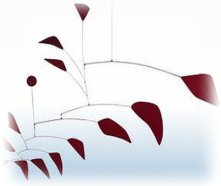


EECS 4101-5101
Advanced Data Structures

Shahin Kamali

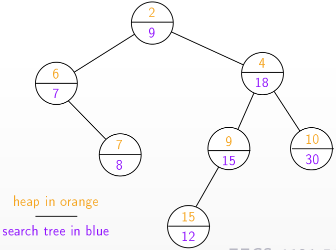
Topic 6: Randomized Data Structures (Treaps)

York University

Picture is from the cover of the textbook CLRS.

Treap Dictionary Data Structure

- A binary tree in which each node has a **key** and a **priority**
  - Keys have binary search tree property: each node's key is larger than its left and smaller than its right.
  - Priorities have heap property: each node priority is smaller than its parent (or could be larger to form a max-treap)



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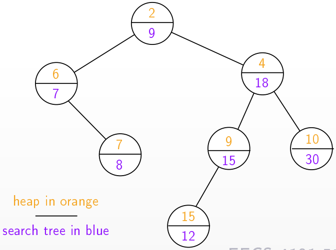
- Treaps are used for implementing dictionaries: keys are dictionary keys and priorities are chosen randomly!



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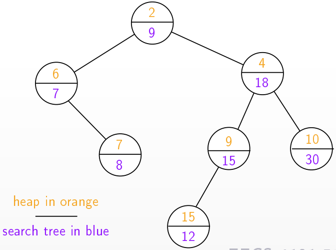
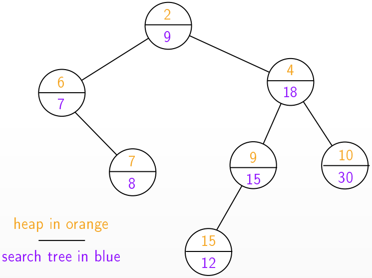
- Treaps are used for implementing dictionaries: keys are dictionary keys and priorities are chosen randomly!
  - *search*(k) is identical to seacrh in a BST $\rightarrow$ it takes $O(h)$, where h is the height of the tree



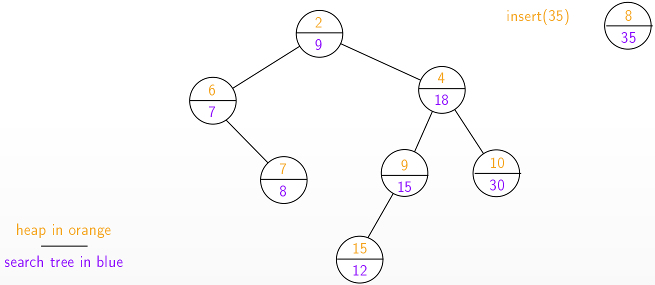
Treap Dictionary Data Structure

- Consider *insert*(k)



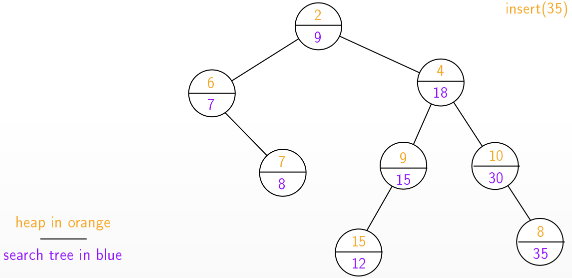
Treap Dictionary Data Structure

- Consider *insert*(k)
  - Choose a random priority (a random number generated say between $[1, n^2]$)



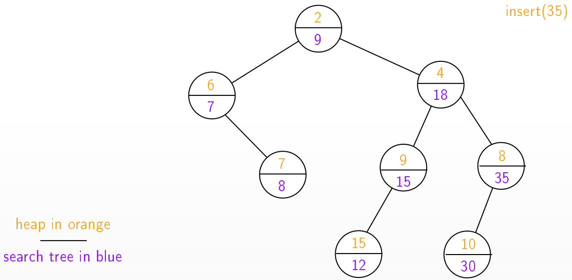
Treap Dictionary Data Structure

- Consider *insert*(k)
  - Choose a random priority (a random number generated say between $[1, n^2]$)
  - Insert as in normal BST



Treap Dictionary Data Structure

- Consider $insert(k)$
  - Choose a random priority (a random number generated say between $[1, n^2]$)
  - Insert as in normal BST
  - Rotate up until heap order is restored (maintaining BST property while rotating)



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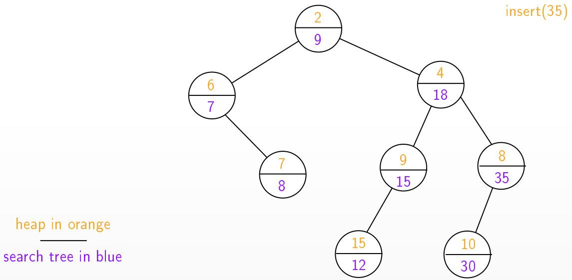
- How long does *insert*(k) take?



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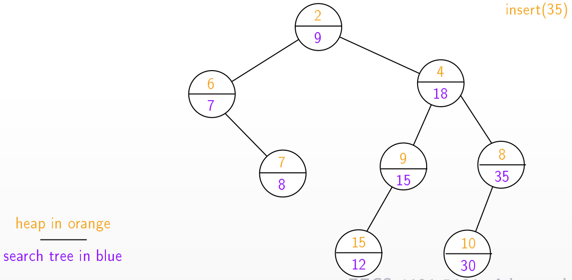
- How long does $insert(k)$ take? $O(h)$, where h is the height of the tree!



Treap Dictionary Data Structure

- Consider *delete*(k)
  - Search for k and delete its node as in regular BSTs
    - Just remove the node if a leaf
    - Replace the node with its child if only one child
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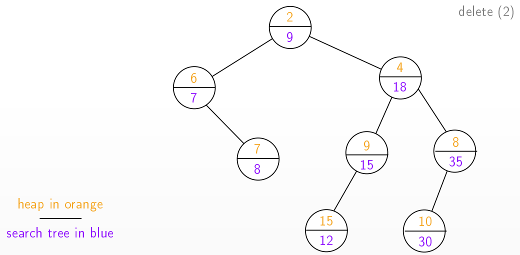

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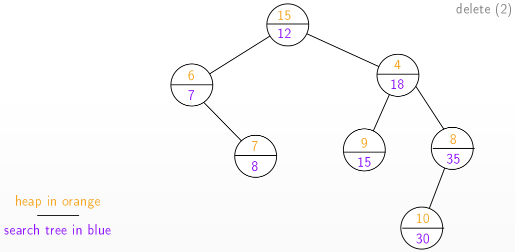

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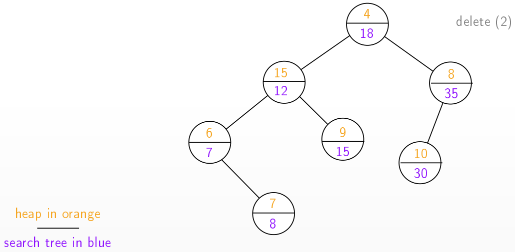

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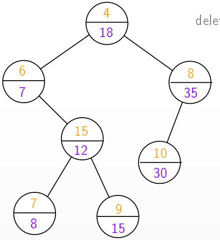

delete (2)

heap in orange

search tree in blue

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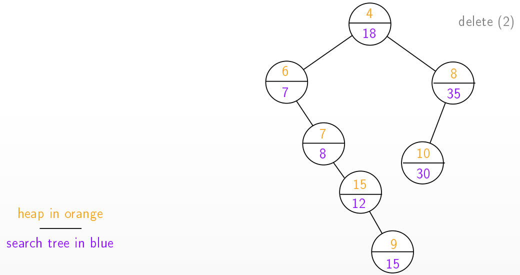

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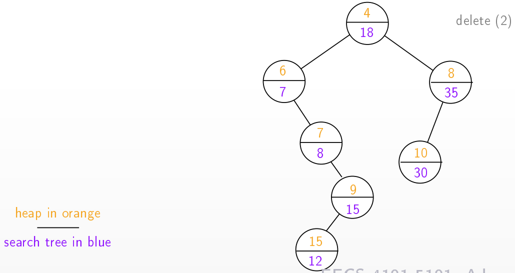

Treap Dictionary Data Structure

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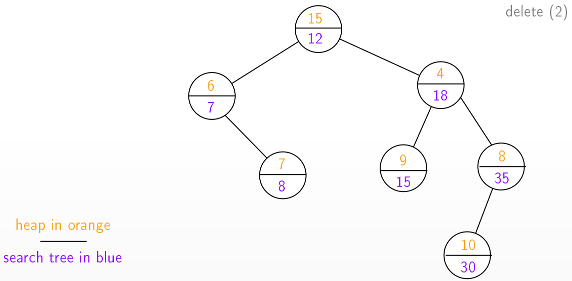

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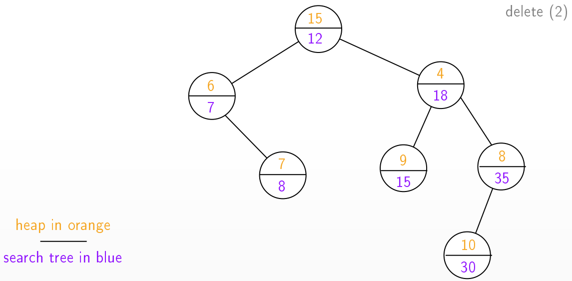

Treap Dictionary Data Structure

- Given that priorities are randomized, all elements have the same chance of being the root (i.e., have the smallest priority).
- The height of a treap is **expected** to be $O(\log n)$.
  - **Search, insert, and delete take $O(\log n)$ expected time in treaps.**
  - Treaps are very simple to implement, little overhead – less than AVL trees