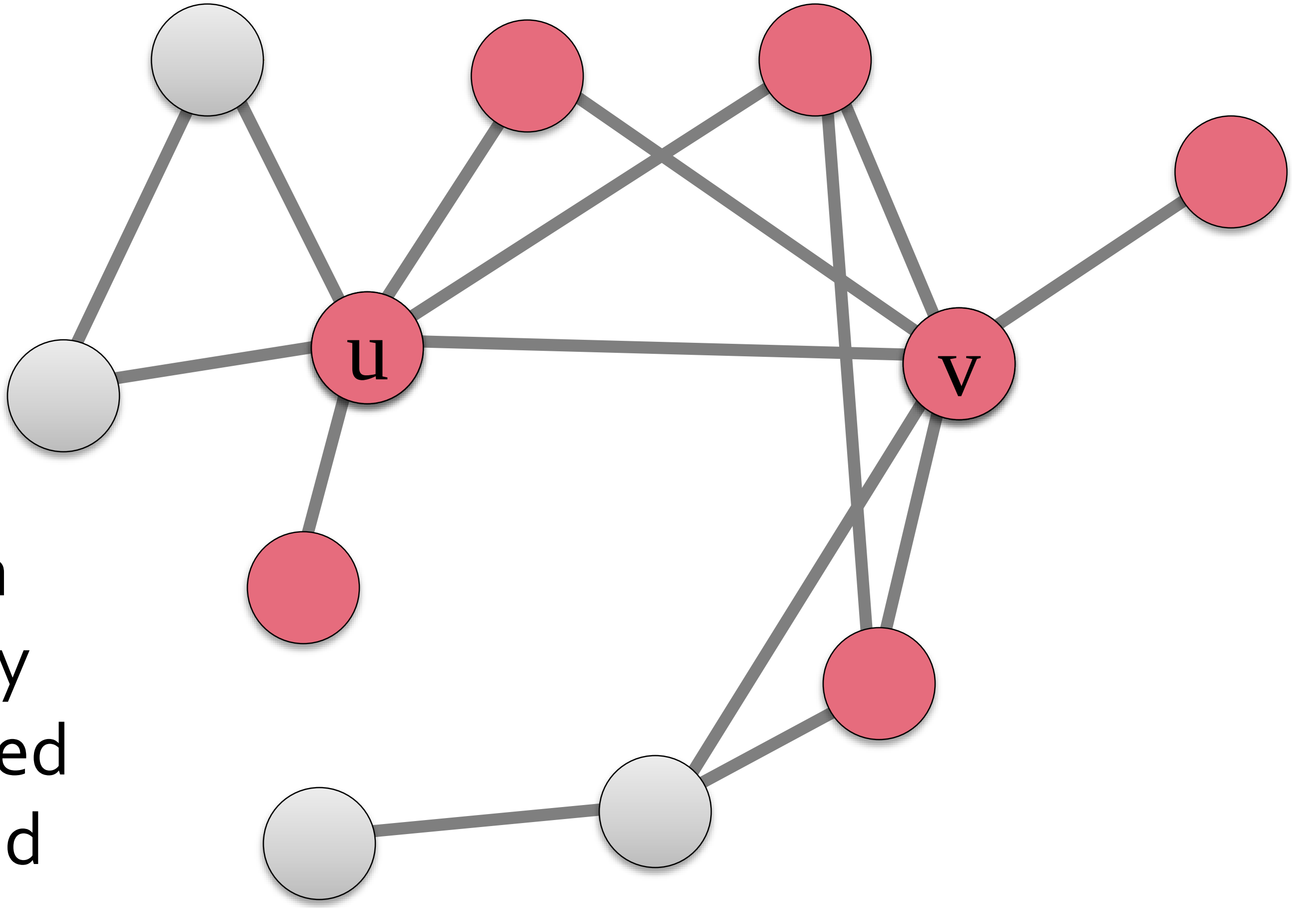

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Example Scenario

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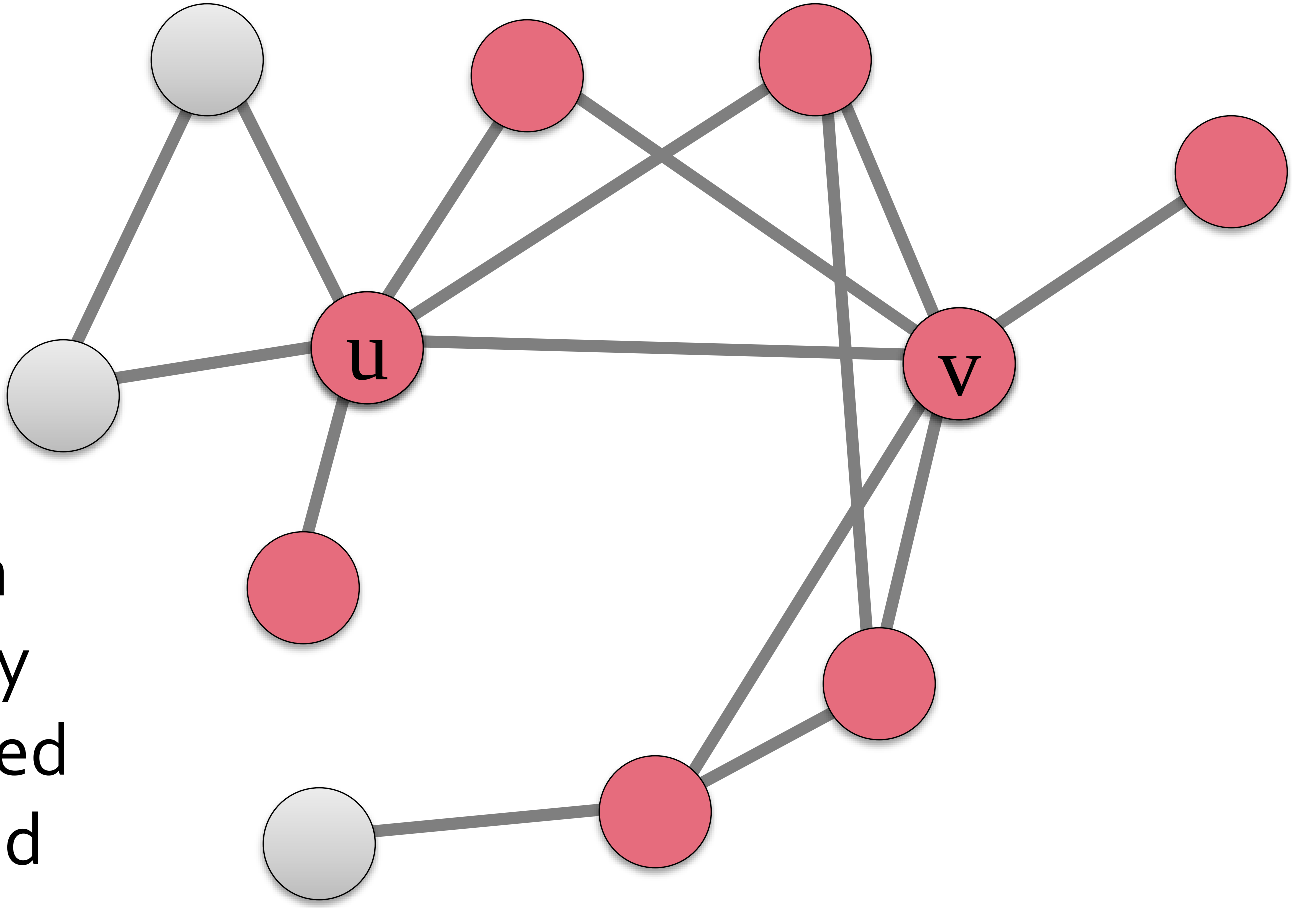

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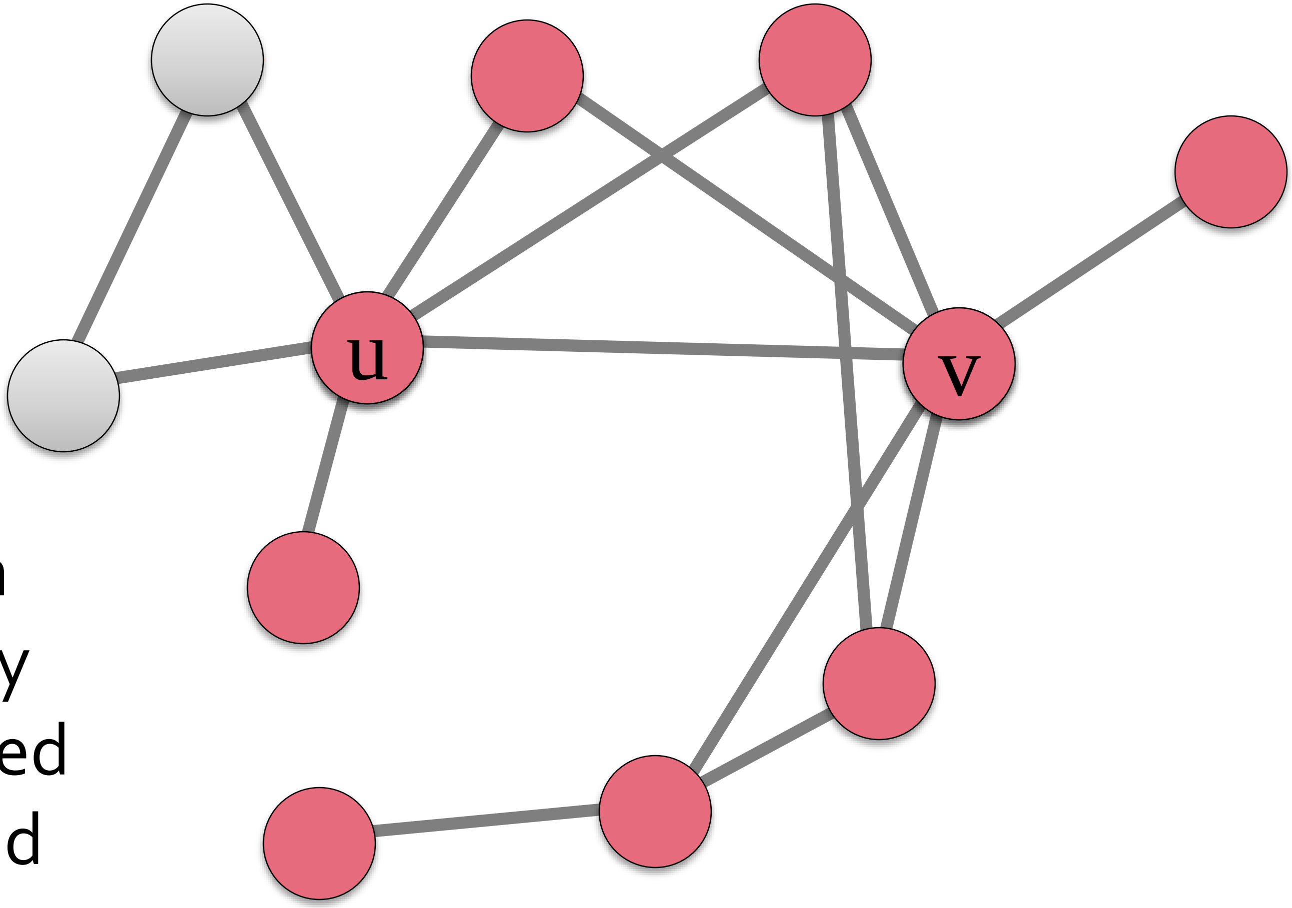


If **more** than **q=**50% of my friends are red I'll also be red

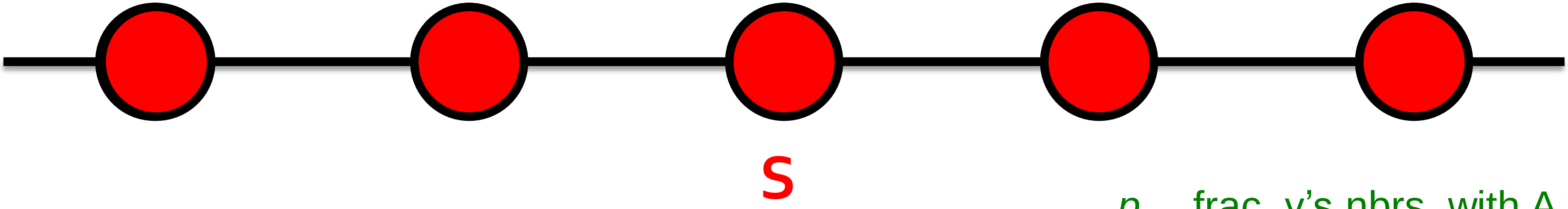
Infinite Graphs

v chooses ***A*** if ***p>q***

$$q = \frac{b}{a+b}$$

- **Consider infinite graph *G***
  - (but each node has finite number of neighbors!)
- We say that a finite set ***S*** causes a cascade in *G* with **threshold *q*** if, when ***S*** adopts ***A***, eventually **every node in G adopts *A***
- Example: **Path**

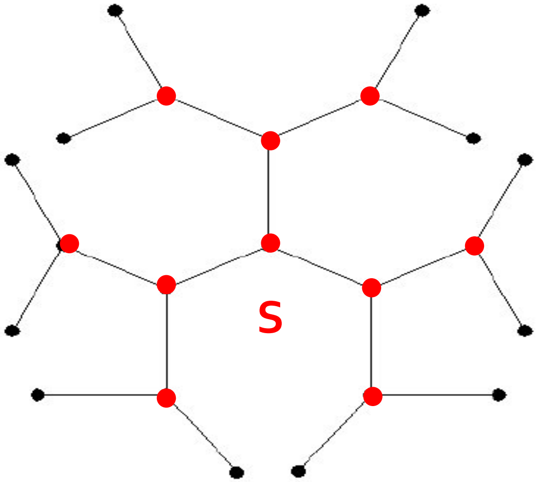
If ***q<1/2*** then cascade occurs



p… frac. v's nbrs. with A
q… payoff threshold

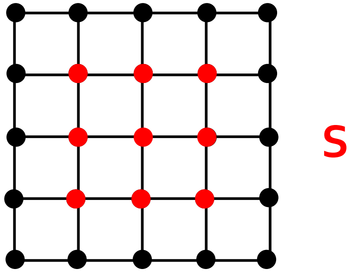
Infinite Graphs

- **Infinite Tree:**



If $q<1/3$ then cascade occurs

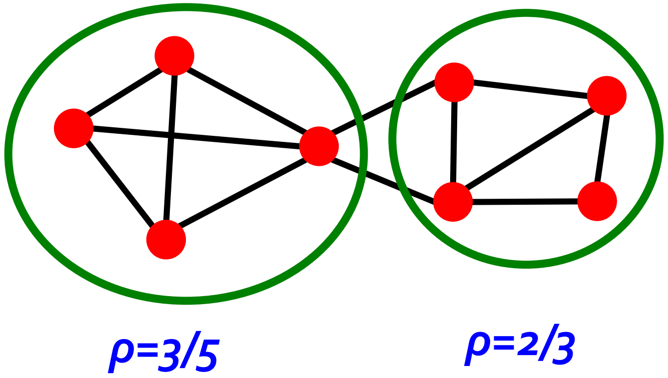
- **Infinite Grid:**



If $q<1/4$ then cascade occurs

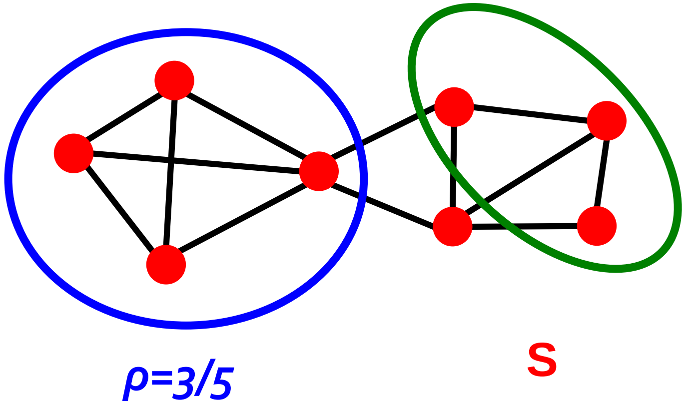
Stopping Cascades

- **What prevents cascades from spreading?**
- Def: **Cluster of density *ρ*** is a **set of nodes *C*** where each node in the set has at least ***ρ*** fraction of edges in ***C***



Stopping Cascades

- **Let S be an initial set of adopters of A**
- All nodes apply threshold ***q*** to decide whether to switch to **A**
- **Two facts:**
  - 1) If ***G\S*** contains a cluster of density ***>(1-q)*** then ***S*** can not cause a cascade
  - 2) If **S** fails to create a cascade, then there is a cluster of density ***>(1-q)*** in **G\S**



No cascade if q>2/5

Extending the Model:
Allow People to Adopt A and B

Cascades & Compatibility

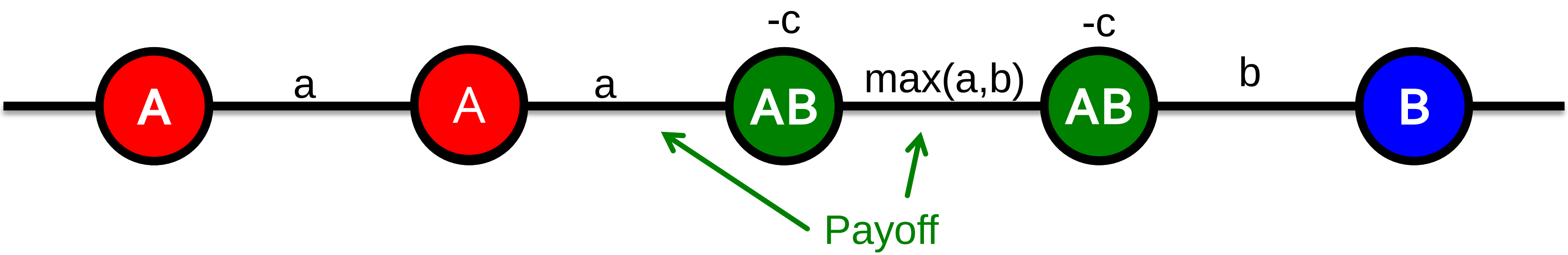
- **So far:**
  - Behaviors ***A*** and ***B*** compete
  - Can only get utility from neighbors of same behavior: ***A-A* get a, *B-B* get *b*, *A-B* get *0***
- **Let an extra strategy "*AB*"**
  - ***AB-A*** : gets ***a***
  - ***AB-B*** : gets ***b***
  - ***AB-AB*** : gets **max(*a*, *b*)**
  - **Also:** Some **cost *c*** for the effort of maintaining both strategies (summed over all interactions)
    - Note: a given node can receive *a* from one neighbor and *b* from another by playing AB, which is why it could be worth the cost *c*

w

		A	B	AB
	A	a, a	o,o	a, a
v	**B**	o,o	b,b	b,b
	AB	a, a	b,b	max(a,b), max(a,b)

Cascades & Compatibility: Model

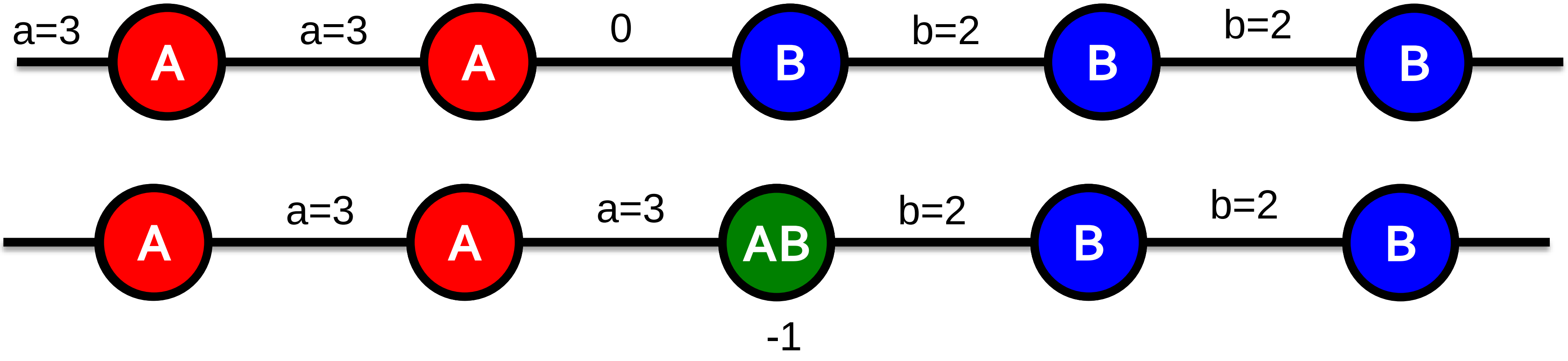
- Every node in an infinite network starts with ***B***
- Then a finite set ***S*** initially adopts ***A***
- Run the model for ***t=1,2,3,…***
  - Each node selects behavior that will optimize payoff (given what its neighbors did in at time ***t-1***)



- **How will nodes switch from *B* to *A* or *AB*?**

Example: Path Graph (1)

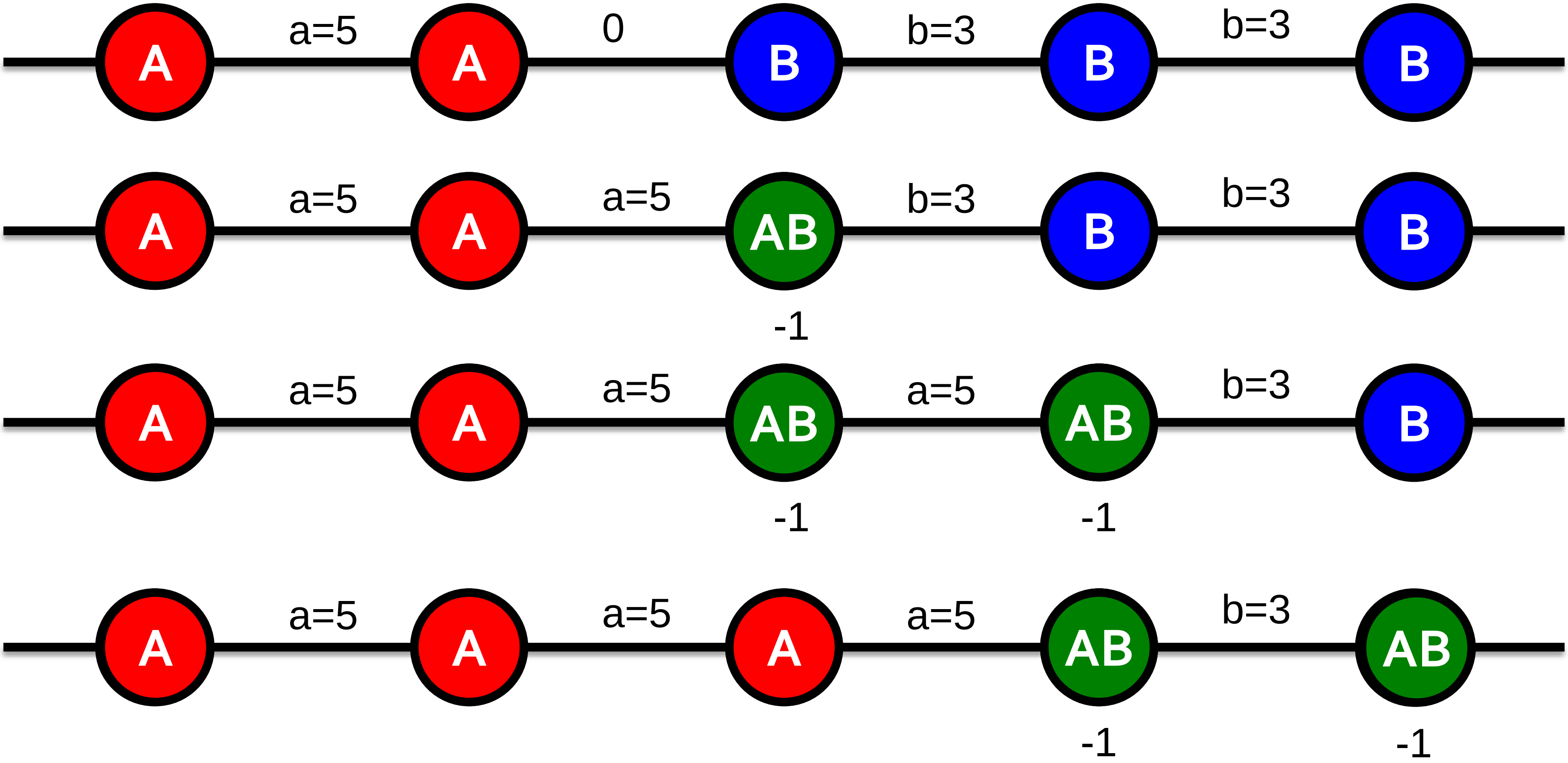
- **Path graph:** Start with all Bs, a > b (A is better)
- **One node switches to A – what happens?**
  - **With just A, B:** A spreads if *a* > *b*
  - **With A, B, AB: Does A spread?**
- **Example: a=3, b=2, c=1**



Cascade stops

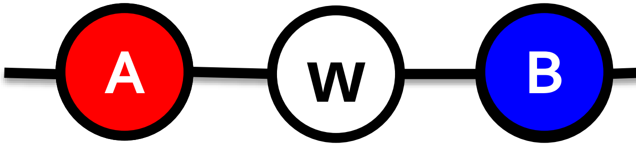
Example: Path Graph (2)

- **Example: a=5, b=3, c=1**

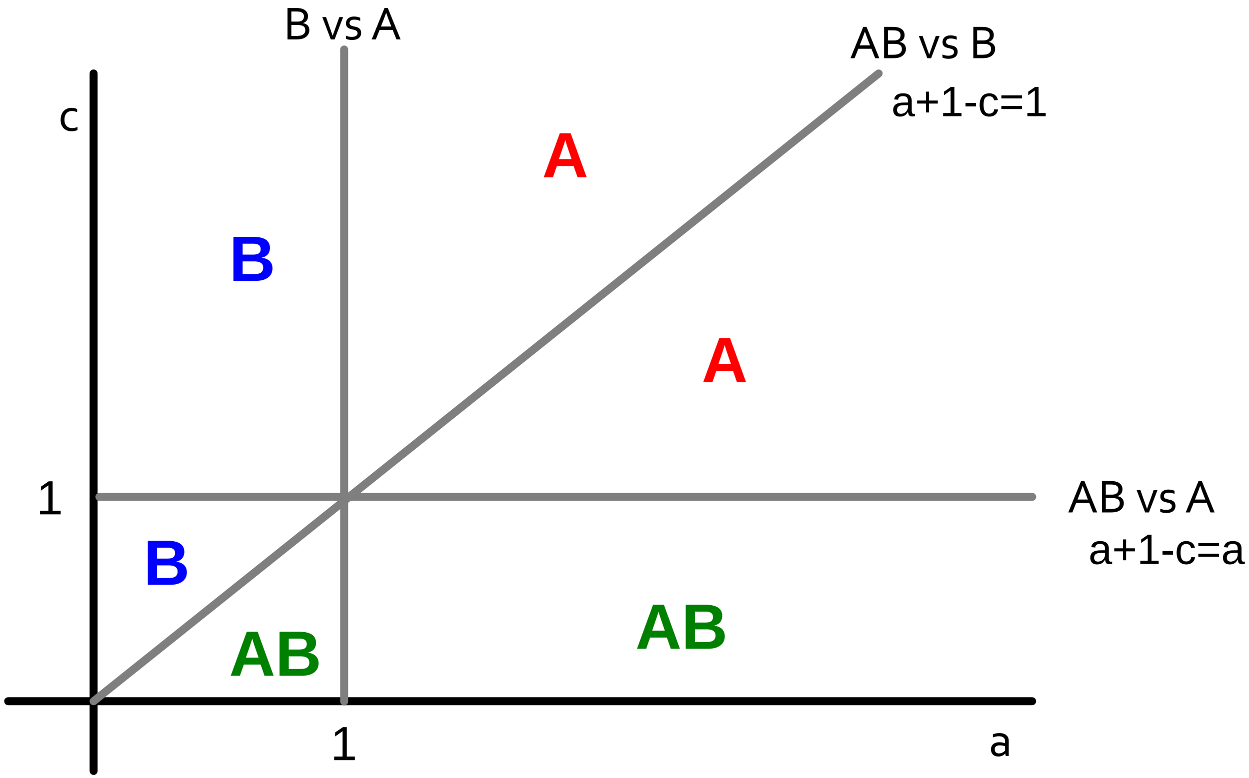


Cascade never stops!

For what pairs (c,a) does A spread?



- **Infinite path, start with all Bs**
- **Payoffs for *w*:** **A**:a, **B**:1, **AB**:a+1-c
- What does node ***w*** in **A-*w*-B** do?



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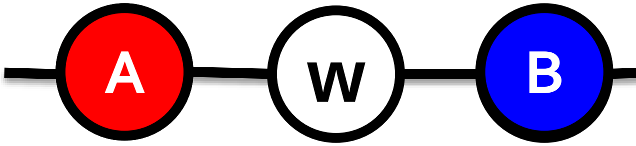

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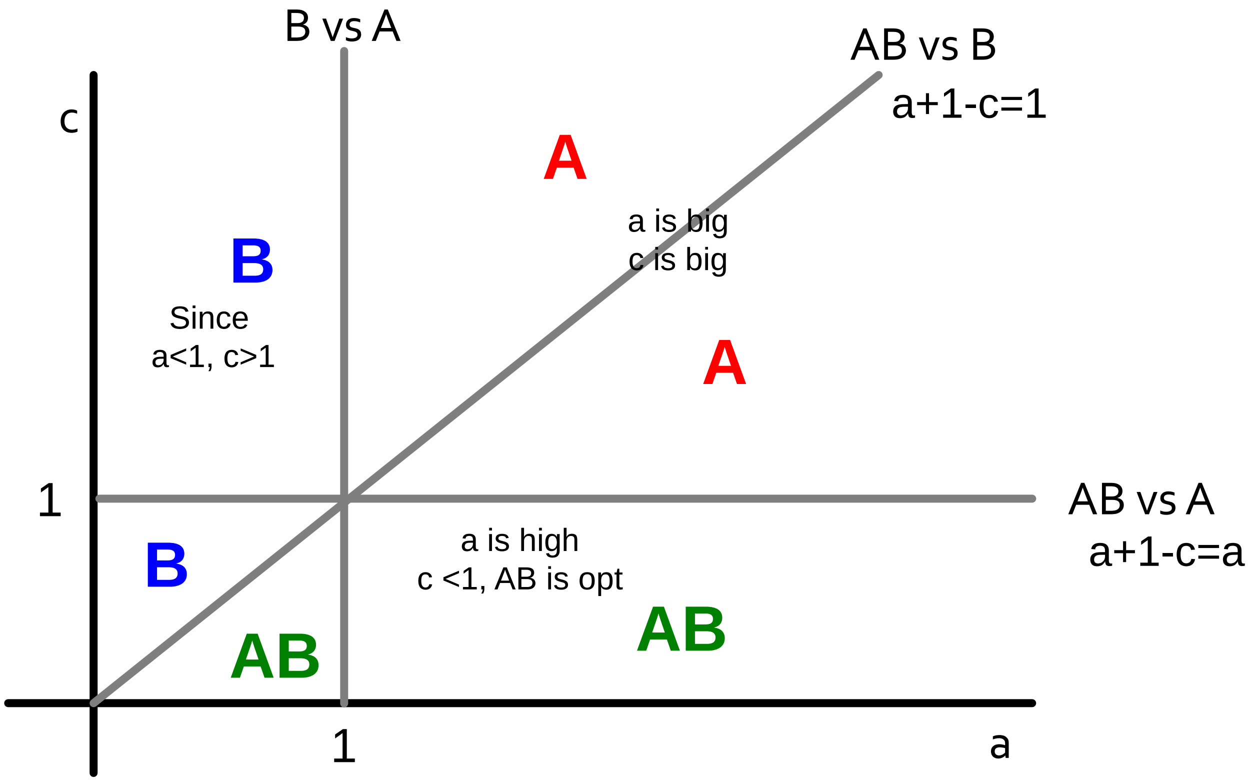
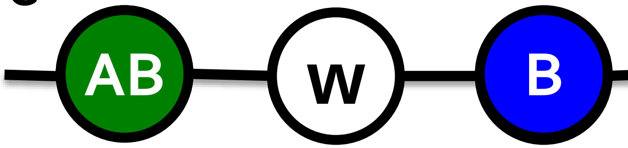
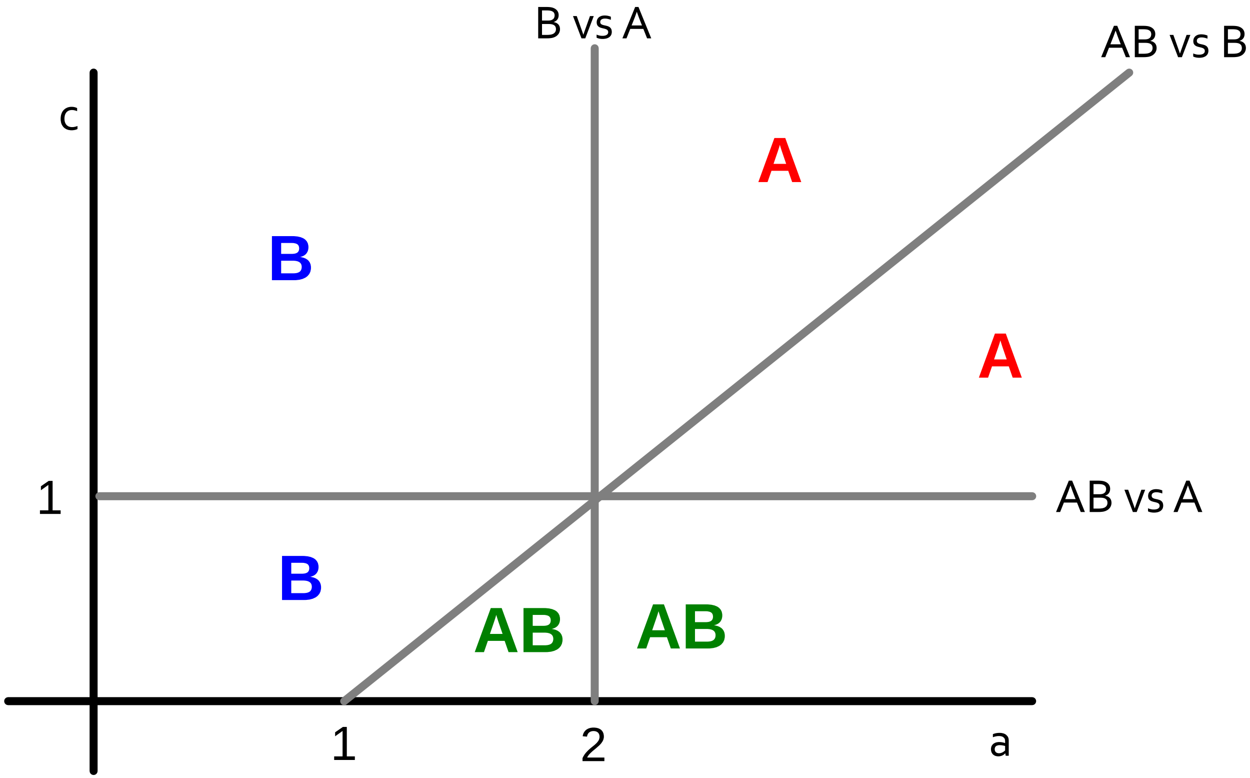


For what pairs (c,a) does A spread?

- **Same reward structure as before but now payoffs for *w* change:** **A**:a, **B**:1+1, **AB**:a+1-c
- **Notice: Now also AB spreads**
- What does node ***w*** in **AB-*w*-B** do?





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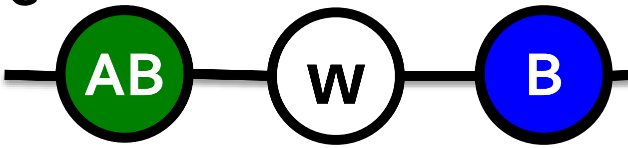

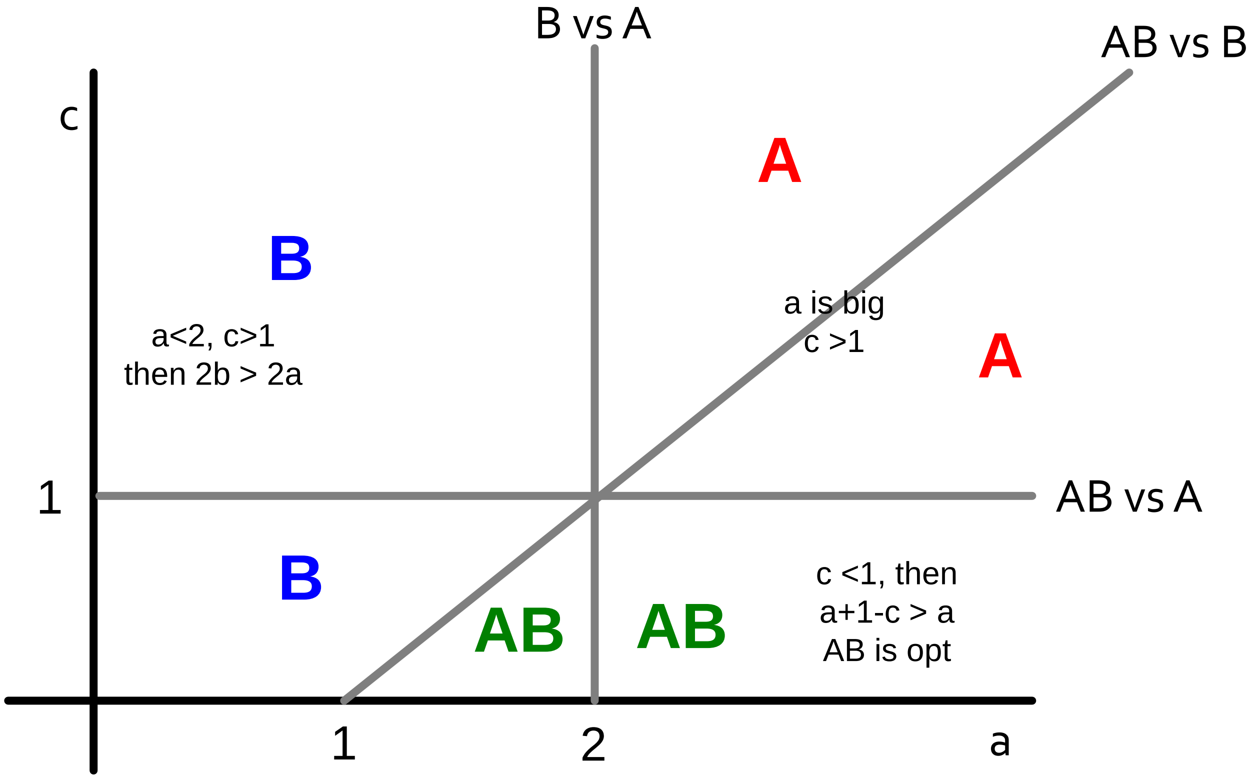

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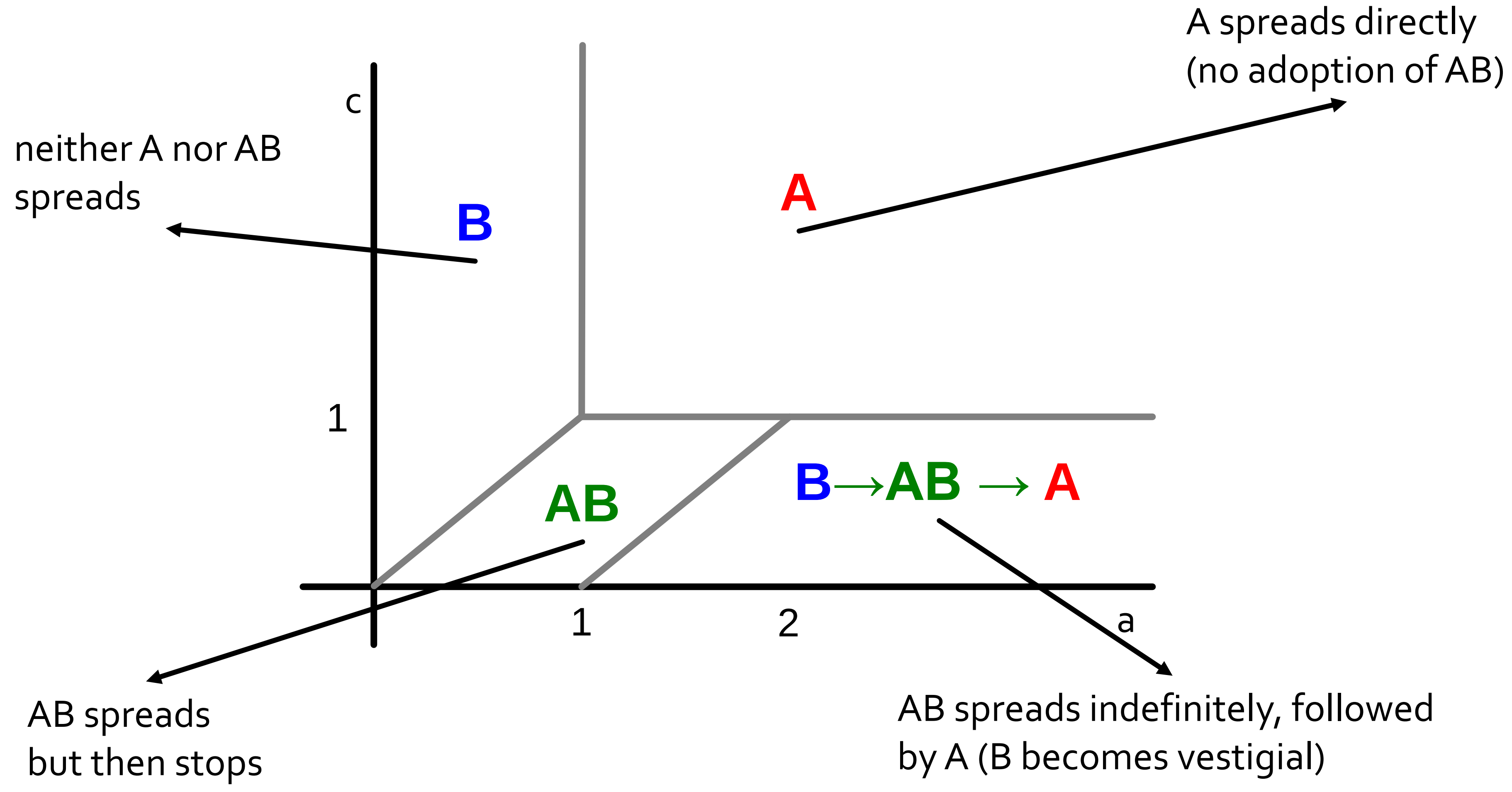
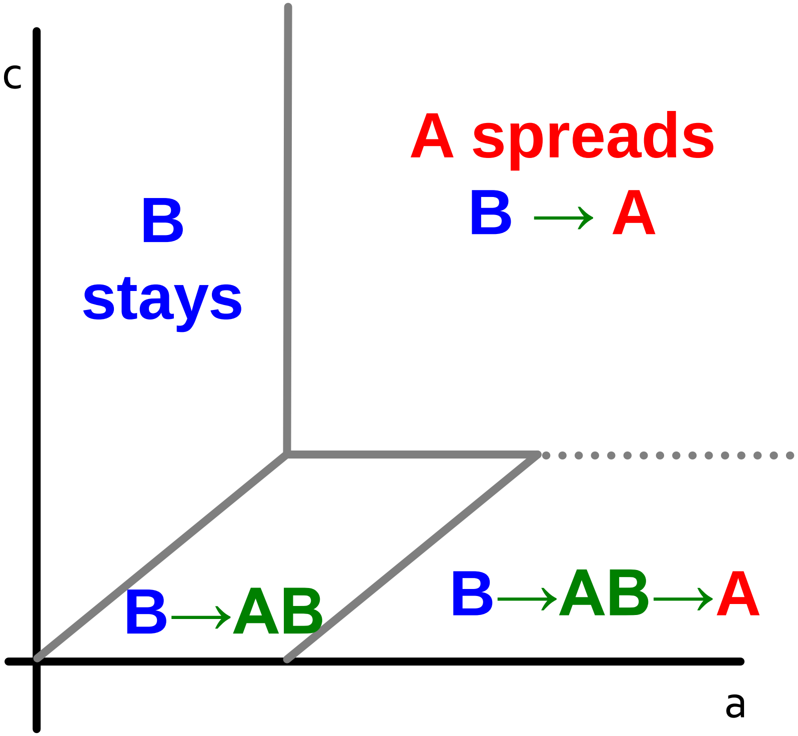
- **Joining the two pictures:**

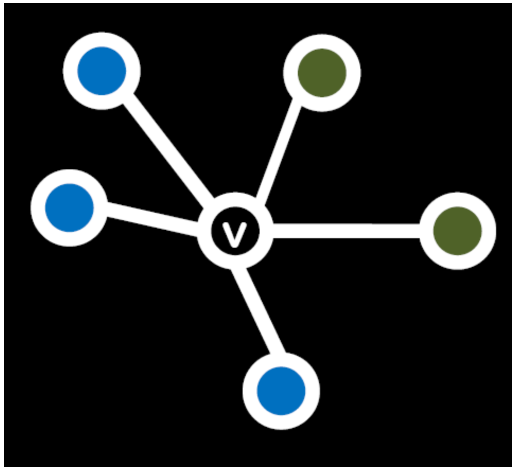


Lesson

- ***B* is the default throughout the network until new/better *A* comes along. What happens?**
  - **Infiltration:** If **B** is too compatible then people will take on both and then drop the worse one **(B)**
  - **Direct conquest:** If **A** makes itself not compatible – people on the border must choose. They pick the better one (**A**)
  - **Buffer zone:** If you choose an optimal level then you keep a static “buffer” between **A** and **B**



Models of Cascading Behavior



- **So far:**
  Decision Based Models
  - Utility based
  - Deterministic
  - "Node" centric: A node observes decisions of its neighbors and makes its own decision
  - Require us to know too much about the data
- **Next: Probabilistic Models**
  - Let's you do things by observing data
  - We lose "why people do things"