Pg 0

$$\Pr\left[N(\mu, \sigma^2) = x\right]$$

$$= \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

Let $x' = \dfrac{x-\mu}{\sigma}$

$$= \frac{1}{\sigma}\left[\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x'-0}{1}\right)^2}\right]$$

$$= \frac{1}{\sigma} \Pr\left[N(0,1) = x'\right]$$

$$= \frac{1}{\sigma} P_N(x')$$

$X_A = T_A + E_A \qquad X_B = T_B + E_B$

$D = T_B - T_A \qquad \text{Let } r = \mathrm{Exp}[E_A] - \mathrm{Exp}[E_B]$

$$\Pr[d] = \Pr\left[D \in [d + dd]\right] / dd$$

= hight of probabilits distaibution

$$\Pr[d \mid X_A = X_B] = \frac{\Pr[X_A = X_B \mid d] \times \Pr[d]}{\Pr[X_A = X_B]}$$

$\Pr[d] \qquad D = T_B - T_A$

$$= N(0, 2\sigma_T^2)$$

$$= \Pr[N(0, 2\sigma_T^2) = d]$$

$$= \frac{1}{\sqrt{2}\sigma_T} \cancel{\Pr[d} \; P_N\left(\frac{d}{\sqrt{2}\sigma_T}\right)$$

$$\Pr[X_A = X_B] = \Pr[X_B - X_A = 0]$$

$$X_B - X_A = \cancel{T_B + E_B} \; [T_B - T_A] + [E_B - E_A]$$

$$\mathrm{Exp}[X_B - X_A] = 0 + r$$

$$\mathrm{Var}[X_B - X_A] = 2\sigma_T^2 + 2\sigma_E^2$$

$$= 1/(2\sigma_T + 2\sigma_E) \; P_N\left(r/(\sqrt{2}\sigma_T + \sqrt{2}\sigma_E)\right)$$

$$Pr\left[X_A = X_B \mid \cancel{T_B} - T_A = d \right]$$

$$X_B - X_A = \underbrace{\left[T_B - T_A \right]}_{\text{Fixed to } d} + \left[E_B - E_A \right]$$

$Exp = d + r$

$Var = 2\sigma_E^2$

Need $X_B - X_A = 0$

$$= Pr\left\{ N\left(d+r, 2\sigma_E^2 \right) = 0 \right]$$

$$= \frac{1}{2\sigma_E} P_N\left(\frac{d+r}{\sqrt{2}\sigma_E} \right)$$

$$Pr\left[d \mid X_A = X_B \right] = \frac{\frac{1}{\sqrt{2}\sigma_E} P_N\left(\frac{d+r}{\sqrt{2}\sigma_E} \right) \times \frac{1}{\sqrt{2}\sigma_T} P_N\left(\frac{d}{\sqrt{2}\sigma_T} \right)}{1/(\sqrt{2}\sigma_T + \sqrt{2}\sigma_E)\, P_N\left(\frac{r}{\sqrt{2}\sigma_E + \sqrt{2}\sigma_T} \right)}$$

Let $\sigma = 1/\sqrt{2}$

$$e^{-\frac{1}{2}\,(d+r)^2 + d^2 - \left(\frac{r}{2}\right)^2} = e^{-\frac{1}{2}\left[2d^2 + 2dr + \frac{3}{4}r^2\right.}$$

$$\Pr[d \mid X_A = X_B] = e^{-\frac{1}{2}\left[2d^2 + 2dr + \frac{3}{4}r^2\right]}$$

$$= e^{-\frac{1}{2}\left[2\left(d + \frac{r}{2}\right)^2\right.}$$

$$2\left(d + r/2\right)^2 + ? = \cancel{2d^2} + \cancel{2dr} + \frac{3}{4}r^2$$

$$\cancel{2d^2} + \cancel{2dr} + \frac{2r^2}{4} + ? =$$

$$? = \left(\frac{r}{2}\right)^2$$

$$= e^{-\frac{1}{2}\left[\left(\sqrt{2}\,d + \frac{r}{\sqrt{2}}\right)^2\right]} \times \underbrace{e^{-\frac{1}{2}\left(r/2\right)^2}}_{\text{constant}}$$

$$e^{-\frac{1}{2}\left(\frac{d + r/2}{1/\sqrt{2}}\right)}$$

$$D' \sim \mathrm{Norm}\left(-r/\sqrt{2},\ \frac{1}{\sqrt{2}}\right)$$

$$\Pr[T_B > T_A \mid X_A = X_B]$$

$$\Pr[D' > 0] = \Pr\left[\mathrm{Norm}\left(-\frac{r}{\sqrt{2}},\ \frac{1}{\sqrt{2}}\right) > 0\right]$$

$$< \frac{1}{2}$$

<u>ocp</u>

Jeff's model

Let $TA \sim TB \sim T$

Let Error be an arbitrary distribution with mean zero

Karan's model

$TA \sim TB \sim T$

Let EA + EB be arbitrary distr

Without loss of generality, ~~Ed~~

$EA \hat{=} EA(Error)$ where EA is some bijection function

$EB \hat{=} EB(Error)$

Let $X = GradeA(T + Error)$
$= GradeB(T + Error)$

$\Rightarrow$

Let $X = Outcome(T, E)$

$= GradeA(T + EA^{-1}(E))$

The GradeB would be fixed to be the same

$\Rightarrow$

Unless the 10^{-100} diget ~~of T is always even~~
of EA " " even
EB " " odd

Then outcome can know A vs B

Karan's Model
is more General

Eg x = Outcome (T, E) = T x E

Jeff's x = GradeA (T + E)
can't compute T x E
from T + E

~~outing~~