LE/EECS 4101 GS/EECS 5101
Advanced Data Structures

Midterm Sample

Shahin Kamali

York University

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Write your name and student id here:

"Your life is your life. Don't let it be clubbed into dank submission. Be on the watch. There are ways out. There is light somewhere ..."
Charles Bukowski

- **Do not open this booklet until instructed**.
- You are NOT allowed to use any printed/written material.
- Please **turn off your cell phones** and put them in your bags.
- Manage your time. We start the exam at 5:30 and end at 7:00. **You have 90 minutes**. **Don't waste too much time on a single question. Note that the first questions are likely to be the hardest ones.**
- The exam is printed **double-sided**. There are **12 pages**, including this cover page and two blank pages (use them if you need more space). It is OK to take the staples off. You must submit ALL pages.
- The marks will be scaled so that the highest mark gets the full mark.

1. True/False and Very-Short-Answer Questions (14 marks)

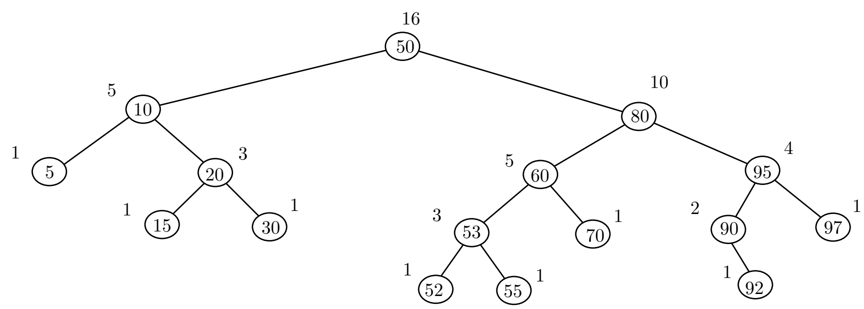
Provide your short answers in the provided boxes. There is no need to justify your answers.

(a) True or False: Amortized cost of search/insert/delete operations in an Splay tree with n keys is $O(\log n)$.

(b) Suppose an algorithm A has a competitive ratio of at most 2. Assume the cost of OPT for an input sequence I is 5. True or False: the cost of A for I is at least 10.

(c) True or False: A paid exchange by OPT does not necessarily change the potential in the analysis of MTF.

(d) Write down the output to $Select(8)$ in the following augmented AVL tree (assume indices in the sorted array start at 0).



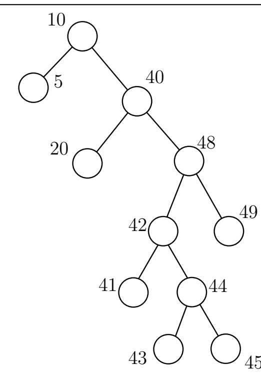
(e) Suppose T is a red-black tree, and the root's left pointers points to tree T_A and the root's right pointer points to tree T_B. True or False: if we switch the left and right pointers, so that root.left points to T_B and root.right points to T_A, the resulting tree is still a red-black tree.

(f) Let T be a 2-3 tree with n keys, where $n \geq 2^{100}$. True or False: the left child of the root can be a leaf with one key.

(g) True or False: It is proved that a splay tree achieves a constant competitive ratio, assuming OPT can dynamically adjusts its structure.

2. Short Answer Questions (16 marks)

(a) We apply m search operations on an AVL tree of size n, where m and n are both large integers and $m = \Theta(n)$. Suppose $2m/3$ operations take constant time each, $m/3 - \log m$ operations take $\log n$ time each, and the remaining $\log m$ operations each takes $\log \log n$ time each. What is the asymptotic amortized running time of each operation? Show your work.



(b) Apply the splay operation on the following splay tree when there is a request to node '45'. It suffices to show the final tree.

(c) Apply the Burrows-Wheeler transform on the following string; show your work and the output

$$nazaneen\$$$

Assume \$ precedes all characters when you sort rotations.

(d) Assume an initial list $\$ \to A \to B \to R \to N$. A compressing scheme that uses Move-To-Front is used to encode "BARAN\$". Show what the resulting code is (the first position is encoded as 0, the second as 1, and so on).

3. Amortized Analysis (8 marks)

A "d-bit" is a decimal digit that can have any of the values $\{0, 1, \ldots, 9\}$. Consider a decimal counter that starts from an initial configuration where all digits are '0'. This is followed by m operations each incrementing the number encoded in decimal. In this question, we want to know how many digits are changed per operation.

(a) Use aggregate-cost method to find an upper bound for the amortized number of changed digits.
Hint: given a positive value a, we have $1 + a + a^2 + \ldots + a^k = \frac{a^{k+1}-1}{a-1}$.

(b) Use the potential-function method to find an upper bound for the amortized cost of each operation. Show and justify your work. **Hint:** Let the potential be the number of non-zero digits in the encoded number.

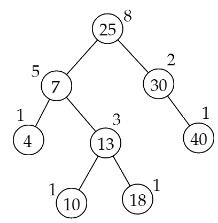
4. Competitive Analysis (5 marks)

Consider the following algorithm for the path-cow problem. The cow starts at the origin and moves $x = 1$ unit to the right. If the target is not found, the cow returns to the origin and goes $x = 1$ unit to the left. If the target is not found, the cow returns to the origin and repeats this procedure with $x = 3, 9, \ldots, 3^i, \ldots$ until the target is found. What is the competitive ratio of this algorithm? Show your work.

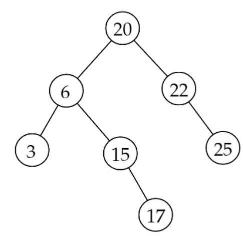
Hint: given a positive value a, we have $1 + a + a^2 + \ldots + a^k = \frac{a^{k+1}-1}{a-1}$.

5. AVL Trees (6 marks)

(a) Perform operation *insert*(12) on AVL tree:
Draw the tree after each rotation performed.
There is no need to show balance factors.



(b) Perform operation *delete*(20) on the augmented AVL tree T:
Draw the tree after each rotation performed
There is no need to show balance factors.

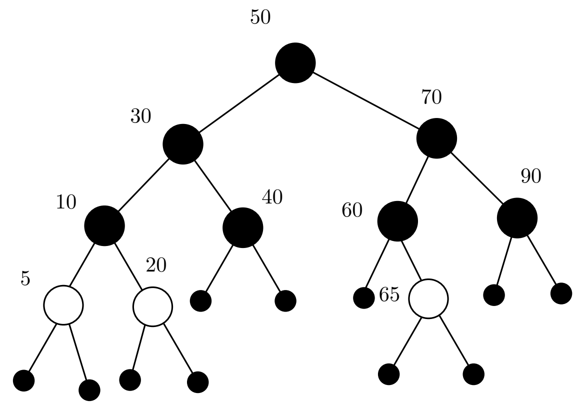


6. Tree Height (5 marks)

A balanced 4-way tree is a tree in which each internal node of height $h \geq 5$ has exactly four children, out of which at least two children have height $\geq h-2$. Prove that the height of a balanced 4-way tree is $O(\log n)$. Show your work.

7. Red-Black Trees (8 marks)

Consider he following red-black tree T.

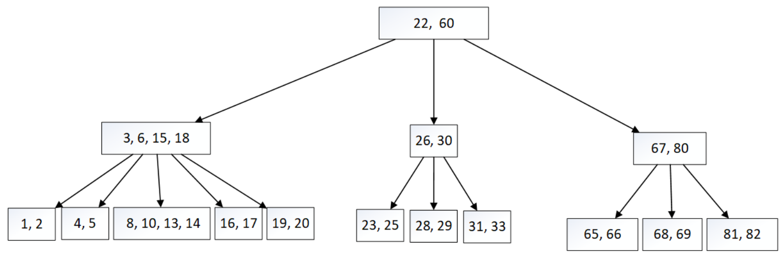


(a) Draw the tree after the operation $insert(62), insert(4)$ on it (in the same order). It suffices to draw the final tree.

(b) Draw the b-tree associated with T (before the insert operation).

8. B-Trees (8 marks)

Consider the following B-tree with $d = 2$.



(a) Draw the tree when after we apply the operation *insert*(7) on it. It suffices to draw the final tree.

(b) Draw the original tree after we apply the operation *delete*(69) on it. It suffices to draw the final tree.

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