Keep changes you like
Changed title

1 Merging Distributions $\langle T, E_A\rangle$ and $\langle T, E_B\rangle$

Added intro
Next sentence is awkward.
Added A and B to T
Merged paragraph
Deleted talk of ranges
Shortened
narrowed probability spaces
Equation of line

Our first step is to understand the probability spaces for the two groups and for comparison merge them into one. Because it is easier, we will start with all the distributions being uniform. Person A and B receive their respective talents T_A and T_B from the same uniform distribution $T = \mathcal{U}(t_{min}, t_{max})$. The B person receives their environment score E_B from $E_B = \mathcal{U}(e_{min}, e_{max})$ while the A person receives E_A from the shifted distribution $E_A = \mathcal{U}(e_{min}+K, e_{max}+K)$. Their performance scores are computed as the sum $X_g = T_g + E_g$. Our assumption is that these two people received the same performance score x. Our goal is to compare their talents, i.e. $Exp[T_A|X_A = x]$ vs $Exp[T_B|X_B = x]$. We will represent the full probability space as the $\langle T, E_A\rangle$ vs $\langle T, E_B\rangle$ rectangles in Figure **??** and **??**. Here talent is on the x-axis and environment on the y. Each tilted green line represents the narrowed probability space when conditioned on the performance score being fixed to $X_A = X_B = x_i$. Note the equation of each line solves $X_g = T_g + E_g$ giving $T_g = x_i - E_g$. Note how the y-intercept, $T_A = x_i - (e_{min}+K)$ vs $T_B = x_i - e_{min}$, is $x_2 - x_1$ higher for x_2 and K lower for group A. Because T, E_g, and X_g are uniform, so is the distribution within each of these green lines. Because we ultimately only care about the talent values, we project these green lines onto the y-axis giving the distribution $[T|X_g = x_i]$. From their ranges, we can deduce that the expected talent for x_2 is greater by this difference $x_2 - x_1$ in performance, namely $Exp[T_g|X_g = x_2] - Exp[T_g|X_g = x_1] = x_2 - x_1$.
In fig a, Change $= x_2$ **to** $= x_1$**.**
In fig, Add A **to** E_A**,** T_A**,** $T_A|X_A = x_2$**. Same** B
In fig, change green line label to $X_A = x_1$**,** $X_A = x_2$**,** $X_B = x_1$**,** $X_B = x_2$**.**

Fig
Intro sentence
same x**-axis**
shorter
Even single authored papers use we instead of I.

For comparison, let us now merge the two groups probability spaces $\langle T, E_A\rangle$ and $\langle T, E_B\rangle$ into one. In order to be able to plot them both on the same x-axis, independently draw an environment score E'_A and E_B from the same distribution $E = \mathcal{U}(e_{min}, e_{max})$. Before we advantaged the A person by computing $E_A = E'_A + K$ and $X_A = T_A + E_A$. Instead lets compute $X'_A = T_A + E'_A$ and $X_A = X'_A + K$. The earlier condition $X_A = X_B = x$ is equivalent to $X_B = x$ and $X'_A = x - K$. As before, A's y-intercept, $T_A = x - (e_{min} + K)$ is K lower than B's $T_B = x - e_{min}$. Projecting these green lines onto the y-axis gives the required result that $Exp[T_B|X_B = x] - Exp[T_A|X_A = x] = K$. The next section will observe that this same result does not occur when the green lines are in the extreme corners.
In fig, Add A to $T_A|X_A = x$. Add B to E_B and $T_B|X_B = x$.
Not X_A but x

Fig
title
over view and shorter
I did not cognition on $G = g$

2 Extreme x and r-values

In previous section, we only considered the cases where the range of the environment is narrower than that of talent. Here we will contrast this to the case where it is wider and hence the noise of the environment makes it impossible to estimate the person's talent. Denote the talent's range by $[t_{min}, t_{max}]$ and the environment's by $[e_{min,g}, e_{max,g}]$. Condition on the fact that the performance score $X_g = T_g + E_g$ is fixed to some value x. Rearranging and considering the environment range gives that $T_g = x - E_g \in [x - e_{max,g}, x - e_{min,g}]$. If x is an *extreme* low value, then this low range $x - e_{max,g}$ is smaller than the talent's low range t_{min} and hence the bound t_{min} kicks in. Similarly, if x is an *extreme* high value, then the high range $x - e_{min,g}$ is trumped by t_{max}. We define $r(x)$ to be the number of endpoint for which this does not happen, i.e. the number of blue $y - axis$ lines that the green line intersects with. Figure **??** gives an example of each of the six cases. In the unextreme $r(x) = 2$ case, the $X_g = x_2$ conditioned talent range is $T_g \in [x - e_{max,g}, x - e_{min,g}]$. In the bottom half-extreme $r(x) = 1$ cases, the $X_g = x_1$ or x_4 conditioned talent range is $T_g \in [t_{min}, x - e_{min,g}]$. In the top half-extreme $r(x) = 1$ cases, the $X_g = x_3$ or x_6 conditioned talent range is $T_g \in [x - e_{max,g}, t_{max}]$. Finally, in the totally-extreme $r(x) = 0$ case, the $X_g = x_5$ conditioned talent range is $T_g \in [t_{min}, t_{max}]$. In each case, the green dot locates the expected value of T_g within the stated range, i.e. half of the sum of its bottom and top limit. Note that $r(x)$ also denotes the number of these limits that contains an x term. Hence, if you increase x by δx, then $Exp(T_g)$ increases by $r(x) \cdot \frac{1}{2} \cdot \delta x$. Figure **??** explains how our conditioning is effectively that $X_B = x$ and $X'_A = x - K$. Because group A's effective x value is lowered by K from B's, $Exp(T_A)$ decreases by $r(x) \cdot \frac{1}{2} \cdot K$. This gives the result

$$Exp[T_B|X_B = x] - Exp[T_A|X_A = x] = \tfrac{1}{2} r(x) K \qquad (1)$$

Fig