York University
EECS 4101/5101, Winter 2023
Assignment 3

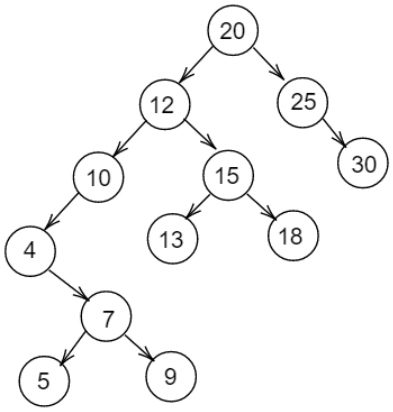
Due Date: March 2nd, at 23:59

Not being heard is no reason for silence ...
Victor Hugo

All problems are written problems; submit your solutions electronically **only via Crowdmark**. You are welcome to discuss the general idea of the problems with other students. However, you must write your answers individually and mention your peers (with whom you discussed the problems) in your solution. Please refer to the course webpage for guidelines on academic integrity.

Problem 1 Splay Trees [15 marks]

a) Apply the splay operation on the following splay tree when there is a request to node '7'. Show your steps.

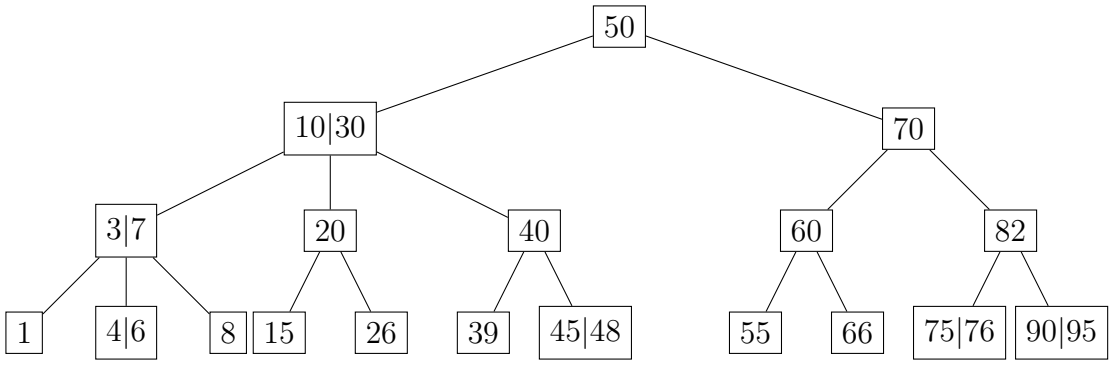


b) Prove or disprove the following statement: "the root of a splay tree always has two children".

c) Prove or disprove: in any splay tree formed by N nodes, there is a sequence of requests with length N that results in the tree having height $\Omega(N)$.

Problem 2 2-3 Trees [10 marks]

This problem will concern operations on the 2-3 tree T shown in the following figure.

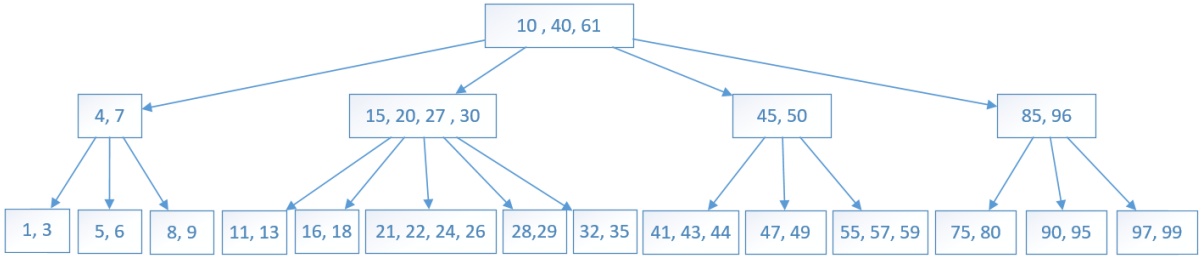


a) Draw the tree after performing operations insert(49) and insert(5).

b) Draw the *original tree* after performing operations delete(76) and then delete(55).

Problem 3 B-Trees [10 marks]

Consider the following B-tree of min-size 2. Note that each node should have at least 2 and at most 4 keys in such a tree.

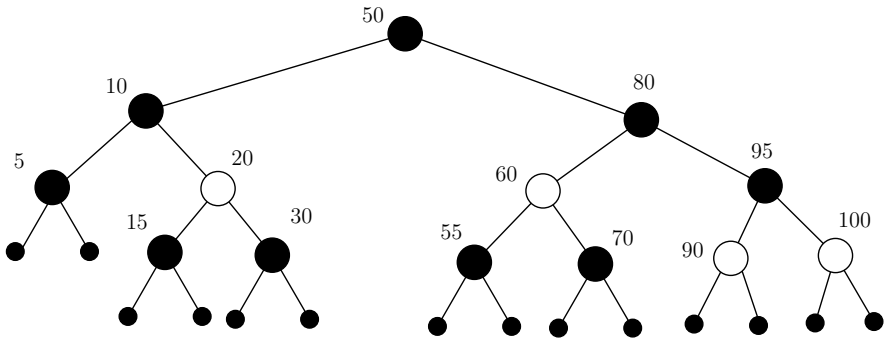


(a) Insert the following keys (in the same order) into the tree: 65, 60, and 25.
You need to draw one tree (the final tree after all insertions).

(b) From the original tree (the above tree), delete the following keys (in the same order): 55, 16, and 57.

Problem 4 Red-Black trees [10 marks]

This problem will concern operations on the red-black T shown in the following figure.



(a) Draw the tree after performing operations insert(52), insert(92), and insert(53) (in the same order). It suffices to draw the final tree.

(b) Draw the original tree after performing operations delete(70), delete (15). For that, take the following steps: i) draw the B-tree associated with T ii) delete the two nodes from the B-tree. Note that after each change to the tree, you must maintain having exactly one black key at each node of the B-tree iii) draw the red-black tree associated with the resulting B-tree.